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2n-dimensional at Fujii model instanton-like solutions and coupling constant's role between instantons with higher derivatives

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Abstract

An instanton-like solution is considered for a class of two-dimensional conformal model with Liouville term. The model expressed here is a new higher-dimensional model with scalar-spinor interaction on even-dimensional spaces, and is a generalization of the even-dimensional Fujii model. A recurrence relation for the Fujii model and coupling constant's indices between relations with interaction constant is extracted. The behavior of Gürsey, Akdeniz-Smailagić and Fujii coupling constants in 4-dimensional conformal models are examined.

Key Words: Instanton, Fujii's model, Liouville term, conformal field theory, coupling constant, scalar-spinor interaction, even-dimensional spaces, two-dimensional conformal model, Gürsey term, scalar-spinor interaction

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1. Introduction

Lagrangian approach permits a perturbative analysis of field theory in powers of the Planck constant and captures some semi-classical non-perturbative effects (such as instantons, merons, and solitons).

Conformal symmetry is not realized in nature. In any realistic model, conformal symmetry needs to be broken. But it does play an important role in all kinds of physics. Conformal Field Theory (CFT) models constitute the essential building blocks of the classical vacua of string theoy, including quantum gravity. CFTs are conformally invariant field theories, primarily in two space-time dimensions and are related to critical phenomena in statistical mechanical systems, and characterize their Renormalization Group (RG) fixed points. For that reason they have numerous condensed matter applications. CFT describes classical string theory background configurations and is the basic tool to study their perturbative physics. There are a large number of interesting mathematical structures. Two-dimensional conformal model [1], higher-dimensional model [2] and

Gürsey model [3]. One possible mechanism could be provided by the nonperturbative quantum fluctuations of the instanton type in the vacuum [4].

The role of instantons in supersymmetric gauge theories has been extensively studied recently [5]. The effect of instantons on vacuum energy density has been studied in the context of nonsupersymmetric Yang-Mills theories [6], in which there exists an infinity of degenerate classical ground states, characterized by integer topological charge. Instantons provide a description of quantum mechanical tunneling between ground states of different topological charge, thereby contributing non-trivially to the vacuum energy density. When massless fermions are added to the theory, the picture changes drastically. Due to the zero modes of the relevant Dirac operator in the topologically non-trivial background, the quantum tunneling between classical vacua of different topological charge is completely suppressed. Therefore, in supersymmetric Yang-Mills theories with massless fermions, single instantons or anti-instantons do not contribute to the vacuum energy. These cannot break supersymmetry, and vacuum energy stays at zero.

Conformal field theories provide toy models for genuinely interacting quantum field theories; they describe two-dimensional critical phenomena, and they play a central role in string theory. Conformal field theories have also had a major impaction various aspects of modern mathematics, in particular the theory vertex operator algebras and Borcherds algebras, finite groups, number theory and low-dimensional topology. It's possible to analyze deformations of conformal field theories that describe integrable massive models [7, 8]. The theory of conformal symmetry is well known and has had important implications in e.g. string theory. Nonperturbative effects are extremely important for our understanding of nature. The most well-known particle physics example is the description of hadrons in Quantum Chromodynamics (QCD), but they play a significant role in many other areas of high energy physics, cosmology and (1+1) dimensions, defined on the half-plane, are used describe the critical properties of various problems in condensed matter physics. With (1+1) dimensions the conformal symmetry algebra becomes infinite dimensional, that is infinitely many conserved quantities will be associated to any 1+1 dimensional CFT. (1+1)-dimensional integrable QFT may be formally viewed as a perturbation of a CFT by means of a relevant field of the CFT itself [5]. CFT can always be thought of as a Renormalization group (RG) critical fixed point. Therefore, its perturbation by means of any relevant operator amounts to "moving" the CFT away from its associated RG fixed point and consequently, to breaking initial conformal invariance. It is a fact that conformal symmetry is an extremely high symmetry which provides every CFT with an infinitive number of local conserved quantities. The hope is that studying CFT and other field theories can lead us closer to the understanding of these problems.

Recall the two-dimensional super-symmetric Liouville model [1] described by the Lagrangian

$$L_{\mathrm{SL}} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{i}{2}\bar{\psi}\ \partial\!\!\!/ \psi - \frac{m^2}{\beta^2}e^{\beta\phi} - \frac{m}{2\sqrt{2}}e^{\beta\phi/2}\,\bar{\psi}\psi. \tag{1}$$

Subsequently, Akdeniz and Dane constructed a new scalar-spinor interaction model in two dimensional conformal-invariant models [1]. Its Lagrangian is

$$L_{\mathrm{AD}} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{i}{2}\bar{\psi}\ \partial\!\!\!/ \psi + \frac{m^2}{\beta^2}e^{\beta\phi} + \frac{m}{2\sqrt{2}}e^{\beta\phi/2}\,\bar{\psi}\psi + g\left(\bar{\psi}\psi\right)^2. \tag{2}$$

It does not contain [2] the so-called Gürsey term [3] but coincides only at 2-dimensions. Aside from the sign of the interaction term, it is different from that of the supersymmetric Liouville model.

Fujii [2] constructed a classical solution of scalar-spinor interaction model with higher derivatives on even-dimensional spaces. (IR^D, D is even-dimension). Their Lagrangian is given by

$$\begin{aligned} F_F &= \tfrac{1}{2}\left(\partial_{\mu D/2}\partial_{\mu(D-2)/2}\cdots\partial_{\mu 1}\phi\right)^2 + \tfrac{1}{2}\left(\partial_{\mu(D-2)/2}\cdots\partial_{\mu 1}\bar{\psi}\right)(i\ \not\partial)\left(\partial_{\mu(D-2)/2}\cdots\partial_{\mu 1}\psi\right) \\ &\quad + \sum_{j=0}^{D}\alpha_j e^{(D-j)\beta\phi/D}\left(\bar{\psi}\psi\right)^j. \end{aligned} \tag{3}$$

This Lagrangian gives the equation of motion

$$\phi = \frac{1}{\beta}\log\left(\frac{2A}{1+X}\right)^D \tag{4}$$

$$\psi = \frac{1}{1+X^2}\left(1 + i\gamma\cdot\mathbf{X}\right)C. \tag{5}$$

Here, $\not\partial$ denotes a Laplacian on IR^D.

One can find the following instanton-like solutions of equations [1] as a result of the conformal symmetry. For this purpose the following ansätze is postulated:

$$\frac{D}{2}2^D\left(D-1\right)! + \beta\sum_{j=0}^{D-1}\alpha_j\frac{D-j}{D}\left(2A\right)^{D-j}\left(\bar{C}C\right)^j = 0, \tag{6}$$

$$-2^D\left(D-1\right)! + \beta\sum_{j=1}^{D}\alpha_j j\left(2A\right)^{D-j}\left(\bar{C}C\right)^{j-1} = 0. \tag{7}$$

Here, C is a constant spinor depending on β, $\alpha_1,\ldots,\alpha_D, g$, and A is a constant number. Many scale invariant theories are known to have classical instanton and meron solutions. Both instantons and merons are vacuum fluctuations: however, these two kinds of vacuum fluctuation have very different properties. Two characteristic properties of an instanton are that it is nonsingular and that its energy-momentum tensor vanishes identically. We construct pure spinor models with higher derivatives in every dimension, which is an extension of the well-known Gursey model in four dimensions, and construct instanton-like solutions [9]. Therefore, in the vicinity of an instanton there is no change even in the local distribution of vacuum energy [10]. Fuji [9, 11] inferred relatively simple algebraic equations which are independent of space variables.

2. Recurrence and the relationships among indices

We want to generalize $\overline{C}C$ on even-dimensional spaces:

$$j = 1,\ldots,D \Rightarrow \left(\bar{C}C\right)^j = \mp\frac{\left(2A\right)^{j+1}\left(D-1\right)!}{2A^D\alpha_{j+1}\left(j+1\right)}, \tag{8}$$

and we want to generalize α_j on even-dimensional spaces:

$$j = 0,\ldots,(D-1) \Rightarrow \alpha_j = \frac{D^2\left(j+1\right)\alpha_{j+1}}{A\beta^2\left(D-j\right)}. \tag{9}$$

And thus the recurrence relations are obtained, with which the Akdeniz-Dane (AD) model and its higher dimensions can be written. Writing the Lagrangian according to these recurrence and the indices relations are

$$L_F = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}\bar{\psi}\left(i\ \not{\partial}\right)\psi + \alpha_0 e^{\beta\phi} + \alpha_1 e^{\beta\phi/2}\left(\bar{\psi}\psi\right) + \alpha_2\left(\bar{\psi}\psi\right)^2, \tag{10}$$

$$\alpha_j = g, \tag{11}$$

$$g = \frac{A\beta^2\alpha_{j-1}}{D^3}. \tag{12}$$

The minimum protected section occurs for $j = D$, hence

$$\alpha_D = \mathrm{g}. \tag{13}$$

If AD model (2) and Fujii Model is compared, its recurrence relations. Generalizing g on even-dimensional spaces, solutions to A are:

a) For a pure solution, in which $g = 0$, without the fermion-fermion interaction, and $\beta^2 \geq 32\mathrm{g} => A$ has real roots,

$$A = -2, 2, 0 \tag{14}$$

$$\bar{C}C = \mp\frac{40}{\beta^2}, 0. \tag{15}$$

b) For a simplified solution [1, 12], in which $g = 0$, without the fermion-fermion interaction, and $\beta 2 \geq 16\mathrm{g} => A$ has real roots,

$$A = -1, 1, 0(15) \tag{16}$$

$$\bar{C}C = \mp\frac{32}{\beta^2}, 0. \tag{17}$$

c) The case $g = \beta^2/32$ [12] =>

$$A = -1/2. \tag{18}$$

Note fluctuations about the classical solutions are not stable.

d) The case $g = 3\beta^2/136$ [12] => another solution in which

$$A = 4. \tag{19}$$

To generalize A on even-dimensional spaces, we write

$$j = 1, \ldots, (D-1) \Rightarrow A = \frac{(j+1)\,\alpha_{j+1}}{2j\alpha_j} = \left[\frac{Dg}{2\,(D-1)\,\alpha_{D-1}}\right]_{D=j+1}. \tag{20}$$

The following solutions were found according to A [12], and from which the pure solution to the model supplied:

$$g = A\alpha_1 = A\left(\frac{m}{2\sqrt{2}}\right) = A\left(\frac{\frac{1}{A\sqrt{2}}}{2\sqrt{2}}\right) = \frac{1}{4}, \tag{21}$$

$$\beta^2 = g2^5 = 8, \tag{22}$$

$$\bar{C}C = \mp\frac{40}{\beta^2} = \mp 5. \tag{23}$$

3. New two-dimensional conformal model

Thus new two-dimensional conformal model Lagrangian is

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial\!\!\!/}\psi + \frac{1}{4}e^{2\sqrt{2}\phi} - \frac{1}{2}e^{\sqrt{2}\phi}\bar{\psi}\psi + \frac{1}{4}\left(\bar{\psi}\psi\right)^2 , \tag{24}$$

with the objective to generalize L on even-dimensional spaces:

$$\begin{aligned} L \;=\; & \tfrac{1}{2}\left(\partial_{\mu D/2}\partial_{\mu(D-2)/2}...\partial_{\mu 1}\phi\right)^2 + \tfrac{1}{2}\left(\partial_{\mu(D-2)/2}...\partial_{\mu 1}\bar{\psi}\right)\left(i\ \partial\!\!\!/\right)\left(\partial_{\mu(D-2)/2}...\partial_{\mu 1}\psi\right) \\ & + \sum_{j=0}^{D} \begin{array}{l} \left(\left(\frac{D^2(j+1)\alpha_{j+1}}{A\beta^2(D-j)}\right)_{j=0,\ldots,(D-1)} + \left(\frac{A\beta^2(D+1-j)\alpha_{j-1}}{D^2 j}\right)_{j=1,\ldots,D}\right) \\ \times e^{(D-j)\beta\phi/D}\left(\bar{\psi}\psi\right)^j . \end{array} \end{aligned} \tag{25}$$

This method can be used for higher dimensional conformal models, i.e., larger n and j; thus constant and free parameters can be written through the AD model in higher dimensions. The condition $m = 0$, therefore, can be understood; but the nature of g and this special value has always been a puzzle. But if $m \neq 0$ (dimensionless), it includes both stable and unstable states.

4. Coupling constant, and our conformal models

Models by Fujii [2], Gürsey [3] and Akdeniz-Smailagić (AS) [13] have similar movements and values (g, $\bar{C}C$) that connect at infinity. It has been apparently observed that since the value of g is quite high, fermion interaction of instanton-like solutions is similar to the strong nuclear force and quarks in the conformal model. It is thus suggested that conformal solutions with mass can be formed at quark level according to these models. Another significant fact is that, although the Gürsey model has a form different from the others, behavior of the coupling constant is the same as other models.

5. Comments and conclusion

As a result, Fujii model has been constructed on the basis of AD model and the recurrence relations of this model (8) and (9) have been inferred. With the AD model in mind, it is found that, whenever $j = D$, the last term is equal to g. After that numerical values of g and the relations between other models (Fujii [2], Gürsey [3], Akdeniz-Smailagić [13] models) was determined. In addition their (g, $\bar{C}C$) results were compared, and higher derivatives were reduced to subderivatives and generalized, and found to have physical meaning.

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