Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2014) 38: 187 – 192

doi:10.3906/fiz-1401-13

# Perturbative approach to the spin-flavor precession of Majorana-type solar neutrinos

**Deniz YILMAZ***
Department of Physics Engineering, Faculty of Engineering, Ankara University, Tandoğan, Ankara, Turkey

**Received:** 23.01.2014 • **Accepted:** 07.03.2014 • **Published Online:** 11.06.2014 • **Printed:** 10.07.2014

**Abstract:** Spin-flavor precession (SFP) of Majorana-type solar neutrinos is investigated in the case of 2 generations. The evolution equation in the SFP framework is solved by using the perturbative method, in which $\mu B$ is accepted to be small. The approximate analytical formula including SFP effect is provided.

**Key words:** Perturbative method, spin-flavor precession, solar neutrinos, Majorana

## 1. Introduction

After neutrino oscillation was confirmed by serious solar, atmospheric, and reactor neutrino experiments [1–10] in the last decades, neutrino oscillation became one of the implications of the physics beyond the Standard Model (SM). Since neutrinos have a mass in a minimal extension of the SM, they also have magnetic moment:

$$\mu_\upsilon = \frac{3eG_f m_\upsilon}{8\pi^2\sqrt{2}} = \frac{3eG_f m_e m_\upsilon}{4\pi^2\sqrt{2}}\mu_B, \tag{1}$$

where $G_f$ is the Fermi constant; $m_e$ and $m_\nu$ are the masses of electrons and neutrinos, respectively; e is the proton charge; and $\mu_B$ is the Bohr magneton. The limits on the neutrino magnetic moment have been obtained by astrophysical arguments [11], solar neutrino experiments combined with KamLAND data [12], and reactor neutrino experiments [13,14]. The new limit recently obtained by the GEMMA experiment on the neutrino magnetic moment is $\mu_\nu < 2.9 \times 10^{-11}\mu_B$ at 90% CL [15]. Detailed discussion on neutrino magnetic moment was given by Balantekin [16].

Neutrinos can be affected by the large magnetic fields throughout the sun due to their magnetic moments. Their spin can flip and a left-handed neutrino becomes a right-handed neutrino when they are passing through the magnetic region of the sun [17–20]. Even though information about the solar magnetic field is not well known, some bounds are given in the literature: $\sim 10^7 G$ at the core and a maximum magnitude of $\sim 10^5 G$ at the bottom of the convective zone [21,22]. The combined effect of the matter and the magnetic field, called spin-flavor precession (SFP), can change a left-handed electron neutrino into the right-handed type of neutrino. In the Majorana case, a right-handed neutrino is called an antineutrino. This can also be responsible for the solar electron neutrino deficit. The SFP effect has been investigated by several studies in different aspects [23–30].

*Correspondence: dyilmaz@eng.ankara.edu.tr

In this paper, neutrinos are assumed to be of the Majorana type. The SFP effect in the case of 2 neutrino generations is studied. The approximate analytical formula is provided by using the perturbative method, in which $\mu B$ is taken to be small.

## 2. Formalism and analysis

The evolution equation for Majorana neutrinos passing through the matter and the magnetic field in the case of 2 generations is given by:

$$i\frac{d}{dt}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \overline{\psi}_e\\ \overline{\psi}_\mu\end{pmatrix} = \mathscr{H}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \overline{\psi}_e\\ \overline{\psi}_\mu\end{pmatrix}, \tag{2}$$

where $\mathscr{H}$ is described as follows [23]:

$$\mathscr{H} = \begin{pmatrix} H & B_{mag}\\ -B_{mag} & \overline{H}\end{pmatrix}. \tag{3}$$

The $2\times 2$ submatrices, $H$ and $\overline{H}$, are

$$H = \begin{pmatrix} \frac{1}{2}\left(V_c - c2_{12}\Delta_{21}\right) & \frac{s2_{12}\Delta_{21}}{2}\\ \frac{s2_{12}\Delta_{21}}{2} & \frac{1}{2}\left(-V_c + c2_{12}\Delta_{21}\right)\end{pmatrix}, \tag{4}$$

$$\overline{H} = \begin{pmatrix} \frac{1}{2}\left(-3V_c - 4V_n - c2_{12}\Delta_{21}\right) & \frac{s2_{12}\Delta_{21}}{2}\\ \frac{s2_{12}\Delta_{21}}{2} & \frac{1}{2}\left(-V_c - 4V_n + c2_{12}\Delta_{21}\right)\end{pmatrix}, \tag{5}$$

where $s2_{12}$, $c2_{12}$, and $\Delta_{12}$ are defined as

$$\begin{aligned} s2_{12} &= sin(2\theta_{12}),\\ c2_{12} &= cos(2\theta_{12}),\\ \Delta_{12} &= \frac{\delta m_{12}^2}{2E}. \end{aligned} \tag{6}$$

The matter potentials used in these equations are

$$V_c = \sqrt{2}G_F N_e \tag{7}$$

and

$$V_n = -\frac{G_F}{\sqrt{2}}N_n, \tag{8}$$

where $N_e$ and $N_n$ are electron and neutron density, respectively, and $G_F$ is the Fermi constant. The magnetic part of Eq. (3) is

$$B_{mag} = \begin{pmatrix} 0 & \mu B\\ -\mu B & 0\end{pmatrix}, \tag{9}$$

where $\mu$ and $B$ are the transition magnetic moment and the magnetic field, respectively. Unlike the Dirac neutrinos that have diagonal and off-diagonal magnetic moments, Majorana neutrinos have only off-diagonal magnetic moments.

By taking into account the definitions given above, we can split the $\mathscr{H}$ into 2 parts:

$$\mathscr{H} = \mathscr{H}_M + \mathscr{H}_B. \tag{10}$$

The matter part, $\mathscr{H}_M$, and the magnetic part, $\mathscr{H}_B$, are given by

$$\mathscr{H}_M = \begin{pmatrix} H & 0 \\ 0 & \overline{H} \end{pmatrix}, \tag{11}$$

$$\mathscr{H}_B = \begin{pmatrix} 0 & B_{mag} \\ -B_{mag} & 0 \end{pmatrix}. \tag{12}$$

The evolution equation for the neutrinos, which is associated with the upper diagonal part of $\mathscr{H}_M$, is

$$i\frac{d}{dt}\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = H \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}, \tag{13}$$

which is the standard MSW equation for 2 neutrino cases. Thus, the solution for $H$ can be chosen as

$$U_H = \begin{pmatrix} \psi_1(t) & -\psi_2^*(t) \\ \psi_2(t) & \psi_1^*(t) \end{pmatrix}, \tag{14}$$

where $\psi_1(t)$ and $\psi_2(t)$ are solutions of Eq. (13) with the initial conditions $\psi_1(t=0) = 1$ and $\psi_2(t=0) = 0$ [31].

Now we can similarly solve the antineutrino part. The evolution equation for the antineutrinos, which is associated with the lower diagonal part of $\mathscr{H}_M$, is

$$i\frac{d}{dt}\begin{pmatrix} \overline{\psi}_e \\ \overline{\psi}_\mu \end{pmatrix} = \overline{H} \begin{pmatrix} \overline{\psi}_e \\ \overline{\psi}_\mu \end{pmatrix}. \tag{15}$$

This equation can be seperated into 2 parts:

$$i\frac{d}{dt}\begin{pmatrix} \overline{\psi}_e \\ \overline{\psi}_\mu \end{pmatrix} = \left[\overline{H}^\alpha + \overline{H}^\beta\right] \begin{pmatrix} \overline{\psi}_e \\ \overline{\psi}_\mu \end{pmatrix}, \tag{16}$$

where

$$\overline{H}^\beta = \begin{pmatrix} -V_c - \frac{\Delta_{21}}{2}c2_{12} & \frac{s2_{12}\Delta_{21}}{2} \\ \frac{s2_{12}\Delta_{21}}{2} & -V_c - \frac{\Delta_{21}}{2}c2_{12} \end{pmatrix}.$$

$\overline{H}^\alpha$ is the phase part, which can be found by subtracting $\overline{H}^\beta$ from $\overline{H}$. The solution including the phases can then be written for $\overline{H}$:

$$\overline{U}_{\overline{H}} = \begin{pmatrix} \overline{\psi_1^\alpha}(t) & -\overline{\psi_2^\alpha}^*(t) \\ \overline{\psi_2^\alpha}(t) & \overline{\psi_1^\alpha}^*(t) \end{pmatrix}, \tag{17}$$

where

$$\alpha = -\frac{V_c}{2} - 2V_n,$$

$$\overline{\psi^\alpha} = e^{i\int \alpha\, dt}\overline{\psi}. \tag{18}$$

Thus, the solution matrix for the $\mathscr{H}_M$ is given by

$$U_M = \begin{pmatrix} U_H & 0 \\ 0 & U_{\overline{H}} \end{pmatrix}. \tag{19}$$

The evolution operator of $\mathscr{H}$ satisfies

$$i\frac{\partial U}{\partial t} = \mathscr{H}U, \tag{20}$$

where $U = U_M U_B$ and $\mathscr{H} = \mathscr{H}_M + \mathscr{H}_B$. Now the complete solution of all of $\mathscr{H}$ can be found by looking at $\mathscr{H}_B$:

$$i\frac{\partial U_B}{\partial t} = (U_M^{\dagger}\mathscr{H}_B U_M)U_B = h_b(t)U_B. \tag{21}$$

Because $\mu B$ is small, we will approximate the solution to this equation to the second order in $\mu B$:

$$U_B = \left[1 - i\int_0^t h_b\left(t'\right) dt' - \int_0^t dt' \int_0^{t'} \left(h_b\left(t'\right) h_b\left(t''\right)\right) dt'' - \mathscr{O}\left(\int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt'''...\right)\right]. \tag{22}$$

Thus, the total evolution is characterized by

$$U = U_M U_B = \begin{pmatrix} \mathrm{A} & \mathrm{C} \\ \mathrm{D} & \mathrm{B} \end{pmatrix}, \tag{23}$$

where $A$, $B$, $C$, and $D$ are $2\times 2$ matrices given by

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \tag{24}$$

and

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}, \tag{25}$$

and so on. The state of the system evolves with the unitary operator of $U$ from the initial state

$$\Psi(t=T) = U\Psi(t=0), \tag{26}$$

where

$$\Psi(t=T) = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \overline{\psi}_e \\ \overline{\psi}_\mu \end{pmatrix}, \quad \Psi(t=0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{27}$$

Therefore, the electron neutrino amplitude, $\psi_e$, is found as

$$\psi_e = A_{11}. \tag{28}$$

The highly oscillating integrations that came out in $A_{11}$ are ignored. The other integrals, such as

$$I = \int_0^{\tau} dt' e^{iQ} g(t) \qquad Q = \int_0^{t'} dt'' A(t''), \tag{29}$$

can be calculated by using the stationary phase approximation method given by Balantekin et al. [32]. The stationary point, $t_R$, is where

$$A(t_R) = 0. \tag{30}$$

Since $t_R$ appears as the SFP resonance point, which takes place before the MSW resonance point, $\psi_2$ and $\overline{\psi}_2$ are rather small at $t_R$. Therefore, we can ignore them at $t_R$ [28]. Thus, $A_{11}$ is found as

$$A_{11} = \psi_1(T)\left(1 - \frac{1}{2}\frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}}|\psi_1(t_R)|^2\right), \tag{31}$$

where

$$\kappa = \frac{\Delta_{21}}{2}c2_{12}, \quad \chi = \frac{G_f}{\sqrt{2}}\left(2N_e - 2N_n\right), \tag{32}$$

and $\psi_e$ can be obtained as

$$\psi_e = \psi_1(T)\left(1 - \frac{1}{2}\frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}}|\psi_1(t_R)|^2\right). \tag{33}$$

One can finally get the survival probability in the SFP framework by ignoring the terms that have higher order than $(\mu B)^2$.

$$P_{2\times2}(\nu_e \to \nu_e, \mu B \neq 0) = P_{2\times2}(\nu_e \to \nu_e, \mu B = 0)\left(1 - \frac{2\pi(\mu B)^2}{|d(\chi-2\kappa)/dt|_{(\chi-2\kappa)=0}}|\psi_1(t_R)|^2\right) \tag{34}$$

## 3. Conclusion

In summary, neutrinos are exposed to SFP resonances due to their magnetic moments and MSW resonance when they are passing through the sun. In this paper, the spin-flavor precession of Majorana-type solar neutrinos was investigated by using the perturbative method, in which $\mu B$ was accepted to be small (in order for SFP to be responsible for the solar neutrino deficit, $\mu B$ must be at least $\sim 10^{-7}\mu_B G$ ). Two neutrino cases were considered. An analytical formula involving the SFP effect was obtained. Since $\mu B$ was accepted to be small, the terms that have higher order than $(\mu B)^2$ were ignored. The final result, Eq. (34), is consistent with the results of Balantekin and Volpe [28]. When $|\psi_1(t_R)|^2$ is taken to be 1, it corresponds to the first 2 terms in the expansion of the exponential factor in Eq. (A31) in the work of Balantekin and Volpe [28]. In the 3 neutrino cases, the situation gets more complex and hard to solve analytically. To solve such complex situations, we may need the perturbative method. One can generalize this study to the 3 neutrino cases.

**Acknowledgment**

Postdoctoral research support from the Council of Higher Education of Turkey is gratefully acknowledged.

## References

[1] Eguchi, K.; Enomoto, S.; Furuno, K.; Goldman, J.; Hanada, H.; Ikeda, H.; Ikeda, K.; Inoue, K.; Ishihara, K.; Itoh, W. et al. *Phys. Rev. Lett.* **2003**, *90*, 021802.

[2] Araki, T.; Eguchi, K.; Enomoto, S.; Furuno, K.; Ichimura, K.; Ikeda, H.; Inoue, K.; Ishihara, K.; Iwamoto, T.; Kawashima, T. et al. *Phys. Rev. Lett.* **2005**, *94*, 081801.

[3] Ahmad, Q. R.; Allen, R. C.; Andersen, T. C.; Anglin, J. D.; Buhler, G.; Barton, J. C.; Beier, E. W.; Bercovitch, M.; Bigu, J.; Biller, S. et al. *Phys. Rev. Lett.* **2001**, *87*, 071301.

[4] Ahmad, Q. R.; Allen, R. C.; Andersen, T. C.; Anglin, J. D.; Buhler, G.; Barton, J. C.; Beier, E. W.; Bercovitch, M.; Bigu, J.; Biller, S. et al. *Phys. Rev. Lett.* **2002**, *89*, 011301.

[5] Fukuda, S.; Fukuda, Y.; Ishitsuka, M.; Itow, Y.; Kajita, T.; Kameda, J.; Kaneyuki, K.; Kobayashi, K.; Koshio, Y.; Miura, M. et al. *Phys. Rev. Lett.* **2001**, *86*, 5651–5655.

[6] Cleveland, B. T.; Daily, T.; Davis, R. Jr.; Distel, J. R.; Lande, K.; Lee, C. K.; Wildenhain, P. S.; Ullman, J. *Astrophys. J.* **1998**, *496*, 505–526.

[7] Abdurashitov, J. N.; Gavrin, V. N.; Girin, S. V.; Gorbachev, V. V.; Gurkina, P. P.; Ibragimova, T. V.; Kalikhov, A. V.; Khairnasov, N. G.; Knodel, T. V.; Mirnov, I. N. *J. Exp. Theor. Phys.* **2002**, *95*, 181–193.

[8] Hampel, W.; Handt, J.; Heusser, G.; Kiko, J.; Kirsten, T.; Laubenstein, M.; Pernicka, E.; Rau, W.; Wojcik, M.; Zakharov, Y. et al. *Phys. Lett. B* **1999**, *447*, 127–133.

[9] Altmann, M.; Balata, M.; Belli, P.; Bellotti, E.; Bernabei, R.; Burkert, E.; Cattadori, C.; Cerichelli, G.; Chiarini, M.; Cribier, M. et al. *Phys. Lett. B* **2000**, *490*, 16–26.

[10] Altmann, M.; Balata, M.; Belli, P.; Bellotti, E.; Bernabei, R.; Burkert, E.; Cattadori, C.; Cerulli, R.; Chiarini, M.; Cribier, M. et al. *Phys. Lett. B* **2005**, *616*, 174–190.

[11] G. G. Raffelt, *Phys. Rept.* **1999**, *320*, 319–327.

[12] Liu, D. W.; Ashie, Y.; Fukuda, S.; Fukuda, Y.; Ishihara, K.; Itow, Y.; Koshio, Y.; Minamino, A.; Miura, M.; Moriyama, S. et al. *Phys. Rev. Lett.* **2004**, *93*, 021802.

[13] Wong, H. T.; Li, H. B.; Lin, S. T.; Lee, F. S.; Singh, V.; Wu, S. C.; Chang, C. Y.; Chang, H. M.; Chen, C. P.; Chou, M. H. et al. *Phys. Rev. D* **2007**, *75*, 012001.

[14] Daraktchieva, Z.; Amsler, C.; Avenier, M.; Broggini, C.; Busto, J.; Cerna, C.; Juget, F.; Koang, D.H.; Lamblin, J.; Lebrun, D. et al. *Phys. Lett. B* **2005**, *615*, 153–159.

[15] Beda, A. G.; Brudanin, V. B.; Egorov, V. G.; Medvedev, D. V.; Pogosov, V. S.; Shevchik, E. A.; Shirchenko, M. V.; Starostin, A. S.; Zhitnikov, I. V. *Phys. Part. Nucl. Lett.* **2013**, *10*, 139–143.

[16] Balantekin, A. B. *AIP Conf. Proc.* **2006**, *847*, 128–133.

[17] Okun, L. B.; Voloshin, M. B.; Vysotsky, M. I. *Yad. Fiz.* **1986**, *44*, 677–680.

[18] Akhmedov, E. K. *Phys. Lett. B* **1988**, *213*, 64–68.

[19] Barbieri, R.; Fiorentini, G. *Nucl. Phys. B* **1988**, *304*, 909–920.

[20] Lim, C. S.; Marciano, W. J. *Phys. Rev. D* **1988**, *37*, 1368–1373.

[21] Antia, H. M.; Chitre, S. M.; Thompson, M. J. *Astron. Astrophys.* **2000**, *360*, 335–344.

[22] Couvidat, S.; Turck-Chieze, S.; Kosovichev, A. G. *Astrophys. J.* **2003**, *599*, 1434–1448.

[23] Balantekin, A. B.; Hatchell, P. J.; Loreti, F. *Phys. Rev. D* **1990**, *41*, 3583–3593.

[24] Bykov, A. A.; Popov, V. Y.; Rashba, T. I.; Semikoz, V. B. arXiv:hep-ph/0002174.

[25] Akhmedov, E. K., Pulido, J. *Phys. Lett. B* **2003**, *553*, 7–17.

[26] Chauhan, B. C.; Pulido, J.; Torrente-Lujan, E. *Phys. Rev. D* **2003**, *68*, 033015.

[27] Chauhan, B. C.; Pulido, J. *JHEP* **2004**, *0406*, 008 (1-16).

[28] Balantekin, A. B.; Volpe, C. *Phys. Rev. D* **2005**, *72*, 033008.

[29] Yilmaz, D.; Yilmazer, A. U. *J. Phys. G* **2005**, *31*, 1123–1131.

[30] Yilmaz, D.; Yilmazer, A. U. *J. Phys. G* **2005**, *31*, 57–69.

[31] Balantekin, A. B.; Yuksel, H. *J. Phys. G* **2003**, *29*, 665–682.

[32] Balantekin, A. B.; Fetter, J. M.; Loreti, F. N. *Phys. Rev. D* **1996**, *54*, 3941–3951.