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Unsteady Free Convection Flow with Thermal Radiation Past a Vertical Porous Plate with Newtonian Heating

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Abstract

This paper considers exact solutions of unsteady free convection flow with thermal radiation past a vertical porous plate with Newtonian heating. The technique of Laplace transform was employed in deriving the solutions to the governing flow and energy equations. A parametric study of all parameters involved was conducted, and a representative set of results showing the effect of the radiation parameter N, the ratio of uniform suction to characteristic velocity parameter R, and the free convection parameter Gr on the velocity and temperature are illustrated graphically. The skin friction or shear and heat flux are discussed quantitatively. It is evident that the results reveal the characteristics of the problem.

Key Words: Boundary layer; Incompressible; Optically thin thermally radiating fluid; Unsteady free convection flow.
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1. Introduction

Natural or free convection flow arises when a heated object is placed in a quiescent fluid; and with flow of heat the density of the fluid varies with temperature. The inducement of free convection fluid motion is due to buoyancy effects. Such flows have been studied most extensively because they are found frequently in nature as well as in engineering and environmental applications. Ghoshdastidar [1] gave various areas of typical applications of free convection such as those found in heat transfer from pipes and transmission lines as well as from various electronic devices, dissipation of heat from the coil of a refrigerator unit to the surrounding air, heat transfer from a heater to room air, heat transfer from nuclear fuel rods to the surrounding coolant, heated and cooled enclosures, quenching, wire-drawing, and extrusion, atmospheric and oceanic circulation.

Buoyancy-induced flows over porous materials enhances heat transfer. These are encountered in a wide range of thermal engineering applications such as in geothermal systems, oil extraction, ground water pollution, thermal insulation, heat exchangers, storage of nuclear wastes, packed bed catalytic reactors, and many more [2, 3, 4]. Most of these applications exhibit high temperature phenomena or high-power radiation sources such as those found in electrical power generation, astrophysical flows, solar power technology, space vehicle reentry, nuclear engineering applications [5, 6, 7].

Chaudhary and Jain [8] investigated the problem of unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating. Their study highlighted the usefulness of unsteady boundary layer flows, and discussed the problem of Newtonian heating and its applications. Lesnic et al. [9], Pop et al. [10], Lesnic et al. [11], Pop et al. [12] and Lesnic et al. [13] have worked extensively on free convection boundary-layer flows over surfaces in porous media with Newtonian heating. In all of these valuable references cited, the effect of radiative heat transfer on the problem of Newtonian heating was not considered. Radiative heat transfer, however, is fundamentally very important in many aspects of practical engineering. Examples are the solar radiation in buildings, foundry engineering and solidification processes, die forging, chemical engineering, composite structures applied in industry. Korycki [14] stated that in textiles (i.e. industrial textiles, textiles designed for use under hermetic protective barrier, multilayer clothing materials, etc.) and in textile structures (i.e. needle heating in heavy industrial sewing), radiative heat transfer problems are encountered as well. These are conjugate heat transfer processes, where Newtonian heating finds its engineering applications. Evidently, high temperature phenomena cannot be ignored. Therefore, it is more realistic to study the problem of unsteady free convection flow with radiative heat transfer past a vertical porous plate with Newtonian heating. This gives a complementary study of the earlier work of Chaudhary and Jain [8], taking into account the effects of radiative heat transfer and suction. This is the objective of the present paper.

In section 2, the mathematical formulation of the problem and the non-dimensional form of the governing equations are established. Solutions to these equations are obtained in section 3. The results of the previous sections are discussed in section 4. In section 5, general concluding remarks of the results of the previous sections are given.

2. Mathematical Formulation

Following the arguments presented by Chaudhary and Jain [8], we consider buoyancy induced unsteady free convection flow with radiative heat transfer of a viscous incompressible fluid past an impulsively started infinite vertical porous plate with Newtonian heating. The x-axis is taken along the plate in the vertically upward direction and y-axis is chosen normal to the plate. Initially, for time $t \leq 0$, the plate and fluid are at the same temperature T_∞ in a stationary condition. At time $t > 0$, the plate is given an impulsive motion in the vertically upward direction against gravitational field with a characteristic velocity U_c, while fluid is sucked from the plate with $v = -v_0$, where $v_0 > 0$ is the constant suction velocity and $v_0 < 0$ is the injection velocity. It is assumed that rate of heat transfer from the surface is proportional to the local surface temperature T. Since the plate is considered infinite in the x direction, hence all physical variables will be independent of x. Therefore, the physical variables are functions of y and t only. With the aid of the Boussinesq approximation, the governing equations of the flow for an optically thin medium are then reduced to the following system of equations:

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \beta\, g \Big(T - T_\infty \Big), \tag{1a}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \alpha_d \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}, \tag{1b}$$

$$\frac{\partial q}{\partial y} = 4\sigma\alpha \Big(T^4 - T_\infty^4 \Big), \tag{1c}$$

where u, v, t, ρ, ν $(= \mu/\rho)$, β, T, c_p, k, σ, α_d $(= k/\rho\, c_p)$, α and q represents the fluid velocity along x-axis, flow along y-axis, time, fluid density, kinematic viscosity, volumetric coefficient of thermal expansion, temperature, heat capacity at a constant pressure, thermal conductivity, the Stefan-Boltzmann's constant, thermal diffusivity, penetration depth or absorption coefficient and heat flux parameter, respectively. g is the acceleration due to gravity.

The corresponding initial and boundary conditions are:

$$t \le 0 : u = 0, \quad T = T_\infty \quad \text{for all} \quad y > 0, \tag{2a}$$

$$t > 0 : u = U_{\mathrm{c}}, \quad \frac{\partial T}{\partial y} = -\frac{h}{k} T \quad \text{on} \quad y = 0, \tag{2b}$$

$$u = 0, \quad T = T_\infty \quad \text{as} \quad y \to \infty, \tag{2c}$$

where h is the convective heat transfer coefficient.

The equation (1c) governing the radiative flux is generally nonlinear. However, under the condition that the radiative flux reflects the notion of an optically thin environment, such as one would find in the intergalactic layers where the plasma gas is assumed to be of low density and $\alpha << 1$ [15], the nonlinearity may be eliminated. In this case there will be no measurable change in temperature elsewhere within the plate and the equilibrium temperature. Thus, in a one space coordinate y, the linear differential approximation of Cogley-Vincenti-Gilles equilibrium model [16] of the radiative flux q

$$\frac{\partial q}{\partial y} = 16\, \sigma\, \alpha\, T_\infty^3 \Big(T - T_\infty \Big) \tag{3}$$

becomes significant. It is emphasized here that equation (3) is widely used in computational fluid dynamics involving radiation absorption problems [17, 18] in expressing the term T^4 as a linear function.

The following non-dimensional quantities are employed to facilitate the analysis:

$$Y = \frac{yU_{\mathrm{c}}}{\nu}\,, \quad U = \frac{u}{U_{\mathrm{c}}}\,, \quad \tau = \frac{tU_{\mathrm{c}}^2}{\nu}\,, \quad \Theta = \frac{T - T_\infty}{T_\infty}\,,$$
$$R = \frac{v_0}{U_{\mathrm{c}}}\,, \quad Gr = \frac{\nu\beta g T_\infty}{U_{\mathrm{c}}^3}\,, \quad Pr = \frac{\nu}{\alpha_d}\,, \quad N = \frac{16\alpha\sigma\nu T_\infty^3}{\rho c_p U_{\mathrm{c}}^2}. \tag{4}$$

The non-dimensional scalings depict that the characteristic velocity U_{c} is defined by $\frac{h\nu}{k}$. Therefore, the parameter R, gives the ratio of uniform suction velocity to characteristic velocity, thereby indicating the significance of suction in the problem. That is, $R > 0$ physically indicates suction and $R < 0$ injection or blowing. The other parameters entering the problem are Gr, the Grashof number; Pr, the Prandtl number; and N, the radiation parameter.

The non-dimensional governing equations are, therefore, given by

$$\frac{\partial U}{\partial \tau} - R\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\,\Theta \tag{5a}$$

$$Pr\,\frac{\partial \Theta}{\partial \tau} - Pr\,R\frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial Y^2} - Pr\,N\Theta, \tag{5b}$$

such that

$$\tau \le 0 : U = 0 = \Theta \quad \text{for all} \quad Y > 0, \tag{6a}$$

$$\tau > 0 : U = 1, \quad \frac{\partial \Theta}{\partial Y} = -(1+\Theta) \quad \text{on} \quad Y = 0, \tag{6b}$$

$$U = 0, \quad \Theta = 0 \quad \text{as} \quad Y \to \infty \tag{6c}$$

are the associated initial and boundary conditions. Equations (5) differ from those in Chaudhary and Jain [8] by way of the additional terms that involve the radiation parameter (N) and the ratio of uniform suction to characteristic velocity parameter (R).

3. Method of Solution

Exact solutions of equations (5) subject to equations (6) are herein deduced by using the Laplace Transform technique [19]. The energy equation (5b) is uncoupled from the momentum equation (5a). One can advance solution for the temperature variable $\Theta(Y,\tau)$ whereupon the solution of $U(Y,\tau)$ is then derived. Therefore, the solution for the velocity and temperature are, respectively, given as follows:

$$\begin{aligned} U(Y,\tau) = e^{-\frac{RY}{2}} \Bigg\{ & \frac{1}{2}\left[e^{\frac{RY}{2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} + \frac{R\sqrt{\tau}}{2} \right) + e^{-\frac{RY}{2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} - \frac{R\sqrt{\tau}}{2} \right) \right] \\ & -\frac{\alpha_9}{2}\left[e^{\frac{RY}{2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} + \frac{R\sqrt{\tau}}{2} \right) + e^{-\frac{RY}{2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} - \frac{R\sqrt{\tau}}{2} \right) \right] \\ & -\frac{\alpha_{10} e^{\alpha_1^2\tau - \alpha_2\tau}}{2}\left[e^{Y\sqrt{\frac{R^2}{4}+\alpha_1^2-\alpha_2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} + \sqrt{\left(\frac{R^2}{4}+\alpha_1^2-\alpha_2\right)\tau} \right) \right. \\ & \left. + e^{-Y\sqrt{\frac{R^2}{4}+\alpha_1^2-\alpha_2}} \mathrm{erfc}\left(\frac{Y}{2\sqrt{\tau}} - \sqrt{\left(\frac{R^2}{4}+\alpha_1^2-\alpha_2\right)\tau} \right) \right] \end{aligned}$$

$$+\frac{\alpha_{11}e^{-\alpha_7\tau}}{2}\left[e^{Y\sqrt{\frac{R^2}{4}-\alpha_7}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2\sqrt{\tau}}+\sqrt{\left(\frac{\mathrm{R}^2}{4}-\alpha_7\right)\tau}\right)\right.$$

$$\left.+e^{-Y\sqrt{\frac{R^2}{4}-\alpha_7}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2\sqrt{\tau}}-\sqrt{\left(\frac{\mathrm{R}^2}{4}-\alpha_7\right)\tau}\right)\right]$$

$$+\frac{\alpha_{12}e^{-\alpha_8\tau}}{2}\left[e^{Y\sqrt{\frac{R^2}{4}-\alpha_8}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2\sqrt{\tau}}+\sqrt{\left(\frac{\mathrm{R}^2}{4}-\alpha_8\right)\tau}\right)\right.$$

$$\left.\left.+e^{-Y\sqrt{\frac{R^2}{4}-\alpha_8}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2\sqrt{\tau}}-\sqrt{\left(\frac{\mathrm{R}^2}{4}-\alpha_8\right)\tau}\right)\right]\right\}$$

$$+\frac{\alpha_9}{2}\left[e^{Y\sqrt{Pr\alpha_2}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\sqrt{\mathrm{Pr}\,\tau}\right)+\mathrm{e}^{-\mathrm{Y}\sqrt{\mathrm{Pr}\alpha_2}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}-\sqrt{\mathrm{Pr}\,\tau}\right)\right]$$

$$+\alpha_{10}e^{-\alpha_2\tau}\left[\frac{e^{-\frac{YPr}{4\tau}}}{\sqrt{\pi\tau}}-\alpha_1 e^{\alpha_1 Y\sqrt{Pr}+\alpha_1^2\tau}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\alpha_1\sqrt{\tau}\right)\right]$$

$$-\frac{\alpha_{11}e^{-\alpha_7\tau}}{2}\left[e^{Y\sqrt{Pr(\alpha_2-\alpha_7)}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\sqrt{(\alpha_2-\alpha_7)\tau}\right)\right.$$

$$\left.+e^{-Y\sqrt{Pr(\alpha_2-\alpha_7)}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}-\sqrt{(\alpha_2-\alpha_7)\tau}\right)\right]$$

$$-\frac{\alpha_{12}e^{-\alpha_8\tau}}{2}\left[e^{Y\sqrt{Pr(\alpha_2-\alpha_8)}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\sqrt{(\alpha_2-\alpha_8)\tau}\right)\right.$$

$$\left.+e^{-Y\sqrt{Pr(\alpha_2-\alpha_8)}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}-\sqrt{(\alpha_2-\alpha_8)\tau}\right)\right], \tag{7a}$$

$$\Theta(Y,\tau)=\frac{e^{-\frac{PrRY}{2}}}{2\sqrt{Pr}\left(\alpha_1+\sqrt{\alpha_2}\right)}\times$$

$$\times\left[e^{\sqrt{Pr\alpha_2}Y}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\sqrt{\alpha_2\tau}\right)+\mathrm{e}^{-\sqrt{\mathrm{Pr}\alpha_2}\mathrm{Y}}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}-\sqrt{\alpha_2\tau}\right)\right]$$

$$+\frac{e^{-\frac{PrRY}{2}-\alpha_2\tau}}{\sqrt{Pr}\left(\alpha_1^2-\alpha_2\right)}\left[\frac{e^{-\frac{PrY^2}{4\tau}}}{\sqrt{\pi\tau}}-\alpha_1 e^{\alpha_1\sqrt{Pr}Y+\alpha_1^2\tau}\mathrm{erfc}\left(\frac{\mathrm{Y}}{2}\sqrt{\frac{\mathrm{Pr}}{\tau}}+\alpha_1\sqrt{\tau}\right)\right], \tag{7b}$$

where

$$\alpha_1 = \frac{R\sqrt{Pr}}{2} - \frac{1}{\sqrt{Pr}}, \quad \alpha_2 = \frac{R^2 Pr}{4} + N, \quad \alpha_3 = \frac{Gre^{-\frac{PrRY}{2}}}{\sqrt{Pr}},$$

$$\alpha_4 = \frac{Pr-1}{\sqrt{Pr}}, \quad \alpha_5 = \frac{2\alpha_2\alpha_4\sqrt{Pr}-1}{\alpha_4^2}, \quad \alpha_6 = \frac{\alpha_2^2 Pr - 1}{\alpha_4^2},$$

$$\alpha_7 = \frac{\alpha_5}{2} - \frac{\sqrt{\alpha_5^2 - 4\alpha_6}}{2}, \quad \alpha_8 = \frac{\alpha_5}{2} + \frac{\sqrt{\alpha_5^2 - 4\alpha_6}}{2}, \quad \alpha_9 = \frac{\alpha_3}{\sqrt{Pr}\alpha_7\alpha_8\left(\alpha_1 + \sqrt{\alpha_2}\right)},$$

$$\alpha_{10} = \frac{\alpha_3}{\sqrt{Pr}\left(\alpha_1^2 - \alpha_2\right)\left(\alpha_1^2 - \alpha_2 + \alpha_7\right)\left(\alpha_1^2 - \alpha_2 + \alpha_8\right)},$$

$$\alpha_{11} = \frac{\alpha_3}{\sqrt{Pr}\alpha_7\left(\alpha_1 + \sqrt{\alpha_2 - \alpha_7}\right)\left(\alpha_8 - \alpha_7\right)},$$

$$\alpha_{12} = \frac{\alpha_3}{\sqrt{Pr}\alpha_8\left(\alpha_1 + \sqrt{\alpha_2 - \alpha_8}\right)\left(\alpha_7 - \alpha_8\right)}. \tag{8}$$

$\mathrm{erfc}(\cdot)$ denotes the complementary error function.

We observe that the solution for the velocity variable given by the equation (7a) is not valid for fluids for which the Prandtl number equals unity. As the Prandtl number is a measure of the relative importance of the viscosity and thermal conductivity of the fluid, the case $Pr = 1$ corresponds to those fluids whose momentum and thermal boundary layer thickness are of the same order of magnitude. The exact solution of the free convection problem when $Pr = 1$ has to be re-derived from equation (5). For lack of space, the solution for the velocity when $Pr = 1$ is not considered.

From the physical point of view, it is necessary to know the skin friction or shear on the plate wall. By virtue of equations (4), it is given by

$$\tau_s = \frac{\partial U}{\partial Y}|_{Y=0}\,. \tag{9}$$

Knowing the temperature distribution, we can calculate the rate of heat flux, q_w, between the fluid and the wall of the plate. This is calculated from

$$q_w = \frac{\partial \Theta}{\partial Y}|_{Y=0}\,, \tag{10}$$

by virtue of equations (4).

It is emphasized here that solution expressions for the skin friction (9) and the rate of heat flux (10) are not given here, but are discussed quantitatively for variations of radiation parameter in the range $1 \le N \le 5$ and suction in the range $1 \le R \le 5$ for the range of time $0.1 \le \tau \le 0.4$ in the next section.

4. Discussion of Results

The problem of an unsteady free convection flow with an optically thin incompressible thermally radiating fluid past a vertical porous plate with Newtonian heating has been solved exactly. It is observed that the inclusion of the radiation and the suction terms to the problem studied by Chaudhary and Jain [8], greatly added value to the solution. Hence, the discussions emanating from this study are on the radiation and suction effects, which happen to be the primary interests. The solutions (7) are discussed with the aid of graphs based on certain range of values of the parameters entering the problem. Typical values of the parameters used for the computations are indicated on the graphs. In particular, as the Prandtl number Pr is a measure of the relative importance of the viscosity and thermal diffusivity of the fluid, it is set equal to a fixed value of 0.71 throughout the investigations, which physically corresponds to an astrophysical body (air) at 20 °C.

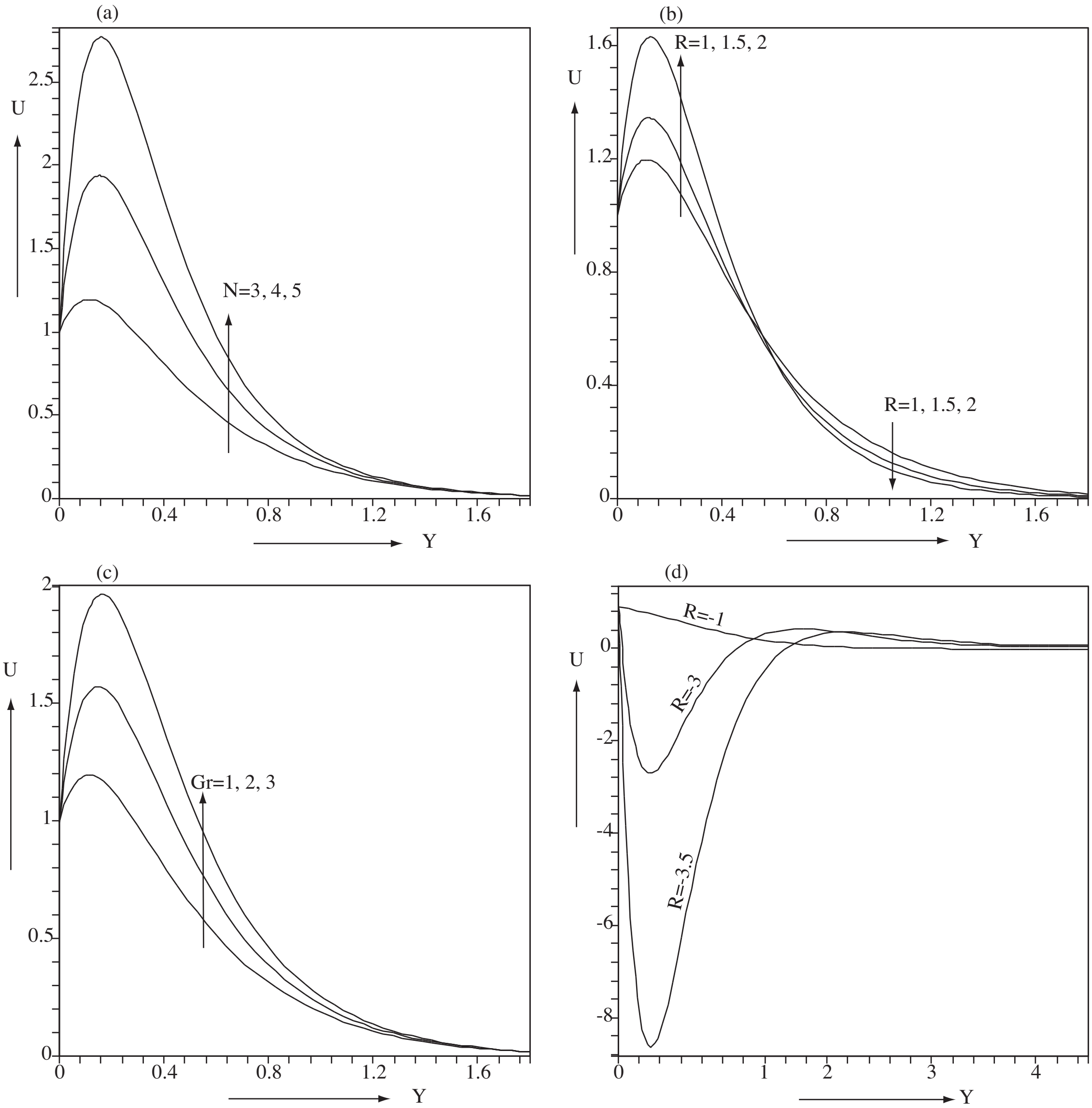

Figure 1. Velocity profiles as a function of Y for variations in (a) radiation parameter: $\tau = 0.4, R = 1, Pr = 0.71, Gr = 1$; (b) suction: $\tau = 0.4, N = 3, Pr = 0.71, Gr = 1$; (c) free convection parameter: $\tau = 0.4, N = 3, Pr = 0.71, R = 1$; (d) blowing: $\tau = 0.4, N = 3, Pr = 0.71, Gr = 1$.

Figure 1 is obtained from the flow solution (7a) for various flow parameters. It shows changes in (a) radiation parameter, (b) suction, (c) free convection parameter affect the fluid flow, and (d) blowing. Evidently, the velocity increases steadily with increase in the parameters, reaches a maximum and then decreases to zero at the edge of the boundary layer. Also, the velocity profiles exhibit large overshoots for increasing values of the parameters. In general, the velocity distribution in the layer tends asymptotically to a similarity form. However, for blowing, where the velocity is negative, the boundary layer is completely drifted away from the plate (Figure 1(d)).

The second set of graphs, Figure 2 displays the evolution of temperature due to variations in (a) radiation parameter, (b) suction, and (c) blowing. It is observed that increase in radiation parameter and suction (see Figure 2(a, b)), decreases the temperature exponentially. Consequently, these parameters produce an inward flow of heating that accelerates the flux of heat to the plate. On the other hand, blowing retards the flux of heat to the plate (see Figure 2 (c)). Thus, as expected, blowing causes a reduction in heat transfer, while suction causes an increase. The asymptotic nature of the graphs is obvious as each of the values of $\Theta \to 0$ as $Y \to \infty$.

Table 1. Skin friction for variations of radiation parameter and time: $R = 1,\ Pr = 0.71,\ Gr = 1$.

N	$\tau = 0.1$ Skin Friction	$\tau = 0.2$ Skin Friction	$\tau = 0.3$ Skin friction	$\tau = 0.4$ Skin friction
1.0	−1.0509	−1.2334	−1.0905	−0.5536
1.5	−2.1146	−1.7053	−1.3546	−0.3619
2.0	−2.2551	−1.7585	−1.3094	0.3526
2.5	−2.2940	−1.7659	−1.1962	1.7354
3.0	−2.3086	−1.7611	−1.0181	4.1340
3, 5	−2.3152	−1.7503	−0.7502	8.3839
4.0	−2.3183	−1.7347	−0.3501	15.9241
4.5	−2.3199	−1.7141	0.2478	29.3182
5.0	−2.3207	−1.6875	1.1425	53.1326

Table 2. Skin friction for variations of suction and time: $N = 3,\ Pr = 0.71,\ Gr = 1$.

R	$\tau = 0.1$ Skin Friction	$\tau = 0.2$ Skin Friction	$\tau = 0.3$ Skin friction	$\tau = 0.4$ Skin friction
1.0	−2.3086	−1.7611	−1.0181	4.1340
1.5	−2.6192	−2.0714	−1.1027	6.9759
2.0	−2.9482	−2.4021	−1.0644	12.5148
2.5	−3.2957	−2.7471	−0.7624	23.9949
3.0	−3.6610	−3.0968	0.0853	49.3456
3, 5	−4.0431	−3.4363	2.0778	109.4048
4.0	−4.4408	−3.7413	6.5613	263.1180
4.5	−4.8535	−3.9690	16.7351	690.3793
5.0	−5.2766	−4.0405	40.6421	1984.9161

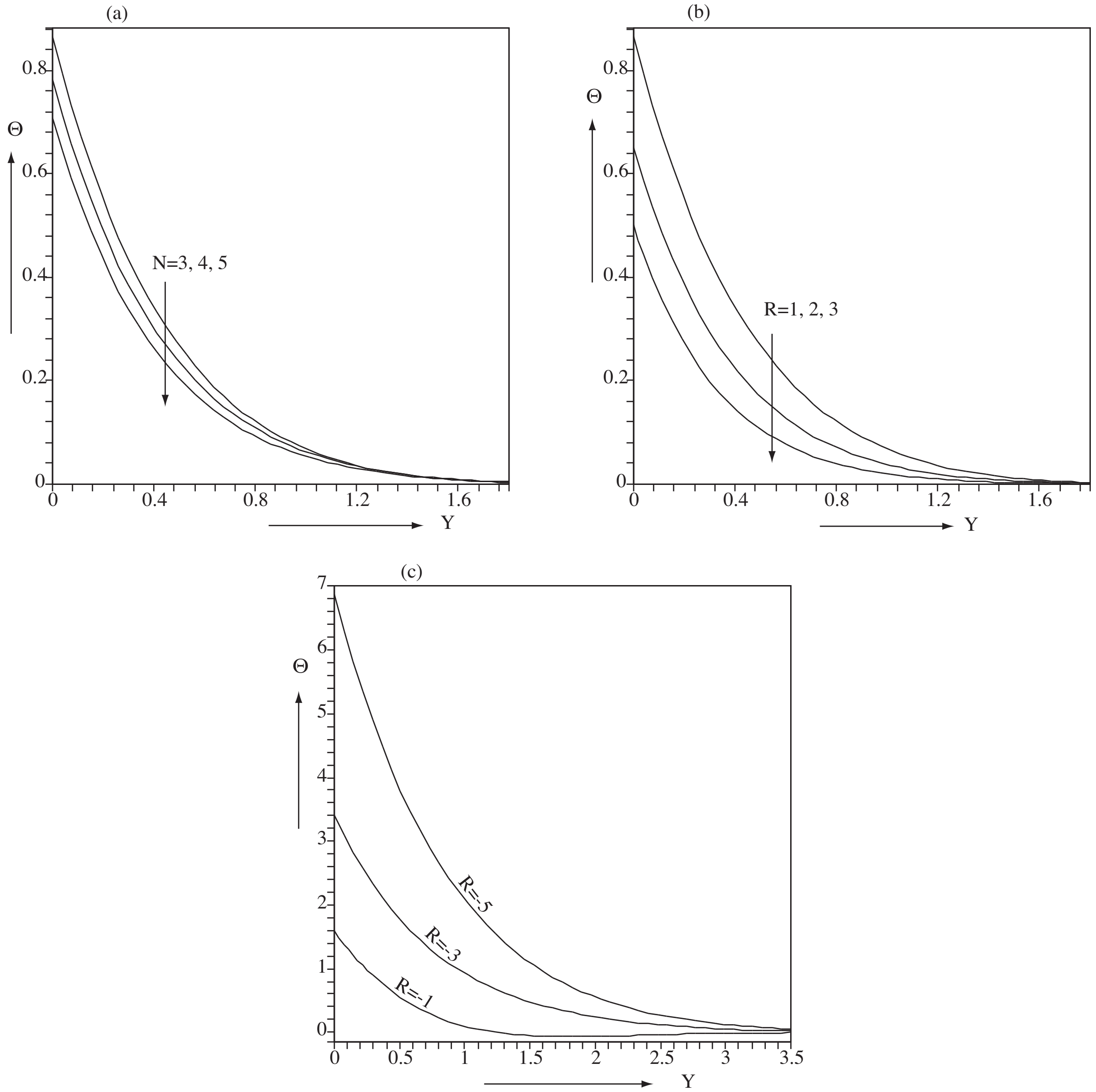

Figure 2. Temperature profiles as a function of Y for variations in (a) radiation parameter: $\tau = 0.4, R = 1, Pr = 0.71$; (b) suction: $\tau = 0.4, N = 3, Pr = 0.71$; (c) blowing: $\tau = 0.4, N = 3, Pr = 0.71$.

Table 1 illustrates the skin friction with respect to radiation parameter and time. It is observed that with increasing radiation parameter in the range $1 \le N \le 5$ at $\tau = 0.1$, and $1 \le N \le 2.5$ at $\tau = 0.2$, skin friction decreases, while for $3 \le N \le 5$ at $\tau = 0.2$, $1.5 \le N \le 5$ at $\tau = 0.3$ and $1 \le N \le 5$ at $\tau = 0.4$, skin friction increases.

Similarly, Table 2 shows the skin friction with respect to variations in suction and time. It is seen that increase in suction decreases the skin friction for $0.1 \le \tau \le 0.2$, while for $1.5 \le N \le 5$ at $\tau = 0.3$ and for $1 \le N \le 5$ at $\tau = 0.4$, the skin friction increases.

Tables 3 and 4, respectively, accounts for effects of variations of radiation parameter and suction with respect to time on heat flux. For $1 \leq N \leq 5$ and $1 \leq R \leq 5$ for time range of $0.1 \leq \tau \leq 0.4$, the heat flux increases. The negative values of the wall temperature gradient, for all values of the dimensionless parameters, are indicative of the physical fact that the heat flows from the plate surface to ambient fluid.

Table 3. Heat flux for variations of radiation parameter and time: $R = 1$, $Pr = 0.71$, $Gr = 1$.

N	$\tau = 0.1$ Heat Flux	$\tau = 0.2$ Heat Flux	$\tau = 0.3$ Heat Flux	$\tau = 0.4$ Heat Flux
1.0	−2.5270	−2.2879	−2.3041	−2.3855
1.5	−2.1267	−2.0358	−2.0991	−2.1922
2.0	−1.9463	−1.9136	−1.9882	−2.0752
2.5	−1.8366	−1.8346	−1.9101	−1.9877
3.0	−1.7604	−1.7765	−1.8490	−1.9167
3, 5	−1.7034	−1.7307	−1.7985	−1.8570
4.0	−1.6585	−1.6928	−1.7554	−1.8056
4.5	−1.6219	−1.6604	−1.7177	−1.7806
5.0	−1.5913	−1.6323	−1.6843	−1.7209

Table 4. Heat flux for variations of suction and time: $N = 3$, $Pr = 0.71$, $Gr = 1$.

R	$\tau = 0.1$ Heat Flux	$\tau = 0.2$ Heat Flux	$\tau = 0.3$ Heat Flux	$\tau = 0.4$ Heat Flux
1.0	−1.7604	−1.7765	−1.8490	−1.9167
1.5	−1.6456	−1.6766	−1.7349	−1.7834
2.0	−1.5588	−1.5947	−1.6402	−1.6742
2.5	−1.4910	−1.5266	−1.5613	−1.5848
3.0	−1.4366	−1.4694	−1.4954	−1.5112
3, 5	−1.3921	−1.4209	−1.4401	−1.4506
4.0	−1.3551	−1.3797	−1.3936	−1.4004
4.5	−1.3238	−1.3443	−1.3542	−1.3586
5.0	−1.2971	−1.3139	−1.3208	−1.3236

5. Concluding Remarks

The problem of unsteady free convection flow of an optically thin incompressible thermally radiating fluid past a vertical porous plate with Newtonian heating has been examined. The effects of radiative heat transfer and suction have been investigated. The results indicate that the radiation parameter and suction have significant influences on velocity and temperature profiles, skin friction and heat flux. The results in this paper are similar to the results of Chaudhary and Jain [8] if there are no radiation effects and the plate is non-porous. It is in general observed that the boundary layers are controlled by the radiation and suction effects. While the boundary layer decreases for suction (similar to radiation effects), the boundary layer increases for injection or blowing.

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