A simple method to extract the parameters of the single-diode model of a PV system

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Abstract: This article suggests a simple method of modeling and simulation of photovoltaic systems. The main goal is to find the parameters \(I_{ph}, I_o, R_s, R_{sh}\), and \(a\) of the single-diode model by adjusting the P-V curve and/or I-V curve at parameters provided by the manufacturer specification sheets \((I_{sc}, V_{oc}, I_{mp}, V_{mp})\). An empirical model is incorporated into the single-diode model in order to remove its implicit nature and to ease the calculations.

Calculations using this method can be extended to generate the I-V curves at temperatures and irradiances other than the standard test condition (STC).

Key words: Array, module, equivalent, model, modeling, photovoltaic (PV)

1. Introduction

Renewable energy is increasingly on demand recently due to the extreme demand on non-renewable traditional fossil fuels energy sources. The Kyoto agreement on global reduction of greenhouse gas emissions dictates that all greenhouse gas emissions related to energy production should be reduced [1]. Among all the renewable energy, such as wind energy, hydrogen and fuel cells, bioenergy, ocean energy etc., solar energy seems to be the most promising. The advantages of utilizing solar energy by using photovoltaic devices are: the short time for designing and installation of a new system, output power matching with peak load demands, static structure, no moving parts, longer lifetime, no noise, and pollution free [2]. A photovoltaic (PV) system directly converts sunlight into electricity, and the basic device of a PV system is the PV cell [3]. Cells may be assembled to form modules or arrays. The power available at the terminal of a PV device (cell/module/array) can provide electricity to either small loads such as a lighting system, or to a very large scale PV system in the range of 10 MW and more.

The performance of photovoltaic cell is normally evaluated under the standard test condition (STC), where an average solar spectrum at AM1.5 is used, the irradiance is normalized to 1000 W/m², and the cell temperature is defined as 25 °C [4]. Special testing equipment, such as an expensive solar simulator and special environment, are necessary to satisfy the requirement of temperature and insulation in the STC.

The mathematical models for PV arrays are based on the theoretical equations that describe the functioning of the PV cells and can be developed using the equivalent circuit of the PV cells. The empirical models rely on different values extracted from the characteristic equation of the solar panels using an analytical function [2]. Many photovoltaic cell models have been developed in the literature, for describing the behavior of a PV

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cell. The most frequently used model is the traditional single-diode model, known also as the five parameters model [5], shown in Figure 1.

\[ I = I_{ph} - I_0 \left\{ \exp \left[ \frac{(V + IR_s)}{V_{tA}} \right] - 1 \right\} - \frac{V + IR_s}{R_{sh}}, \]  

(1)

where \( I_{ph} \) and \( I_0 \) are the photo-generated current and the dark saturation current of the PV system, respectively, and \( V_t = N_s kT/q \) is the thermal voltage of the PV system with \( N_s \) cells connected in series, \( R_s \) and \( R_{sh} \) are the cell series resistance and the cell shunt resistance, respectively, \( a \) is the diode quality factor, \( q \) is the electronic charge \( 1.6 \times 10^{-19} \text{C} \), \( k \) is the Boltzmann’s constant \( 1.38 \times 10^{-23} \text{J/K} \), and \( T \) is the ambient temperature, in Kelvin.

Equation (1) describes the single-diode model shown in Figure 1. Some authors have proposed more sophisticated models that account for better accuracy and may serve for different purposes. For example, in a two-diode model, one diode is used to represent the effect of the recombination of carriers [6–10]. A three-diode model is also proposed to include the influence of effects that are not considered by the previous models [11]. All these models are implicit functions, i.e. have the form \( I = f(V, I) \). Such implicit nature of the models makes the method for adjusting the model parameters more difficult.

Manufacturers of PV arrays provide only limited experimental data about electrical and thermal characteristics. Unfortunately, the manufacturer’s datasheet does not include some of the parameters required for adjusting PV array models, such as the photocurrent, diode reverse saturation current, the diode ideality factor, the series resistance, and the shunt resistance. Datasheets of all PV arrays bring basically the following parameters: the open-circuit voltage \( V_{oc} \), the short-circuit current \( I_{sc} \), the voltage at the maximum power point \( V_{mp} \), the current at the maximum power point \( I_{mp} \), the open-circuit voltage/temperature coefficient \( \beta_{V_{oc}} \), the short circuit current/temperature coefficient \( \alpha_{I_{sc}} \), and the maximum experimental peak output power \( P_{maxe} \). These parameters are always provided with reference to the nominal condition or to the standard test condition STC of temperature and solar irradiation. Some manufacturers provide I-V curves at various irradiation and temperature conditions. Such curves make the adjustment and the validation of the desired mathematical I-V equation easier. Basically, this is all the information that one can get from datasheets of PV arrays.

Some other models that can generate the IV curves from the manufacturer’s datasheet have been suggested. Among these models is the model expressed in the following function:

\[ I_{TA} = I_{sc} - C_1 \exp \left( -\frac{V_{oc}}{C_2} \right) \left[ \exp \left( \frac{V}{C_2} \right) - 1 \right]. \]

(2)
This is derived and simplified from [12]. Also, the coefficients of the model equation $C_1$ and $C_2$ can be obtained using the approximations

$$C_1 \approx I_{sc},$$

$$C_2 \approx \frac{V_{mp} - V_{oc}}{\ln \left(1 - \frac{I_{mp}}{I_{sc}}\right)},$$

or, by solving simultaneously the following equations:

$$C_1 = \frac{I_{sc}}{\left[1 - \exp\left(-\frac{V_{oc}}{C_2}\right)\right]},$$

$$C_1 = \frac{I_{sc} - I_{mp}}{\left[\exp\left(\frac{V_{mp}}{C_2}\right) - 1\right]\left[\exp\left(-\frac{V_{oc}}{C_2}\right)\right]}.$$

The determination of PV system cell parameters from measured I-V characteristics is of vital importance for the quality control and the evaluation of its performance. Several authors [13–22] have suggested methods to extract the parameters that describe the non-linear model of PV systems.

Ishibashi et al. have recently introduced a method to extract all the parameters of a solar cell at one constant illumination level [13]. However, this method relies on calculating the differential value $dV/dI$ from the experimental data, which requires a very smooth I-V curve. Thus, techniques to smooth the experimental data such as the polynomial approximation method are inevitable.

A widely used method to extract the solar cell parameters is the curve fitting approach [14–17]. The least squares approach, which is a common method used in curve fitting, extracts the parameters of a solar cell by minimizing the squared error between the calculated target variable and the experimental data. However, the current in equation (1) constitutes an implicit function, i.e. includes the dependent and independent variables ($I$, $V$) at the same time. Such implicit nature of equation (1) increases the complexity and difficulty of the parameters extraction.

Explicit analytic expressions for $I$ or $V$ can be obtained with the help of the Lambert W function [18–21]. Jain et al. [18] have used the Lambert W function to study the properties of solar cells. However, their study is validated only on simulated I-V characteristics instead of extracting the parameters from the experimental I-V data. Later, Ortiz-Conde et al. [21] proposed a method to extract the solar cell parameters from the I-V characteristics based on the Lambert W function. They first calculated the Co-content (CC) function from the exact explicit analytical expressions, and then extracted the device parameters by curve fitting [21]. However, the CC remains a function of $I$ and $V$, and thus the fitting process is a bi-dimensional fitting process. Recently, Zhang et al. [22] have suggested a method for the extraction of all the parameters of a solar cell from a single current-voltage I-V curve under the constant illumination level using the Lambert W function. They have reduced the number of the parameters, so that the expression for $I$ only depends on $a$, $R_s$, and $R_{sh}$. Then, the analytic expression was directly used to fit the experimental data and extract the device parameters.

In this article a simple method is proposed to estimate the PV system parameters. The simplicity of this method for adjusting the parameters and the improvements proposed make the method handy for power electronics designers who are looking for an easy and effective method for the simulation of PV devices with power converters.
2. Modeling of a photovoltaic module

Evaluating equation (1) for the open circuit condition gives

\[ 0 = I_{ph} - I_0 \left\{ \exp \left[ \frac{V_{oc}}{V_t a} \right] - 1 \right\} - \frac{V_{oc}}{R_{sh}} \]  \hspace{1cm} (7)

Equation 7 can be rearranged to the form

\[ I_0 = \frac{I_{ph} - \frac{V_{oc}}{R_{sh}}}{\exp \left( \frac{V_{oc}}{V_t a} \right) - 1} \]  \hspace{1cm} (8)

Equation (8) can be approximated as

\[ I_0 \approx \frac{I_{ph}}{\exp \left( \frac{V_{oc}}{V_t a} \right) - 1} \]  \hspace{1cm} (9)

The ideality factor \( a \) depends only on cell temperature, but not on irradiance. At a given temperature the value of \( a \) may be arbitrarily chosen. Methods to estimate the correct value of this constant have been discussed by many authors [23, 24]. Usually, \( 1 \leq a \leq 1.5 \) for silicon solar cells and the choice depends on other parameters of the I-V model [3]. Some values of \( a \) are found in [25] based on empirical analyses. The value of \( a \) affects the curvature of the I-V curve, while the value of the short circuit current remains unchanged. Because \( a \) expresses the degree of ideality of the diode and it is totally empirical, any initial value of \( a \) can be chosen in order to adjust the model at a given manufacturer’s data. The value of \( a \) can be later readjusted in order to improve the model fitting to the experimental data.

2.1. Adjusting the method

To ease the calculations, the implicit nature of equation (1) is removed by substituting equation (2) instead of current in the right hand side of equation (1) to obtain:

\[ I = I_{ph} - I_0 \left\{ \exp \left[ \frac{(V + I_{ TA } R_s)}{V_t a} \right] - 1 \right\} - \frac{V + I_{ TA } R_s}{R_{sh}} \]  \hspace{1cm} (10)

One can start by using the approximation for the photocurrent, \( I_{ph} \approx I_{sc} \), and the approximation for the saturation current \( I_0 \) given in equation (9). Two parameters are still unknown in equation (10), which are \( R_s \) and \( R_{sh} \). One can use a method for adjusting \( R_s \) and \( R_{sh} \) based on the fact that there is only one pair \( \{R_s, R_{sh}\} \) that gives \( P_{maxm} = P_{maxe} = V_{mp} I_{mp} \) at the maximum power point of the I-V curve [3]. In other words, the maximum power \( (P_{maxm}) \) calculated by the I-V model of equation (10) is equal to the maximum experimental power \( (P_{maxe}) \) from the datasheet. Usually, modeling methods found in the literature take care of the I-V curve but ignore that the P-V curve must match the experimental data, too. Few works gave attention to the necessity of matching the power curve but with different or simplified models [26, 27]. For example, in [26] the series resistance of the array model is neglected.

Multiplying both sides of equation (10) with \( V \) one obtains the power \( P = IV \) to get

\[ P_{maxm} = V_{mp} \left( I_{ph} - I_0 \left\{ \exp \left[ \frac{(V_{mp} + I_{mp} R_s)}{V_t a} \right] - 1 \right\} - \frac{V + I_{mp} R_s}{R_{sh}} \right) \]. \hspace{1cm} (11)
The relation between $R_s$ and $R_{sh}$, the only remaining unknowns of equation (10), may be found by either by making $P_{max,m} = P_{max,e}$ and solving the resulting equation for $R_s$ to obtain

$$R_{sh} = \frac{V_{mp} (V_{mp} + I_{mp} R_s)}{V_{mp} I_{ph} - V_{mp} I_0 \exp \left( \frac{(V_{mp} + I_{mp} R_s)}{V_t a} \right) + V_{mp} I_0 - P_{max,e}}.$$  \hspace{1cm} (12)

or from equation (10) at the maximum power point

$$R_{sh} = \frac{V_{mp} I_{ph} - V_{mp} I_0 \exp \left( \frac{(V_{mp} + I_{mp} R_s)}{V_t a} \right) + V_{mp} I_0 - I_{mpe}}{V_{mp} + I_{mp} R_s}.$$ \hspace{1cm} (13)

2.2. Iterative solution of $R_s$ and $R_{sh}$

The aim is to find the value of the pair $(R_s, R_{sh})$ that makes the maximum power point $(V_{mp}, I_{mp})$ coincide with the experimental maximum power point $(V_{mpe}, I_{mpe})$. In other words, one searches for the pair $(R_s, R_{sh})$ that makes the peak of the mathematical P-V curve coincide with the experimental peak power point at $(V_{mpe}, I_{mpe})$. This requires an iteration process until $V_{mp} = V_{mpe}$ and $I_{mp} = I_{mpe}$ or $P_{max,m} = P_{max,e}$. Such iterative process can be easily done using Mathcad 14 by slowly increasing $R_s$ starting from $R_s = 0$. Adjusting the I-V curve or the P-V curve to match the experimental data requires finding the curve for several values of $R_s$ and $R_{sh}$. In fact, plotting the I-V curve or the P-V curve is not necessary, as only the maximum power point or the peak power value is required.

Figures 2 and 3 illustrate how such iterative process works. In Figure 2, as $R_s$ increases, the P-V curve moves to the left and the peak power ($P_{max,m}$) goes toward the experimental maximum power point. For every P-V curve of Figure 2, there is a corresponding I-V curve in Figure 3. As expected from Figure 3, all I-V curves cross the desired experimental MPP point at $(V_{mp}, I_{mp})$. The I-V points are easily obtained from equation (10) for a set of $V$ values and obtaining the corresponding set of $I$ points. Obtaining the P-V points is straightforward by calculating the power $P = IV$.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure2.png}
\caption{P-V curves plotted for different values of $R_s$ and $R_{sh}$ for MSX-60 solar module.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure3.png}
\caption{I-V curves plotted for different values of $R_s$ and $R_{sh}$ for MSX-60 solar module.}
\end{figure}
Figure 2 shows a plot of $P_{\text{max},m}$ as a function of $V$ for several values of $R_s$. It is clear that there is only one point, corresponding to a single value of $R_s$ that satisfies the imposed condition $P_{\text{max},m} = V_{mp}I_{mp}$ at the maximum power point. An alternative way for graphically finding the solution for $R_s$ is illustrated in Figure 4. This figure shows a plot of $P_{\text{max},m}$ as a function of $R_s$ for $I = I_{mp}$ and $V = V_{mp}$. The desired value of $R_s$ is the value at the minimum of the curve.

![Figure 4](image)

**Figure 4.** $P_{\text{max}} = f(R_s)$ with $I = I_{mp}$ and $V = V_{mp}$ for MSX-60 solar module.

### 2.3. Further improving the method

The method developed so far can be further improved by repeating the calculations using the value of $I_0$ given by equation (8), and introducing the value $I_{\text{ph}}$ given by equation (14).

$$I_{\text{ph}} = \frac{R_{sh} + R_s}{R_{sh}}I_{sc}. \quad (14)$$

Finally, the diode quality factor $a$ is readjusted by increasing its value slightly, if necessary, to obtain the best fit between the experimental and the model data. Values of the parameters from datasheet for RTC France Si solar cell and MSX-60 solar module are shown in Table 1. Table 2 shows a comparison between the values of the parameters extracted using the current method and some other methods available in literature for RTC France Si solar cell and MSX-60 solar module.

**Table 1.** Parameters available from datasheet.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RTC Si solar cell</th>
<th>MSX-60 module</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{sc}$, A</td>
<td>0.760</td>
<td>3.81</td>
</tr>
<tr>
<td>$V_{oc}$, V</td>
<td>0.5728</td>
<td>21.1</td>
</tr>
<tr>
<td>$I_{mp}$, A</td>
<td>0.69119</td>
<td>3.5</td>
</tr>
<tr>
<td>$V_{mp}$, V</td>
<td>0.45</td>
<td>17.14</td>
</tr>
<tr>
<td>$T$, °C</td>
<td>33</td>
<td>25</td>
</tr>
<tr>
<td>$N_s$</td>
<td>1</td>
<td>36</td>
</tr>
</tbody>
</table>
Table 2. Extracted parameters by the current method and the previous work.

<table>
<thead>
<tr>
<th>PV system</th>
<th>$I_{ph}$ (A)</th>
<th>$I_o$ ($\mu$A)</th>
<th>$R_s$ (Ω)</th>
<th>$R_{sh}$ (kΩ)</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si (RTC France)</td>
<td>0.7608</td>
<td>0.3223</td>
<td>0.0364</td>
<td>0.0538</td>
<td>1.484</td>
</tr>
<tr>
<td>Previous work$^a$</td>
<td>0.7609</td>
<td>0.4039</td>
<td>0.0364</td>
<td>0.0495</td>
<td>1.504</td>
</tr>
<tr>
<td>Previous work$^b$</td>
<td>0.77</td>
<td>0.20</td>
<td>1.037</td>
<td>0.032</td>
<td>1.4</td>
</tr>
<tr>
<td>Previous work$^c$</td>
<td>0.7611</td>
<td>0.2422</td>
<td>0.0373</td>
<td>0.042</td>
<td>1.4561</td>
</tr>
<tr>
<td>This work</td>
<td>0.7609</td>
<td>0.11912</td>
<td>0.0423</td>
<td>0.05575</td>
<td>1.388</td>
</tr>
<tr>
<td>MSX60 solar module</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Previous work$^c$</td>
<td>3.812</td>
<td>0.1866</td>
<td>0.178</td>
<td>0.358569</td>
<td>1.358</td>
</tr>
<tr>
<td>This work</td>
<td>3.812</td>
<td>0.1802</td>
<td>0.171</td>
<td>0.337671</td>
<td>1.277</td>
</tr>
</tbody>
</table>

$^a$See Ref. 14., $^b$See Ref. 15., $^c$See Ref. 13., $^d$See Ref. 22., $^e$See Ref. 28.

Figures 5 and 6 show the I-V and P-V curves of the MSX-60 solar module adjusted with the parameters obtained with proposed method. The model curves match well with the experimental data. Similarly, Figures 7 and 8 show the I-V and P-V curves of the RTC Si solar cell adjusted with the parameters obtained with this method. Also, the model curves match well with the experimental data. The variations of absolute error of current and power with respect to voltage for MSX-60 module and RTC Si solar cell are illustrated in Figures 9 and 10, respectively.

2.4. PV system characteristics at various temperatures

To predict a PV module or solar cell I-V characteristics curve at temperatures where data or I-V curves are not available, parameter temperature coefficients are necessary. Temperature coefficients are the rate of change of different photovoltaic parameters with respect to temperature. These coefficients can be determined for $I_{sc}$ ($\alpha_{Isc}$), $V_{oc}$ ($\beta_{Voc}$), $I_{mp}$ ($\alpha_{Imp}$), and $V_{mp}$ ($\beta_{Vm p}$). It has been published [12, 29] that these four temperature coefficients are necessary to accurately model the effect of temperature on the I-V characteristics of a module. The set of equations necessary to calculate the module parameters at temperature values other than SRC are:

$$V_{oc}(T) = V_{oc}(T_r) - \beta_{Voc}(T_r - T),$$

Figure 5. I-V curves for MSX-60 module at STC.

Figure 6. P-V curves for MSX-60 module at STC.
The diode curve fitting factor, the reverse saturation current, and the photocurrent vary with temperature according to the following equations:

$$a_T = \frac{T}{T_r},$$  \hspace{1cm} (19)

$$I_0(T) = \left( I_{sc}(T) - \frac{V_{oc}(T) - I_{sc}(T) \cdot R_s}{R_{sh}} \right) \cdot \exp \left( - \frac{V_{oc}(T)}{a_T V_t} \right),$$  \hspace{1cm} (20)

$$I_{ph}(T) = I_0(T) \cdot \exp \left( \frac{V_{oc}(T)}{a_T V_t} \right) + \frac{V_{oc}(T)}{R_{sh}}.$$  \hspace{1cm} (21)

Sandia National laboratory provides an online Microsoft Access database for various PV modules and arrays [30]. This database contains values of various temperature coefficients as well as values of other
parameters. For example, applying equations (15)–(18) to MSX-60 module with SRC parameter values $I_{sc}(T_r)$, $V_{oc}(T_r)$, $I_{mp}(T_r)$, and $V_{mp}(T)$ at temperature $T_r = 25 \, ^\circ\text{C}$, one can determine $I_{sc}(T)$, $V_{oc}(T)$, $I_{mp}(T)$, and $V_{mp}(T)$ at various temperatures. Then, by applying the calculated values into equations (19)–(21) the values of $a_T$, $I_0(T)$, and $I_ph(T)$ can be easily obtained, then the I-V curve at the corresponding temperature can be generated. Figures 11 and 12 show the results of generating these curves at three different temperature values (0 °C, 50 °C, and 75 °C) from the data at SRC ($T = 25 \, ^\circ\text{C}$). It is clear from the figure that the calculated curves match the data well.

![Figure 11](image1.png)

**Figure 11.** Current vs. voltage at various temperatures for MSX-60 module. Points are measured data. Lines are calculated data.

![Figure 12](image2.png)

**Figure 12.** Power vs. voltage at various temperatures for MSX-60 module. Points are measured data. Lines are calculated data.

### 2.5. PV system characteristics at various irradiances

Masters [31] reports that the short-circuit current $I_{sc}$ is directly proportional to the solar insolation. The relationship is described by the relation

$$I_{sc} = I_{scr} \left( \frac{G}{G_r} \right),$$

where $G$ is the irradiance at STC = 1000 (W/m$^2$), and $I_{scr}$ is the short-circuit current at STC. Cutting irradiance in half, for instance, leads to a drop in $I_{sc}$ by half. Decreasing irradiance also reduces $V_{oc}$, but it does so following a logarithmic relationship that results in relatively modest changes in $V_{oc}$. It is worth noting that while irradiance does not affect $V_{oc}$ significantly, temperature increase also causes insignificant increase in $I_{sc}$. Figure 13 depicts the results of the generated curves at different irradiance values (800 W/m$^2$, 500 W/m$^2$, 300 W/m$^2$, and 100 W/m$^2$) related to the data at STC (1000 W/m$^2$) for MSX-60 solar module. These curves were obtained using a set of translation equations for current and voltage. Such equations allow one to translate the entire current versus voltage curve from temperature $T_1$ to $T_2$ and irradiance $G_1$ to $G_2$. These equations
are \[ I_2 = I_1 + I_{sc1} \left( \frac{G_2}{G_1} - 1 \right) + \alpha (T_2 - T_1) \] (23)

\[ V_2 = V_1 - R_s (I_2 - I_1) - I_2 K (T_2 - T_1) + \beta (T_2 - T_1), \] (24)

where \( K \) is a curve-shape correction factor.

**Figure 13.** I-V characteristics calculated at various irradiance levels, 25 \(^\circ\)C for MSX-60 module.

### 3. Conclusion

A simple method for modeling and simulation of photovoltaic systems is proposed in this article. The main objective is to find the parameters \( I_{ph}, I_o, R_s, R_{sh} \) and \( \alpha \) of the single-diode model by adjusting the P-V curve and/or the I-V curve parameters provided by the manufacturer specification sheets (\( I_{sc}, V_{oc}, I_{mp} \) and \( V_{mp} \)). In order to ease the calculations, an empirical model is incorporated into the single-diode model in order to remove its implicit nature.

Calculations using the current method are extended to generate the I-V curves at temperatures and irradiances other than the standard test condition (STC).

### References

http://www.sandia.gov/pv/docs/Database.htm