Theoretical investigations of Pt$_3$X (X = Al, Sc, Hf, Zr) ground state

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Abstract: The electronic structure of Pt$_3$X compounds showed that Pt$_3$Hf and Pt$_3$Zr were more stable for the D0$_{24}$ structure, rather than L1$_2$. The compound Pt$_3$Al was predicted to be the hardest and most ductile, but not with the L1$_2$ structure at ground state. The density of states showed that Pt$_3$Hf, Pt$_3$Zr, and Pt$_3$Al can be stabilized to the L1$_2$ phase with suitable element addition. All calculations were done within the density functional theory framework.

Key words: DFT, hardness, ductile, density of states

1. Introduction

Nickel-based superalloys (NBSAs) are efficient materials that have been used in engines, plants, and production processes since World War II. They traditionally have good elevated temperature strengths and high resistance to oxidation and corrosion [1]. Unfortunately, this class of alloys is presently used near their melting points. For higher system efficiency and less environmental pollution, a new class of alloys with higher melting points is required. To seek a substitute, alloys based on platinum, iridium, and rhodium have been suggested [2-4], primarily because of their fcc structure and higher melting points than nickel. A major drawback with iridium and rhodium alloys is their brittleness [5]. On the other hand, platinum is a recyclable and highly ductile metal [6,7].

The only limitations to the use of platinum are the cost and weight, but these can be reduced by alloying Pt with other metals. An assessment of systems with Pt as the major constituent showed that Pt–Al-based alloys had the highest potential due to precipitation strengthening from Pt$_3$Al and high oxidation resistance [8]. At high temperature (>1000 °C), Pt$_3$Al has the L1$_2$ structure [9]. At lower temperatures, there is some uncertainty about the structure of Pt$_3$Al. The L1$_2$ structure is reported to transform to $t'H16-U_3$Si (D0$_{24}$) structure before another transformation to the $t'I16$-Ir$_3$Si (D0$_{24}$') structure [10], while other researchers [11] reported that the low temperature phase was $tP16$-Pt$_3$Ga type. Both L1$_2$ and D0$_{24}$ structures have been reported in Pt$_3$Hf and Pt$_3$Zr [9,12,13], while the structure for Pt$_3$Sc has been reported to be L1$_2$. All these structure types are shown in Figure 1.

It is now routine to compliment experimental alloy development with theoretical modeling. It not only guides, but can also aid in avoiding the time consuming and expensive “trial and error” approaches that go with traditional material preparation. The fundamental particles that determine the nature of matter are the electrons and nuclei [14]. As the formation of a condensed material involves thousands to millions of atoms,

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the theory describing these particles is a many-body problem. The intrinsic properties of materials in the large
system can be evaluated using a statistical approach. Another strategy for theoretically obtaining material
properties relies on approximations that include quantitative theories such as density functional theory (DFT)
[15–18].

Figure 1. From left to right: L1_2, tP16-Pt₃Ga, D0₁', and D0₂₄ structures. Images from

The purpose of the present work is to consider the importance of the addition of Al, Sc, Hf, and Zr to
Pt in view of the need to achieve low density in the final Pt₃X stoichiometry and their potential use under
extreme conditions. Although the present work is theoretical, understanding the ground state structure and
mechanical properties of these compounds will add to the knowledge required for the engineering of a new class
of high temperature superalloys.

2. Methods
All the calculations are based on the Hohenberg–Kohn [17] and Kohn–Sham [18] DFT formalism. The ground
state energy $E$ of a system can be expressed as a function of the density $n$ as:

$$E (n) = T_s (n) + E_H (n) + \mu_{xc} (n) + \int n (r) V_{ext} (r) \, dr$$

In terms of the expansion wave-functions $\{\psi\}$, the noninteracting kinetic energy $T_s (n)$ is expressed as:

$$T_s (n) = \frac{\hbar^2}{2m} \sum_i \int \psi_i^* (r) \nabla^2 \psi_i (r) \, dr$$

The functional, $\mu_{xc} (n)$, is unknown and is usually approximated. $V_{ext}$ is an external potential energy. The
repulsive coulomb interaction between electrons is given by the Hartree energy as:

$$E_H (n) = \int \frac{n (r) n (r)}{r - r} \, drdr$$

For a many-particle system without any electric or magnetic fields, the external potential in Eq. (1) can be
expressed by:

$$||V_{ext} (r) = - \sum_{\alpha=1}^{N} \frac{Z_{\alpha}}{r - R_{\alpha}}$$
where \( N \) = number of the atoms, \( Z_\alpha \) = charge of the \( \alpha \)th atom, and \( R_\alpha \) is the position of the \( \alpha \)th atom.

The solution of Eq. (1) was achieved with the Vienna Ab-initio Simulation Package (VASP) [19]. VASP is a package for performing quantum mechanical calculations on molecules and solids. It uses an iterative variational method [20] to solve the Kohn–Sham equation of a system. The functional, \( \mu_{xc} \), was obtained using the generalized gradient approximation (GGA) [21]. The projector augmented wave method [22] was used during wave-function expansions and “high” precision was sought in the calculation of the kinetic energy. Integration of the Brillouin zones during the self-consistency procedure was performed according to the Monkhorst–Pack scheme [23]. Forces on atoms were less than 0.001 eV/Å during geometrical optimizations. For all the structures considered, the elastic constants, \( c_{ij} \), were calculated using the approach in Mehl et al. [24]. For later use, we define in Eqs. 5-17 the relationship between the bulk modulus, shear modulus, and the elastic constants. In a cubic crystal, 3 elastic constants, namely \( c_{11}, c_{12}, \) and \( c_{44} \), are important [25]. According to the Voigt [26] and Reuss [27] approximations, the relationships between the bulk and shear moduli and these elastic constants for cubic structures are:

\[
B_V = \frac{c_{11} + 2c_{12}}{3}, \quad G_V = \frac{c_{11} - c_{12} - 3c_{44}}{5}, \quad B_R = \frac{c_{11} + 2c_{12}}{3}, \quad G_R = \frac{5(c_{11} - c_{12})c_{44}}{[4c_{44} + 3(c_{11} - c_{12})]^3},
\]

where \( B_V \) and \( G_V \) are the bulk and shear moduli as defined by the Voigt notation and \( B_R \) and \( G_R \) are the bulk and shear moduli according to the Reuss notation.

For a tetragonal lattice:

\[
B_V = \frac{1}{9}(2c_{11} + c_{12}) + c_{33} + 4c_{13}, \quad G_V = \frac{1}{30}(M + 3c_{11} - 3c_{12} + 12c_{44} + 6c_{66}),
\]

where

\[
M = c_{11} + c_{12} + 2c_{33} - 4c_{13},
\]

and the Reuss bounds are:

\[
B_R = \frac{C^2}{M}, \quad G_R = 15 \left[ \frac{18B_V}{C^2} \left( \frac{6}{c_{11} - c_{12}} \right) + \frac{6}{c_{44}} + \frac{3}{c_{66}} \right]^{-1},
\]

where

\[
C^2 = (c_{11} + c_{12})c_{33} - 2c_{13}^2.
\]

In this study, the averages between the Voigt and Reuss bounds (Hill averages) for all the crystals considered were obtained by:

\[
B = \frac{1}{2}(B_V + B_R), \quad G = \frac{1}{2}(G_V + G_R)
\]
The Poisson’s ratio \( (\nu) \) was calculated from the relation:

\[
\nu = \frac{3B - 2G}{2(3B + G)} \quad (17)
\]

### 3. Results

#### 3.1. Phase stability and mechanical properties

The results of the calculations are given in the Table. Based on the equilibrium energy \( E_0 \) data, the most stable compound was D0\(_{24}\)-Pt\(_3\)Hf. Because of its lowest \( E_0 \) value, the D0\(_{24}\) structure will be more readily formed in Pt\(_3\)Hf by thermodynamic means than the L1\(_2\) structure. At ground state, both Pt\(_3\)Hf and Pt\(_3\)Zr are more stable in the D0\(_{24}\) phase than in the L1\(_2\) phase. The Pt\(_3\)Al compound is predicted to be more stable in the \( tP16-Pt_3Ga \) structure than in the L1\(_2\) structures.

As the hardness of a crystal relates to the bulk modulus [28–30], the hardest compound according to the results was Pt\(_3\)Al in the \( tP16-Pt_3Ga \) structure. The least hard compound was D0\(_{24}\)-Pt\(_3\)Hf. D0\(_{24}\)-Pt\(_3\)Zr is predicted to be harder than D0\(_{24}\)-Pt3Hf, and the available experimental data [31] showed that the Vickers microhardness of D0\(_{24}\)-Pt\(_3\)Hf and D0\(_{24}\)-Pt\(_3\)Zr is 796 HV and 811 HV, respectively. With regards to their densities, Pt\(_3\)Al and Pt\(_3\)Sc are less dense compared with Pt\(_3\)Hf and Pt\(_3\)Zr. According to the Pugh criterion [32], a ductile compound should have a B/G > 1.75. The results showed that D0\(_{24}\)-Pt\(_3\)Hf is brittle, while the other compounds are predicted to be ductile, with Pt\(_3\)Al (\( tP16-Pt_3Ga \) type) being the most ductile compound.

#### 3.2. Density of states

The density of states (DOS) of a system describes the number of states per energy interval at each energy level that are available for occupation by electrons. When a compound is formed, electron redistribution occurs, creating different types of electronic structures. The correlations between the electronic structure of a material and its various physical properties are important, and, in some cases, can be used to predict new structures [33–35]. The DOS for all the phases in the Table are given in Figures 2a–2d.

The DOS plots further confirmed that the structures of Pt\(_3\)Al, Pt\(_3\)Hf, and Pt\(_3\)Zr will not be L1\(_2\) or D0\(_c\)'. For L1\(_2\)-Pt\(_3\)Al, the Fermi level falls on the antibonding (increasing energy) part of the spectrum—an indication of a metastable phase. The Fermi level on the DOS of D0\(_c\)'-Pt\(_3\)Al was on a peak, which indicates an unstable phase. The Fermi level on the DOS of Pt\(_3\)Al (\( tP16-Pt_3Ga \) type) was on a plateau, showing that this is the most stable of all the structures ever reported for Pt\(_3\)Al.

The Fermi levels on the DOS of both D0\(_{24}\)-Pt\(_3\)Hf and L1\(_2\)-Pt\(_3\)Hf were on the pseudogap.

Moreover, the Fermi level of D0\(_{24}\)-Pt\(_3\)Hf was more on the nonbonding (reducing energy) part of the spectrum and this phase is predicted to be more stable than the L1\(_2\)-Pt\(_3\)Hf phase that had its Fermi level more on the antibonding (increasing energy) side of the DOS spectrum. Likewise, D0\(_{24}\)-Pt\(_3\)Zr is predicted to be more stable than L1\(_2\)-Pt\(_3\)Zr. The DOS spectrum of D0\(_{24}\)-Pt\(_3\)Zr had its Fermi level exactly on the pseudogap, while that of L1\(_2\)-Pt\(_3\)Zr was on the antibonding side of the DOS spectrum. At ground state, therefore, the phases will be D0\(_{24}\) for Pt\(_3\)Hf and Pt\(_3\)Zr and \( tP16-Pt_3Ga \) for Pt\(_3\)Al, agreeing with previous works [31,38].
Figure 2. a) Total electronic density of states for Pt$_3$Al in D0$_{22}$ and tP16-Pt$_3$Ga structures. Energies relative to Fermi level are indicated by the dotted vertical line. b) Total electronic density of states for Pt$_3$Al and Pt$_3$Hf in the L1$_2$ structures. Energies relative to Fermi level are indicated by the dotted vertical line. c) Total electronic density of states for Pt$_3$Zr (D0$_{24}$) and L1$_2$-Pt$_3$Sc structures. Energies relative to Fermi level are indicated by the dotted vertical line. d) Total electronic density of states for structures of Pt$_3$Zr (L1$_2$) and Pt$_3$Hf (D0$_{24}$). Energies relative to Fermi level are indicated by the dotted vertical line.
Table. Calculated bulk modulus (\(B\)), shear modulus (\(G\)), \(B/G\) ratio, Poisson's ratio (\(v\)), equilibrium energy (\(E_0\)), and density (\(\rho^{\text{cal}}\)) for Pt\(_3\)X compounds. Experimental data are in brackets and all moduli are in GPa.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Structure</th>
<th>(B) (GPa)</th>
<th>(G) (GPa)</th>
<th>(B/G)</th>
<th>(v)</th>
<th>(E_0) (eV/at.)</th>
<th>(\rho^{\text{cal}}) (g.cm(^{-3}))</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pt(_3)Sc</td>
<td>L(_{12})</td>
<td>250</td>
<td>103</td>
<td>2.43</td>
<td>0.29</td>
<td>7.178</td>
<td>12.36</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L(_{12})</td>
<td>280</td>
<td>108</td>
<td>2.59</td>
<td>0.30</td>
<td>6.256</td>
<td>12.24</td>
<td>-</td>
</tr>
<tr>
<td>Pt(_3)Al</td>
<td>D(_0^\prime)</td>
<td>309</td>
<td>98</td>
<td>3.15</td>
<td>0.30</td>
<td>6.297</td>
<td>12.68</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Pt(_3)Ga</td>
<td>378</td>
<td>71</td>
<td>5.32</td>
<td>0.34</td>
<td>6.354</td>
<td>12.68</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L(_{12})</td>
<td>290</td>
<td>120</td>
<td>2.42</td>
<td>0.28</td>
<td>8.129</td>
<td>18.88</td>
<td>-</td>
</tr>
<tr>
<td>Pt(_3)Hf</td>
<td>D(_0^{24})</td>
<td>151</td>
<td>111</td>
<td>1.36</td>
<td>0.31</td>
<td>8.135</td>
<td>19.59 [19.62]</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>L(_{12})</td>
<td>282</td>
<td>113</td>
<td>2.50</td>
<td>0.29</td>
<td>7.693</td>
<td>14.31</td>
<td>-</td>
</tr>
<tr>
<td>Pt(_3)Zr</td>
<td>D(_0^{24})</td>
<td>280</td>
<td>106</td>
<td>2.64</td>
<td>0.31</td>
<td>7.701</td>
<td>16.96 [18.13]</td>
<td>36,37</td>
</tr>
</tbody>
</table>

However, the results in the Table showed that Pt\(_3\)Hf and Pt\(_3\)Zr had higher hardness (indicated by \(B\)) and lower density in the L\(_{12}\) structure than in the D\(_0^{24}\) phase. Moreover, the ductility was much better in the L\(_{12}\) structure. An assessment of the DOS plots showed that for these compounds to exist in the L\(_{12}\) structures, energy reduction in the system is important, and this can be achieved by the addition of low electron elements. The DOS results also showed that an additional element will be required to attain a more stable structure for L\(_{12}\)-Pt\(_3\)Al. In this case, however, elements with many electrons such as palladium, cobalt, and chromium are predicted to play a beneficial role. The Fermi level is far from the pseudogap; therefore, more electrons are required to shift the Fermi level to the pseudogap.

4. Conclusions

Both the elastic and electronic structure properties of platinum alloys in the stoichiometry Pt\(_3\)X (X = Al, Sc, Hf, Zr) were evaluated within the framework of DFT. The most stable compound was D\(_0^{24}\)-Pt\(_3\)Hf. The compressibility of the compounds was in the order of D\(_0^{24}\)-Pt\(_3\)Hf > L\(_{12}\)-Pt\(_3\)Sc > L\(_{12}\)-Pt\(_3\)Al/ D\(_0^{24}\)-Pt\(_3\)Zr > L\(_{12}\)-Pt\(_3\)Zr > L\(_{12}\)-Pt\(_3\)Hf > D\(_0^\prime\)-Pt\(_3\)Al > Pt\(_3\)Al (tP16- Pt\(_3\)Ga). Except for D\(_0^{24}\)-Pt\(_3\)Hf, all compounds are predicted to be ductile. Pt\(_3\)Al had the highest potential on the account of hardness, ductility, and density. The DOS showed that additional elements will be required to obtain Pt\(_3\)Al, Pt\(_3\)Hf, and Pt\(_3\)Zr as L\(_{12}\) structures at ground state.

Acknowledgments

We acknowledge support from the Department of Science and Technology, and National Research Foundation, South Africa, and African Materials Science and Engineering Network (AMSEN - a Carnegie-RISE Network). We profoundly thank Prof JE Lowther and Prof IA Fuwape for useful discussions.

References


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