Turk J Phys
34 (2010) , 149 – 157.

doi:10.3906/fiz-0909-14

On the s-wave Jost solution for Coulomb-distorted nuclear potential

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Received 14.09.2009

Abstract

A closed form expression of the off-shell Jost solution for Coulomb plus separable nuclear potential is derived using interacting Green's functions and their integral transforms. As an application of our result the off-shell Jost function for the proton-proton scattering in the $^1S_0$ channel is computed for various laboratory energies to investigate the role of long range Coulomb interaction in off-shell scattering.

Key Words: Interacting Green's functions, Coulomb-nuclear potential, integral transform, off-shell Jost solution

1. Introduction

There exist experimental situations which involve scattering by additive interactions [1], some of which must for various physical reasons be treated exactly, whereas others may be relatively small perturbation. A typical example of this kind is the scattering of particles under the combined influence of Coulomb and nuclear forces. A short range local potential superimposed on Coulomb potential is often treated by the two-potential formula of Gell-Mann and Goldberger [2]. In this approach one computes the value of physical quantities like the scattering phase-shifts, transition matrices etc. by making use of judicious approximation methods. The present text addresses itself to the study of Coulomb distorted nuclear scattering by replacing short range local potential by a finite rank separable nonlocal interaction [3]. Such an approach is, however, no loss of generalization since, on the one hand, the short range local potentials can be represented by finite rank separable potentials in a mathematically well defined sense [4]. In view of the importance of experiments which involve charged hadrons, the interest in studying potentials consisting of the sum of a short-range finite-rank separable potential and Coulomb potential is increased.

It is well known that in absence of the Coulomb force the two-body Schrödinger equation for separable potentials can be solved in simple analytical form [5]. In view of this, nonlocal separable two-body interactions have often been used in the nuclear matter calculation [6–7]. They have also been used very systematically with

Faddeev equations for the three-body problem [8]. Thus, it is of considerable interest to study the Coulomb-nuclear problem within the framework of a separable model for nuclear interaction with particular emphasis on half- and off-shell effects which will provide an adequate and convenient starting point for rigorous calculations on few-body systems with charges. Here we shall deal with Coulomb plus Yamaguchi [9] potential and construct exact analytical expression for proton-proton off-shell Jost solution and function.

Concentrating on s-wave problem in recent few papers [10–13] we have constructed expressions for off-shell Jost solutions for coulomb and Coulomb-like (Coulomb plus separable) potentials via different approaches to the problem. As it is of some importance to have in the literature a relatively non-complicated mathematical prescription the objective of the present work is to look for a straightforward method to derive an expression for off-shell Jost solution $f(k,q,r)$ for Coulomb plus separable nuclear potential. This will be achieved by exploiting the particular solution of an inhomogeneous Schrödinger equation satisfied by $f(k,q,r)$ with judicious use of associated irregular Green's function together with certain properties and integral representations of higher transcendental functions. In this course of study it will be observed that the merit of the present approach is its simplicity. Section 2 is devoted to construct an exact analytical expression for off-shell Jost solution $f(k,q,r)$ for motion in Coulomb-Yamaguchi potential. Related numerical results, discussion and concluding remarks are presented in Section 3.

2. The off-shell Jost solution

Separable potentials have been, since the appearance of Yamaguchi's original paper [3], an immensely popular tool in dynamical calculations. In the representation space a one term separable potential is introduced as

$$V(r,r') = \lambda g(r)g(r') = \lambda e^{-\alpha r}e^{-\beta r'}, \tag{1}$$

to describe the nucleon-nucleon system. Here, α, λ and β stand for the strength and inverse range parameters. The off-shell Jost solution $f(k,q,r)$ for Coulomb plus separable potential satisfies the inhomogeneous differential equation

$$\left[\frac{d^2}{dr^2} + k^2 - V_C(r)\right] f(k,q,r) - \lambda g(r)\int_0^\infty ds g(s) f(k,q,s) = (k^2-q^2)e^{iqr} \tag{2}$$

with the Coulomb potential $V_C(r) = 2k\eta/r$, where η is the well known Sommerfeld parameter.

According to Fuda and Whiting [14] the particular integral of equation (2) represents the off-shell Jost solution. We thus write

$$f(k,q,r) = (k^2-q^2)\int_r^\infty dr' e^{iqr'} G^{(I)}(r,r'), \tag{3}$$

where $G^{(I)}(r,r')$ stands for the irregular Green's function [15] for motion in Coulomb plus separable potential and is given by

$$G^{(I)}(r,r') = \begin{cases} 0 & \text{for} \quad r' < r \\ \frac{1}{f(k)}\left[\phi(k,r')f(k,r) - \phi(k,r)f(k,r')\right] & \text{for} \quad r' > r. \end{cases} \tag{4}$$

Here, $\phi(k,r)$ and $f(k,r)$ are the regular and irregular solutions of the homogeneous part of equation (2) and read as [12–16]

$$\begin{aligned}\phi(k,r) &= re^{ikr}\Big[\Phi(1+i\eta,2;-2ikr) - \tfrac{\lambda}{2ikD(k)(\beta^2+k^2)}\left(\tfrac{(\beta-ik)}{(\beta+ik)}\right)^{i\eta} \\ &\quad \sum_{n=0}^{\infty}\tfrac{\rho^n}{n!}\theta_{n+1}(1+i\eta,2;-2ikr)\Big]\end{aligned} \tag{5}$$

and

$$\begin{aligned}f(k,r) &= f^C(k,r) - \tfrac{\lambda e^{\pi\eta/2} r e^{ikr}}{D(k)(\beta-ik)\Gamma(2+i\eta)} F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)}{(\beta-ik)}\right) \\ &\quad \Big[\tfrac{1}{(\beta-ik)(1+i\eta)}\,F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)}{(\beta-ik)}\right)\Phi(1+i\eta,2;-2ikr) + \tfrac{2ik\Gamma(1+i\eta)}{(\beta^2+k^2)} \\ &\quad \left(\tfrac{(\beta-ik)}{(\beta+ik)}\right)^{i\eta}\Psi(1+i\eta,2;-2ikr) + \tfrac{1}{2ik}\sum_{n=0}^{\infty}\tfrac{\rho^n}{n!}\theta_{n+1}(1+i\eta,2;-2ikr)\Big],\end{aligned} \tag{6}$$

with $\rho = \frac{(\beta+ik)}{2ik}$, $f^C(k,r)$ the Coulomb on-shell Jost or irregular solution [15] and $D(k)$, the Fredholm determinant [17] written as

$$\begin{aligned}D(k) &= 1-\lambda\int_0^{\infty}\int_r^{\infty} dr dr' g(r) G^{C(I)}(r,r') g(r') \\ &= \tfrac{1}{(1+i\eta)(\beta-ik)}\Big[\tfrac{1}{(\alpha^2+k^2)}\left(\tfrac{(\alpha-ik)}{(\alpha+ik)}\right)^{i\eta} F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)}{(\beta-ik)}\right) \\ &\quad -\tfrac{1}{(\alpha+\beta(\alpha-ik)}\,F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)(\alpha+ik)}{(\beta-ik)(\alpha-ik)}\right)\Big].\end{aligned} \tag{7}$$

Obviously, $f(k)$ is the corresponding Jost function. Substitution of equation (4) together with equations (5), (6) and (7) in equation (3) involves certain tedious indefinite integrals . To circumvent these difficulties in calculation we shall express the irregular Green's function for Coulomb plus separable potential in terms of pure Coulomb Green's function and their integral transforms as

$$G^{(I)}(r,r') = G^{C(I)}(r,r') + \frac{\lambda}{D(k)} I(k,\beta,r)\int_r^{\infty} dr' g(r') G^{C(I)}(r,r'), \tag{8}$$

where

$$I(k,\beta,r) = \int_r^{\infty} dr' g(r') G^{C(I)}(r,r'). \tag{9}$$

From equations (8) and (3) the off-shell Jost solution $f(k,q,r)$ is expressed as

$$f(k,q,r) = f^C(k,q,r) + \lambda\frac{(k^2-q^2)}{D(k)} I(k,\beta,r)\tilde{G}^{C(I)}(\beta,q) \tag{10}$$

with the Coulomb off-shell Jost solution [10-11]

$$f^C(k,q,r) = (k^2-q^2)\int_r^{\infty} dr' e^{iqr'} G^{C(I)}(r,r') \tag{11}$$

and

$$\tilde{G}^{C(I)}(\beta,q)=\int_0^\infty\int_r^\infty drdr'g(r)G^{C(I)}(r,r')g(r'). \tag{12}$$

Now our aim is to evaluate the indefinite integral in equation (11). Combination of equations (4) and (11) with pure Coulomb potential and rearrangement of terms leads to

$$\begin{aligned} f^C(k,q,r) &= \tfrac{(k^2-q^2)}{f^C(k)}\left[f^C(k,r)\int_0^\infty e^{iqr}\varphi^C(k,r)dr-\varphi^C(k,r)\int_0^\infty e^{iqr}f^C(k,r)dr\right. \\ &\left.+f^C(k)\int_0^r e^{iqr'}G^{C(R)}(r,r')dr'\right], \end{aligned} \tag{13}$$

where $\varphi^C(k,r)$, $f^C(k,r)$, $f^C(k)$ [15] and $G^{C(R)}(r,r')$, the regular Green's function for the Coulomb potential given by

$$\varphi^C(k,r)=re^{ikr}\Phi(1+i\eta,2;-2ikr), \tag{14}$$

$$f^C(k,r)=-2ike^{\pi\eta/2}\Psi(1+i\eta,2;-2ikr), \tag{15}$$

$$f^C(k)=\frac{e^{\pi\eta/2}}{\Gamma(1+i\eta)} \tag{16}$$

and

$$G^{C(R)}(r,r')=\begin{cases}\frac{1}{f^C(k)}\left[\phi^C(k,r)f^C(k,r')-\phi^C(k,r')f^C(k,r)\right] & \text{for} \quad r'<r,\\ 0, & \text{for} \quad r'>r,\end{cases} \tag{17}$$

With the help of equations (14), (15) and (16) together with the relation [18]

$$\Psi(a,c;z)=\frac{\Gamma(1-c)}{\Gamma(a-c+1)}\Phi(a,c;z)+\frac{\Gamma(c-1)}{\Gamma(a)}\bar{\Phi}(a,c;z), \tag{18}$$

equation (17) is expressed as

$$\begin{aligned} G^{C(R)}(r,r') &= 2ikrr'e^{ik(r+r')}\left[\Phi(1+i\eta,2;-2ikr')\bar{\Phi}(1+i\eta,2;-2ikr)-\right. \\ &\left. Phi(1+i\eta,2;-2ikr)\,\bar{\Phi}(1+i\eta,2;-2ikr')\right]. \end{aligned} \tag{19}$$

Substituting equation (19) in (13) together with equations (14), (15) and (16), the Coulomb off-shell Jost solution [10–11] is obtained as

$$\begin{aligned} f^C(k,q,r) &= 2ik\Gamma(1+i\eta)re^{ikr}\left[\tfrac{(q-k)}{2k\Gamma(2+i\eta)}\,F\left(1,i\eta;2+i\eta;\tfrac{(q-k)}{(q+k)}\right)\Phi(1+i\eta,2;-2ikr)\right. \\ &\left.\left(\tfrac{(q+k)}{(q-k)}\right)^{i\eta}\Psi(1+i\eta,2;-2ikr)\right]-\tfrac{(k^2-q^2)}{2ik}re^{ikr}\sum_{n=0}^{\infty}\tfrac{1}{n!}\left(\tfrac{(k-q)}{2k}\right)^n\theta_{n+1}(1+i\eta,2;-2ikr). \end{aligned} \tag{20}$$

In deriving the above result, we have used the following standard integrals, relation and integral representation [18, 19]:

$$\int_0^\infty e^{-ax}x^{s-1}\Psi(b,d;\mu x)dx = \frac{\Gamma(1+s-d)\Gamma(s)}{a^s\Gamma(1+b+s-d)}F\left(b,s;1+b+s-d;1-\mu/a\right), \tag{21}$$

$$\int_0^\infty e^{-\lambda z}z^\nu\Phi(a,c;pz)dz = \frac{\Gamma(\nu+1)}{\lambda^{\nu+1}}F\left(a,\nu+1;c;\frac{p}{\lambda}\right) \tag{22}$$

$$F(a,b;c;z) = (1-z)^{c-a-b}F(c-a,c-b;c;z) \tag{23}$$

and

$$\theta_\sigma(a,c;z) = \frac{1}{(c-1)}\left[\Phi(a,c;z)\int_0^z e^{-z'}z'^{(\sigma+c-2)}\bar{\Phi}(a,c;z')dz' - \bar{\Phi}(a,c;z')\int_0^z e^{-z'}z'^{(\sigma+c-2)}\Phi(a,c;z')dz'\right]. \tag{24}$$

The quantities $\tilde{G}^{C(I)}(\beta,q)$ and $I(k,\beta,r)$ can easily be obtained from the results given in Equations (7) and (20), respectively. Thus the off-shell Jost solution $f(k,q,r)$ for motion in Coulomb plus separable potential is expressed as

$$\begin{aligned} f_(k,q,r) \;=\;& f^C(k,q,r) - \tfrac{\lambda r e^{ikr}}{D(k)(1+i\eta)(\beta-ik)}\left[\left(\tfrac{(q+k)}{(q-k)}\right)^{i\eta}F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)}{(\beta-ik)}\right)\right.\\ &\left.+\tfrac{e^{i\pi/2}(q-k)}{(\beta-iq)}F\left(1,i\eta;2+i\eta;\tfrac{(q-k)(\beta+ik)}{(q+k)(\beta-ik)}\right)\right]\\ &\left\{\tfrac{1}{(1+i\eta)(\beta-ik)}\;F\left(1,i\eta;2+i\eta;\tfrac{(\beta+ik)}{(\beta-ik)}\right)\Phi(1+i\eta,2;-2ikr)\right.\\ &\left.+\tfrac{2ik\Gamma(1+i\eta)}{(\beta^2+k^2)}\left(\tfrac{\beta-ik}{\beta+ik}\right)^{i\eta}\Psi(1+i\eta,2;-2ikr)+\tfrac{1}{2ik}\sum_{n=0}^{\infty}\tfrac{\rho^n}{n!}\theta_{n+1}(1+i\eta,2;-2ikr)\right\}. \end{aligned} \tag{25}$$

The expression in above equation is in exact agreement with the previous results for Coulomb plus Yamaguchi (with $\alpha = \beta$) potential [13–20]. A couple of useful checks is made on the expression for Coulomb plus Yamaguchi off-shell Jost solution with particular emphasis on their limiting behavior and on-shell discontinuity. For example, in absence of Yamaguchi potential, i.e. $\lambda = 0, f(k,q,r)$ goes to pure Coulomb off-shell Jost solution [10–11]. Secondly, in the limit of no Coulomb field, $\eta = 0$, Yamaguchi off-shell Jost solution is obtained [21]. When both λ and η goes to zero, $f(k,q,r) = e^{ikr}$. Other useful checks consists in showing that

$$f(k,q,r)_{r\to 0} \to f(k,q) \tag{26}$$

and

$$f(k,r) = Lt_{q\to k}\left[e^{\pi\eta/2}\Big/\Gamma(1+i\eta)\right]\;[(q-k)/(q+k)]^{i\eta}f(k,q,r). \tag{27}$$

It has been checked that the above results are in order and are in agreement with that of van Haeringen [22] and Talukdar et al [23].

3. Results-discussion and conclusion

The results for Yamaguchi and Coulomb plus Yamaguchi off-shell Jost functions (hereby denoted as $f_Y(k,q)$ and $f_{CY}(k,q)$, respectively) for $q < k$ and $q > k$ have been computed and presented in Tables 1, 2 and 3 as a function of off-shell momentum q for laboratory energies 10, 20 and 30 MeV, respectively, for (p-p) system in the $^1S_0$ channel. Here, the numbers refer to a repulsive Coulomb potential with $(2k\eta)^{-1} = 28.8\ fm$ and attractive short range potential with $\beta = 1.1\ fm^{-1}$ and $\lambda = -2.405\ fm^{-3}$. The values of $f_Y(k,q)$ and $f_{CY}(k,q)$ provide a basis for investigating the role of long range Coulomb interaction in pp scattering off the energy shell. Real parts of both $f_Y(k,q)$ and $f_{CY}(k,q)$ are positive while imaginary parts of them are negative over entire range of q. The quantity $f_Y(k,q)$ is a continuous function of q. Real part of $f_Y(k,q)$ increases whereas $\mathrm{Im}\, f_Y(k,q)$ decreases smoothly with q for E_{Lab}=10, 20 & 30 MeV. But $f_{CY}(k,q)$ exhibits a discontinuity at the on-shell point $q = k$. For $q < k$, $\mathrm{Re} f_{CY}(k,q)$ increases with q and reaches discontinuity at $q = k$. Beyond

Table 1. Off-shell Jost function for Yamaguchi and Coulomb plus Yamaguchi potentials as a function of off-shell momentum q for E_{Lab}=10 MeV. The number in the braces stand for the power of 10 with which entries in the table should be multiplied.

$q\ fm^{-1}$	Re $f_Y(k,q)$	Im$f_Y(k,q)$	Re$f_{CY}(k,q)$	Im$f_{CY}(k,q)$
0.01	9.8017(-02)	-8.1998(-03)	1.5026(-01)	-4.6087(-02)
0.05	9.9802(-02)	-4.0918(-02)	1.5471(-01)	-7.2271(-02)
0.10	1.0534(-01)	-8.1333(-02)	1.6281(-01)	-1.0448(-01)
0.15	1.1441(-01)	-1.2076(-01)	1.7381(-01)	-1.3563(-01)
0.20	1.2681(-01)	-1.5876(-01)	1.8783(-01)	-1.6510(-01)
0.25	1.4225(-01)	-1.9494(-01)	2.0524(-01)	-1.9199(-01)
0.30	1.6039(-01)	-2.2898(-01)	2.2770(-01)	-2.1387(-01)
0.32	1.6833(-01)	-2.4194(-01)	2.3989(-01)	-2.1901(-01)
0.33	1.7242(-01)	-2.4827(-01)	2.4797(-01)	-2.1918(-01)
0.34	1.7661(-01)	-2.5450(-01)	2.6035(-01)	-2.1390(-01)
q=k	1.7968(-01)	-2.5895(-01)	——	——
0.35	1.8087(-01)	-2.6063(-01)	2.3322(-01)	-1.7753(-01)
0.37	1.8963(-01)	-2.7258(-01)	2.2000(-01)	-2.1450(-01)
0.40	2.0329(-01)	-2.8971(-01)	2.2309(-01)	-2.4226(-01)
0.45	2.2726(-01)	-3.1612(-01)	2.3792(-01)	-2.7798(-01)
0.50	2.5240(-01)	-3.3982(-01)	2.5772(-01)	-3.0780(-01)
0.55	2.7835(-01)	-3.6082(-01)	2.8000(-01)	-3.3351(-01)
0.60	3.0478(-01)	-3.7921(-01)	3.0381(-01)	-3.5574(-01)
0.65	3.3140(-01)	-3.9508(-01)	3.2853(-01)	-3.7487(-01)
0.70	3.5795(-01)	-4.0858(-01)	3.5369(-01)	-3.9117(-01)
0.75	3.8421(-01)	-4.1986(-01)	3.7894(-01)	-4.0490(-01)
0.80	4.1001(-01)	-4.2909(-01)	4.0402(-01)	-4.1627(-01)
0.85	4.3519(-01)	-4.3644(-01)	4.2871(-01)	-4.2552(-01)
0.90	4.5966(-01)	-4.4210(-01)	4.5285(-01)	-4.3284(-01)
0.95	4.8332(-01)	-4.4623(-01)	4.7632(-01)	-4.3845(-01)
1.00	5.0611(-01)	-4.4899(-01)	4.9904(-01)	-4.4253(-01)

Table 2. Off-shell Jost function for Yamaguchi and Coulomb plus Yamaguchi potentials as a function of off-shell momentum q for E_{Lab}=20 MeV. The number in the braces stand for the power of 10 with which entries in the table should be multiplied.

$q\ fm^{-1}$	Re $f_Y(k,q)$	Im$f_Y(k,q)$	Re$f_{CY}(k,q)$	Im$f_{CY}(k,q)$
0.01	1.4076(-01)	-7.8113(-03)	1.8792(-01)	-4.2041(-02)
0.05	1.4246(-01)	-3.8979(-02)	1.9129(-01)	-6.8787(-02)
0.10	1.4773(-01)	-7.7479(-02)	1.9818(-01)	-1.0178(-01)
0.15	1.5638(-01)	-1.1504(-01)	2.0801(-01)	-1.3388(-01)
0.20	1.6819(-01)	-1.5124(-01)	2.2064(-01)	-1.6468(-01)
0.25	1.8289(-01)	-1.8571(-01)	2.3596(-01)	-1.9377(-01)
0.30	2.0018(-01)	-2.1813(-01)	2.5381(-01)	-2.2074(-01)
0.35	2.1969(-01)	-2.4828(-01)	2.7418(-01)	-2.4511(-01)
0.40	2.4105(-01)	-2.7598(-01)	2.9736(-01)	-2.6603(-01)
0.45	2.6388(-01)	-3.0114(-01)	3.2519(-01)	-2.8079(-01)
0.46	2.6859(-01)	-3.0586(-01)	3.3203(-01)	-2.8208(-01)
0.49	2.8297(-01)	-3.1941(-01)	3.7636(-01)	-2.5285(-01)
q=k	2.8348(-01)	-3.1987(-01)	——	——
0.50	2.8783(-01)	-3.2371(-01)	3.2270(-01)	-2.5614(-01)
0.51	2.9272(-01)	-3.2792(-01)	3.2030(-01)	-2.6942(-01)
0.53	3.0259(-01)	-3.3602(-01)	3.2273(-01)	-2.8657(-01)
0.55	3.1255(-01)	-3.4742(-01)	3.2824(-01)	-2.9974(-01)
0.60	3.3773(-01)	-3.6124(-01)	3.4675(-01)	-3.2589(-01)
0.65	3.6308(-01)	-3.7636(-01)	3.6814(-01)	-3.4673(-01)
0.70	3.8837(-01)	-3.8922(-01)	3.9078(-01)	-3.6391(-01)
0.75	4.1339(-01)	-3.9996(-01)	4.1392(-01)	-3.7815(-01)
0.80	4.3796(-01)	-4.0875(-01)	4.3714(-01)	-3.8985(-01)
0.85	4.6196(-01)	-4.1576(-01)	4.6016(-01)	-3.9933(-01)
0.90	4.8526(-01)	-4.2115(-01)	4.8275(-01)	-4.0685(-01)
0.95	5.0780(-01)	-4.2508(-01)	5.0479(-01)	-4.1264(-01)
1.00	5.2952(-01)	-4.2771(-01)	5.2615(-01)	-4.1689(-01)

this, on-shell point ($q > k$) Re$f_{CY}(k,q)$ reaches its minimum at $q = 0.37$ (10 MeV), 0.51(20 MeV) and 0.62 (30 MeV) and then increases smoothly with q. In contrast, for $q < k$ Im$f_{CY}(k,q)$ initially decreases with q, reaches its minimum values at $q = 0.33$ (10 MeV), 0.46 (20 Mev) and 0.57 (30 MeV) then increases and reaches its discontinuity at $q = k$. For $q > k$, Im$f_{CY}(k,q)$decreases with q. The values of $f_Y(k,q)$ and $f_{CY}(k,q)$ differ much for lower values of energy and momentum (q) the difference becomes insignificant for large energy and momentum values as the Coulomb distortion is predominant [24] at low and intermediate range of these parameters. The results for $f_{CY}(k,q)$ exhibit characteristic discontinuity at the energy-shell, arising from the fact that the Coulomb potential distorts not only the scattered wave but also the incident plane wave [25]. Sharma and Jain [26] and Kok et al [27] confirmed that off-shell effects arte sizeable for (α, 2α) reaction. The class of reactions like (p, 2p) and (p, p) Bremsstrahlung are believed to probe the off-shell two-nucleon force directly. The results in Tables 1, 2 and 3 are in order for (p, 2p) reaction in which a single proton is knocked out of the nucleus and the momentum transfer distribution is measured [28].

Table 3. Off-shell Jost function for Yamaguchi and Coulomb plus Yamaguchi potentials as a function of off-shell momentum q for E_{Lab}=30 MeV. The number in the braces stand for the power of 10 with which entries in the table should be multiplied.

$q\ fm^{-1}$	Re $f_Y(k,q)$	Im$f_Y(k,q)$	Re$f_{CY}(k,q)$	Im$f_{CY}(k,q)$
0.01	1.7963(-01)	-7.4579(-03)	2.2464(-01)	-3.8040(-02)
0.05	1.8126(-01)	-3.7216(-02)	2.2747(-01)	-6.4317(-02)
0.10	1.8629(-01)	-7.3973(-02)	2.3368(-01)	-9.6749(-020
0.15	1.9454(-01)	-1.0983(-01)	2.4277(-01)	-1.2834(-01)
0.20	2.0582(-01)	-1.4440(-01)	2.5460(-01)	-1.5870(-01)
0.25	2.1986(-01)	-1.7730(-01)	2.6899(-01)	-1.8748(-01)
0.30	2.3637(-01)	-2.0826(-01)	2.8570(-01)	-2.1438(-01)
0.35	2.5499(-01)	-2.3705(-01)	3.0450(-01)	-2.3911(-01)
0.40	2.7538(-01)	-2.6350(-01)	3.2521(-01)	-2.6139(-01)
0.45	2.9719(-01)	-2.8751(-01)	3.4770(-01)	-2.8089(-01)
0.50	3.2005(-01)	-3.0907(-01)	3.7212(-01)	-2.9698(-01)
0.55	3.4365(-01)	-3.2817(-01)	3.9960(-01)	-3.0777(-01)
0.57	3.5323(-01)	-3.3514(-01)	4.1263(-01)	-3.0888(-01)
0.60	3.6769(-01)	-3.4490(-01)	4.5192(-01)	-2.8007(-01)
q=k	3.68389-01)	-3.4534(-01)	——	——
0.61	3.7253(-01)	-3.4796(-01)	4.0316(-01)	-2.8065(-01)
0.62	3.7737(-01)	-3.5094(-01)	4.0146(-01)	-2.9305(-01)
0.65	3.9190(-01)	-3.5933(-01)	4.0745(-01)	-3.1367(-01)
0.70	4.1604(-01)	-3.7161(-01)	4.2520(-01)	-3.3560(-01)
0.75	4.3993(-01)	-3.8187(-01)	4.4554(-01)	-3.5180(-01)
0.80	4.6339(-01)	-3.9026(-01)	4.6670(-01)	-3.6455(-01)
0.85	4.8630(-01)	-3.9695(-01)	4.8801(-01)	-3.7467(-01)
0.90	5.0855(-01)	-4.0209(-01)	5.0911(-01)	-3.8263(-01)
0.95	5.3007(-01)	-4.0585(-01)	5.2980(-01)	-3.8875(-01)
1.00	5.5080(-01)	-4.0836(-01)	5.4993(-01)	-3.9329(-01)

In contrast to our earlier methods to off-shell Jost solution [12, 13] for coulomb-like potential the present approach is entirely different and much more simpler. It represents a straightforward approach to deal with off-shell scattering by Coulomb and Coulomb-like potentials. The method is applicable for a Coulomb plus separable potential of arbitrary rank.

References

[1] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley and Sons. 1964).

[2] M. Gell-Mann and M. L. Goldberger, *Phys. Rev.*, **91**, (1953), 398.

[3] Y. Yamaguchi, *Phys. Rev.*, **95,** (1954), 1628.

[4] H. van Haeringen and R. van Wageningen, *J. Math. Phys.*, **16**, (1975), 1441.

[5] K. M. Watson, J. Nuttall and J. S. R. Chisholm, *Topics in Several Particle Dynamics* (Holden-Day. 1967).

[6] G. E. Brown, *Rev. Mod. Phys.*, **43**, (1971), 1.

[7] J. M. Eisenberg and W. Greiner, *Microscopic Theory of the Nucleus* (North-Holland Publishing Company. 1972).

[8] A. N. Mitra, in *Advances in Nuclear Physics*, ed. by M. Baranger and E. Vogt (Plenum Press. 1969) Vol. 3.

[9] U. Laha, Ph. D Thesis, Visva-Bharati University, India (1987).

[10] U. Laha and B. Kundu, *Phys. Rev.*, **A71,** (2005), 032721.

[11] U. Laha, *J. Phys. A: Math. Gen.*, **38,** (2005), 6141.

[12] U. Laha, *Phys. Rev. A***, 74,** (2006), 012710.

[13] U. Laha, *Pramana J. Phys.*, **72,** (2009), 457.

[14] M. G. Fuda and J. S. Whiting, *Phys. Rev. C,* **8,** (1973), 1255.

[15] R. G. Newton, *Scattering Theory of Waves and Particles*, (McGraw-Hill. 1982).

[16] B. Talukdar, D. K. Ghosh and T. Sasakawa, *J. Math. Phys.*, **23,** (1982), 1700.

[17] U. Laha and B. Talukdar, *Pramana J. Phys.*, **36,** (1991), 289.

[18] A. Erdelyi, *Higher Transcendental Functions,* **vol. 1,** (McGraw-Hill. 1953).

[19] A. W. Babister, *Transcendental Functions Satisfying Non-Homogeneous Linear Differential equations,* (Macmillan. 1967).

[20] H. van Haeringen, *J. Math. Phys.*, **24,** (1983), 2467.

[21] D. K. Ghosh, S. Saha, K. Niyogi and B. Talukdar, *Czech. J. Phys.*, **B33**, (1983), 528.

[22] H. van Haeringen, *J. Math. Phys.*, **24**, (1983), 246.

[23] B. Talukdar, D. K. Ghosh and T. Sasakawa, *J. Math. Phys.*, **25**, (1984), 323.

[24] W. Plessas, L. Streit and H. Zingl, *Acta Phys. Austr.*, **40**, (1974), 272.

[25] W. F. Ford, *Phys. Rev.*, **B133**, (1964), 1116.

[26] N. R. Sharma and B. K. Jain, *Nucl. Phys.*, **A377**, (1982), 201.

[27] L. P. Kok, J. E. Holwerda and J. W. de Maag, *Phys. Rev. C*, **27**, (1983), 2548.

[28] I. E. McCarthy, *Introduction to Nuclear Theory,* (John Wiley. 1968).