

# DWBA of the Reaction ${}^9\text{Be}(p, \alpha){}^6\text{Li}$ at $E_p = 18.6 \sim 50$ MeV

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## Abstract

The compound nucleus produced in the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  in the incident energy range  $E_p = 18.6 \sim 50$  MeV is mainly occurred when its excitation energy is greater than the  $\alpha$ -particle threshold separation energy by about 20 MeV, strongly suggesting the reaction mechanism is a direct one-step process. To understand this phenomenon, we employ semi-microscopic zero-range distorted-wave Born approximation (DWBA) theory to analyse experimental data obtained in this energy range for the lower three  ${}^6\text{Li}$ -states: ground state ( $1^+ ; 0$ ); 2.186 MeV ( $3^+ ; 0$ ); and 3.563 MeV ( $0^+ ; 1$ ). The Kurath-Millener spectroscopic-factor amplitudes for the triton-cluster transfer are used in the analysis. Both the experimental angular distributions and the absolute values of the differential cross-sections are found to have good correlation with corresponding theoretical predictions. Good coincidence is obtained between the theoretically predicted spectroscopic factors  $S_{JT}^{(p, \alpha)}$  and both experimental and theoretical integrated cross-sections ( $0^\circ$ – $90^\circ$ ) for  $E_p = 22.5, 31$  and  $45$  MeV. Further, the experimental integrated cross-sections for the investigated  ${}^6\text{Li}$ -states agree well with the corresponding theoretical values at  $E_p \geq 22.5$  MeV. In addition, a suitable coincidence is obtained between the calculated and the experimental excitation energies. The agreement between experimental and theoretical data of this reaction can be attributed to the use of the Cohen-Kurath wave functions and the model of calculations. The present study shows that, the incident energy at which the reactions begin to proceed via the direct-component mechanism (i.e., the critical incident energy) varies from state to another.

**Key Words:** Direct mechanism; Differential Cross-sections; Spectroscopic-factor amplitudes; Dependence of the integrated cross-sections on energy.

## 1. Introduction

Nuclear reactions at low incident energies tend to proceed through the compound nucleus formation mechanism. On increasing the incident energy increases, the compound nucleus mechanism component decreases as the direct mechanism increases. At sufficient higher incident energies, the pure direct mechanism dominates. An example is found in the reaction  $(p, \alpha)$  on different 1p shell [1–5] and 2s-1d shell targets [3, 6–9], especially those which lead to the formation of both compound nuclei and final nuclei of  $\alpha$ -type. The incident energy at which the change from compound to direct mechanism begins to appear is called the *critical incident energy*. This critical incident energy mainly corresponds to an excitation energy above the threshold separation energy of the  $\alpha$ -particle by  $\sim 20$  MeV [1–9]. In such cases, all one-particle channels are

opened and production associated with the compound nucleus mechanism decreases remarkably, especially in the backward direction. This may be reflected in the absolute values of the cross-sections and in the shapes of the angular distributions, which normally have forward peaks. The same phenomena is also found for the  $(p, \alpha)$  reaction on other types of targets nuclei with the same characteristics for angular distributions and absolute values of cross-sections [10–13].

Direct transfer nuclear reactions can be described with the help of the distorted-wave Born approximation (DWBA) theory, giving information on the structure of nucleons in the final states of the rest nucleus and on the reaction's mechanism. Among direct transfer reactions, most one-step particle transfer (i.e. stripping or pick-up) processes can be successfully described in view of zero-range DWBA. However, in other cases one must use finite-range DWBA theory. Traditionally, the DWBA-calculations need what is known as spectroscopic-factor amplitudes for the residual-nucleus's final-states. These amplitudes are usually calculated according to one of the known nuclear models (an example is the shell-model). Through these description processes, the forms of angular distributions can be exactly reproduced, and the experimental transition-strengths will be ready for comparison with the theoretical predicted spectroscopic factors. Both processes may be considered as two valid tests for the accuracy of the wave functions used in the spectroscopic-factor calculations as well as for the nuclear model itself. In a previous work [14], the zero-range DWBA [15] is employed to investigate the  $^{11}\text{B}(p, \alpha)^8\text{Be}$  reaction for incident proton energies  $E_p = 15 - 45$  MeV. It is found that, the reaction mechanism changes from pure compound nucleus to the direct mechanism at  $E_p \approx 15$  MeV. In the present work, the same theory is utilized to analyze the experimental data of the  $^9\text{Be}(p, \alpha)^6\text{Li}$  reaction at  $E_p = 18.6 - 50$  MeV. A good fit between experimental angular distributions and the predicted theoretical curves is obtained for the first three  $^6\text{Li}$ -states at higher incident energies. Good fit is also obtained between the experimental and theoretical total cross-sections in the forward direction with the bare spectroscopic factors calculated by Kurath and Millener [16] using the shell-model wave functions from Cohen and Kurath [17], for these three states.

## 2. Experimental Data and Its Previous Analyses

The incident energies of the experimental data for the reaction  $^9\text{Be}(p, \alpha)^6\text{Li}$  which are analysed here in the range from  $E_p = 15.6$  to 50 MeV. The angular distributions for the lower two  $^6\text{Li}$ -states were obtained by Maxson at  $E_p = 15.6$  and 18.6 MeV [18]. Gambarini et al. and Guazzoni et al. [19, 20] used the predictions of the plane-wave Born approximation theory (PWBA) for the four direct mechanisms, namely pick-up, knock-out, heavy particle pick-up and heavy-particle knock-out. They compared them with the experimental angular distributions for the lower two  $^6\text{Li}$ -states obtained in the reaction  $^9\text{Be}(p, \alpha)^6\text{Li}$  at  $E_p = 26.7$  and 38 MeV. The predictions of these four mechanisms were not able to reproduce the patterns of the experimental angular distributions at both of the two incident energies.

The angular distributions for the lower three  $^6\text{Li}$ -states were obtained also by Weng [4, 5] and Walz [12, 13] at  $E_p = 22.5, 31$  and 41 MeV; and by Devries et al. [21] at  $E_p = 45$  MeV. Analysing the experimental data using a cluster form factor in the finite-range DWBA [22], Walz and Devries et al. reproduced the shapes of the ground-state angular distributions in the  $^9\text{Be}(p, \alpha)^6\text{Li}$  reaction, especially at relatively high incident energies. These calculations could be done by the addition of the Heavy particle pick-up (HPPU) process to the normal Light particle pick-up (LPPU) process and show the importance for the inclusion of the heavy-particle pick-up process in the mechanisms of this reaction.

Gurevich et al. [23] carried out the  $^9\text{Be}(p, \alpha)^6\text{Li}$  reaction at  $E_p = 50$  MeV and used a theoretical code via finite-range DWBA to analyse the angular distributions for the lower two  $^6\text{Li}$ -states. Their calculations involve the direct triton cluster (LPPU) and the heavy particle pick-up (HPPU) mechanisms, finding disagreement between experimental and theoretical curves in this case.

Yamashita et al. [24] used a finite-range DWBA code to analyse the experimental data measured by Hauser et al. and Devries et al. [13, 21] and Gurevich et al. [23] for the reaction  $^9\text{Be}(p, \alpha)^6\text{Li}$  at 45

and 50 MeV, respectively. Their formalism includes both direct and exchange modes, in which all one-step DWBA-terms involving transfers of a single particle or bound clusters are treated coherently. The results of Yamashita et al. are inconsistent with the results found previously by Hauser et al. [13] and Devries et al. [21], indicating the heavy-particle pick-up (HPPU) process must be included in the mechanisms of this reaction.

Pellegrini et al. [25] performed the reaction at  $E_p = 25$  and 30 MeV and obtained angular distributions for the lower three  ${}^6\text{Li}$ -states (0.0, 2.186 and 3.563 MeV) at both incident energies. They analysed the experimental angular distributions using a finite-range DWBA code for the one-step three- and five-nucleon transfer processes. The energy dependence of the integrated cross-sections is in good agreement with that predicted via DWBA for the g.s. and 2.186 MeV  ${}^6\text{Li}$ -states, while a disagreement is observed between the integrated cross-section curves for the 3.563 MeV  ${}^6\text{Li}$ -state.

The experimental angular distributions for the lower three  ${}^6\text{Li}$ -states for incident energies between  $E_p = 15$  to 50 MeV are shown in Figures 1(a–c).

### 3. The Optical Model Parameters

To calculate the differential cross-sections via DWBA for any direct nuclear reaction, we must know the parameters of the optical model potentials for the incident and outgoing channels and the bound state. These parameters are usually used in the total potential in the form of

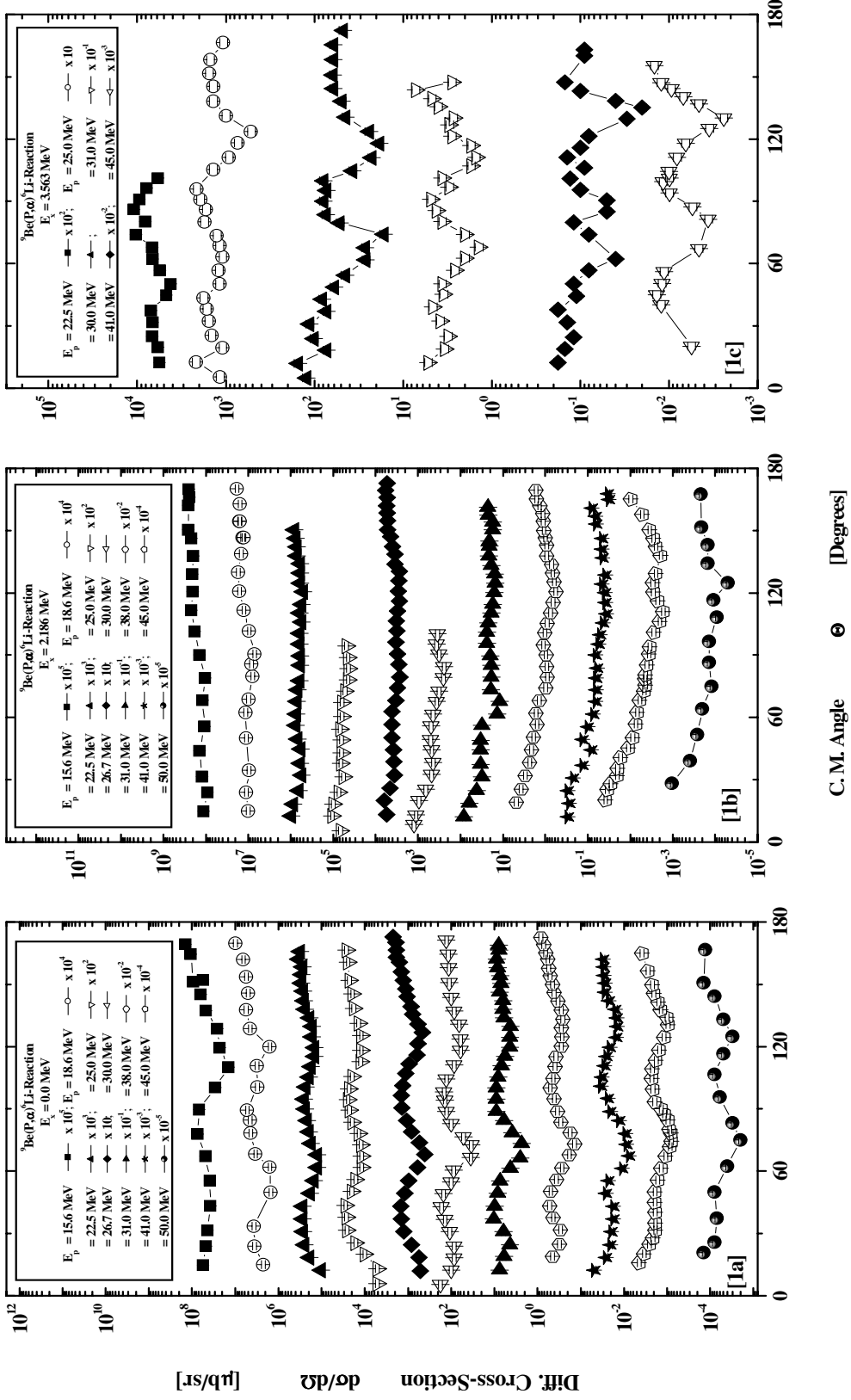
$$\begin{aligned}
 U = V_c(r_c) & - V_o f(x_o) + \left(\frac{\hbar}{m_\pi c}\right)^2 \cdot V_{LS}(L \cdot \sigma) \cdot \frac{1}{r} \cdot \frac{d}{dr} f(x_{LS}) \\
 & - i \left[ W_v \cdot f(x_v) - 4W_D \frac{d}{dx_D} f(x_D) \right].
 \end{aligned}
 \tag{1}$$

Here,  $V_c(r_c)$  is the Coulomb potential,  $V_o$  is the depth of the volume term and  $V_{LS}$  is that of the spin-orbit term for the real part of the potential.  $W_v$  is the depth of the volume term and  $W_D$  denotes the depth of surface term for the imaginary part of the potential.  $f(x_i)$  stands for the radial function of a Woods-Saxon potential and is given by

$$f(x_i) = (1 - e^{x_i})^{-1} \text{ with } x_i = (r - r_i A^{1/3})/a_i,
 \tag{2}$$

where  $r_i$ ,  $a_i$  and  $A$  are the radius parameter, the diffuseness parameter and the atomic mass number respectively.  $f(x_i)$  is a function used to calculate the radial parts of the different potential terms, where subscript  $i$  refers to each of the various contributing terms, LS, for the spin-orbit term; D, for the surface term  $v$ , for the volume term.

In this work, we have used the optical parameters from literature. We have considered works such as Satchler [26], who analysed the elastic and inelastic scattering of 46 MeV protons off  ${}^9\text{Be}$ ; and the work of Bingham et al. [27], who analysed the elastic scattering of 18.54 MeV  $\alpha$ -particles off  ${}^6\text{Li}$ . We also have in mind that there is no measured data for the scattering of tritons off  ${}^6\text{Li}$ ; and thus instead consider the data obtained by Luedecke et al. [28] for the scattering of 16 MeV  ${}^3\text{He}$  off  ${}^6\text{Li}$ . Sometimes modifications of these parameters are essential; and in our case, modification is done for the Proton's parameters in the incident channel by using the program SUCH [29]. Table 1 shows the used optical model parameters in the case of the present reaction.



**Figure 1.** The experimental angular distributions for the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  at the corresponding incident energies for: (a) the  ${}^6\text{Li}$  ground state; (b) the  ${}^6\text{Li}$  2.186 MeV state; (c) the  ${}^6\text{Li}$  3.563 MeV state. (The errors for the experimental cross-section values in the Figures 1, 2 and 5 are taken to be  $\pm 10\%$  and the curves in this figure are guides for the eye.)

## 4. The Theoretical Calculations

According to previously obtained results for the (p,  $\alpha$ ) reactions on light nuclei [1–4, 7, 8], the mechanism for the  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction begins to change from compound-nucleus to direct mechanism at about  $E_p = 19.861$  MeV. This incident energy corresponds to an excitation energy of the compound nucleus ( ${}^{10}\text{B}$ ) of about 20 MeV above the threshold separation energy of the  $\alpha$ -particle [3, 4]. In this work, the experimental angular distributions for  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction have been classified into two different groups. The first contains those at two lower incident energies,  $E_p = 15.6$  and 18.6 MeV, which correspond to the compound-nucleus formation mechanism. However, the second contains those for  $E_p \geq 22.5$  MeV, which are associated with the one-step process mechanism. In this way, all experimental angular distributions associated with the second group can be described by a theory of the direct one-step processes as, for example, the zero-range DWBA. According to this theory, the differential cross-section for the stripping reaction  $A(a, b)B$  is given by [30, 31]

**Table 1.** The Optical potential parameters used in the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$ . The depths of potential-terms are given in MeV and the radii and diffuseness are in units of fm.

Channel	${}^9\text{Be}+p$ [26]	(*)	${}^6\text{Li}+\alpha$ [27]	${}^6\text{Li}+\tau$ [28]
$V_o$	66.9		194.2	
$r_o$	0.801	1.101	2.0	1.2
$a_o$	0.71	0.15	0.43	0.72
$W_v$	4.96		—	
$r_v$	2.049		—	
$a_v$	0.629		—	
$W_D$	—		18.2	
$r_D$	—		2.0	
$a_D$	—		0.43	
$V_{LS}$	2.95		10.2	
$r_{LS}$	0.801		2.0	
$a_{LS}$	0.71		0.43	
$r_c$	1.2		2.0	1.3

(\*) The modified radius and diffuseness values for the incident channel.

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi \hbar^2)^2} \cdot \left(\frac{K_b}{K_a}\right) \cdot \frac{1}{[(2s_a + 1) \cdot (2J_A + 1)]} \cdot \sum_{M_A M_B M_a M_b} \left| T_{(a,b)}^{DWBA} \right|^2, \quad (3)$$

where the  $\mu_a(\mu_b)$  and  $K_a(K_b)$  are the reduced masses and wave numbers in the initial (final) state, respectively.  $s_a$  and  $J_A$  are the spins of the incident particle (a) and the target nucleus (A), respectively.  $M_A$ ,  $M_B$ ,  $M_a$  and  $M_b$  are the target, rest-nucleus, projectile and ejectile magnetic quantum numbers, respectively.  $T_{(a,b)}^{DWBA}$  is the transition amplitude and given by

$$T_{(a,b)}^{DWBA} = J \int d\mathbf{r}_b \int d\mathbf{r}_a \chi_f^{-*}(\mathbf{k}_b \mathbf{r}_b) \langle bB | V | aA \rangle \chi_i^+(\mathbf{k}_a, \mathbf{r}_a) \quad (4)$$

where  $\chi_i^+$  and  $\chi_f^-$  are the distorted waves for the elastic scattering on the optical potentials in initial and final channels, respectively.  $r_a$  and  $r_b$  are the relative coordinates for the two systems (a, A) and (b, B), respectively; and J is the Jacobian of the transformation to these coordinates. The quantity  $\langle bB | V | aA \rangle$  is the form factor for the reaction and must contain a delta function for the coordinates  $r_a$  and  $r_b$ .

Finally, the theoretical differential cross-section for a transferred particle (in a state with the quantum numbers  $L$ ,  $S$  and  $J$ ) may be calculated with the expression

$$\frac{d\sigma}{d\Omega} = \frac{(2J_B + 1)}{(2J_A + 1)} \cdot \frac{4\pi}{(E_a E_b)} \cdot \left(\frac{K_a}{K_b}\right) \cdot \left(\frac{B}{A}\right)^2 \cdot \frac{1}{(2s_a + 1)} \cdot \sum_{J,M,M_a,M_b} \left\{ \frac{1}{(2J + 1)} \cdot \left| \sum_{L,S} \left[ \sqrt{(2L + 1)} \cdot \beta_{LSJ} \cdot S_{LSJ}^{MM_a M_b} \right] \right|^2 \right\}, \quad (5)$$

where  $E_a$  and  $E_b$  are the centre of mass energies for the incident and exit channels, respectively; the quantity  $\beta_{LSJ}$  is a measure for the strength of the interaction; and  $S_{LSJ}^{MM_a M_b}$  is a kinematics-part. The quantity calculated by DWUCK-IV for the (a, b)-reaction (where  $a \neq b$ ) is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{DW4}^{LSJ} = \frac{4\pi}{(E_a E_b)} \cdot \left(\frac{K_a}{K_b}\right) \cdot \left(\frac{B}{A}\right)^2 \cdot \frac{1}{(2s_a + 1)} \cdot (2L + 1) \cdot \sum_{M,M_a,M_b} \left| S_{LSJ}^{MM_a M_b} \right|^2, \quad (6)$$

and the final relation between the experimental and theoretical cross-sections for the pick-up reaction ( $p$ ,  $\alpha$ ) can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{exp}^{(p,\alpha)} = \frac{(2s_\alpha + 1)}{(2s_p + 1)} \cdot \frac{D_o^2}{10^4} \cdot S^{(P,H^3)} \cdot \sum_J S_{JT}^{(p,\alpha)} \cdot C_T^2 \cdot \frac{1}{(2J + 1)} \cdot \left(\frac{d\sigma}{d\Omega}\right)_{DW4}^{LSJ}, \quad (7)$$

where  $s_p$  and  $s_\alpha$  are the spins of incident Proton and outgoing  $\alpha$ -particles, respectively.  $S^{(P,H^3)} (= 2)$  is the spectroscopic factor for the formation of the outgoing  $\alpha$ -particle (formed from the incident Proton and the transferred triton). The quantity  $S_{JT}^{(p,\alpha)}$  refers to the spectroscopic factor for the transferred three-particles (cluster), calculated by Kurath-Millener [16].  $C_T^2$  is an isospin coupling coefficient between initial and final states. The quantity  $\left(\frac{d\sigma}{d\Omega}\right)_{DW4}^{LSJ}$  is a kinematics part for the differential cross-section and given by the formula (6).  $J$  and  $T$  are the total angular momentum and isospin quantum numbers for the transferred cluster state. The quantity  $D_o$  is the zero-range normalization factor, which has for the ( $p$ ,  $\alpha$ )-reaction the value  $506 \text{ MeV} \cdot \text{fm}^{3/2}$  [32].

## 5. The Spectroscopic Factors

Starting from Proton's incident energy of  $\sim 20$  MeV, the mechanism of the  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction could be considered, to a higher expectation, as a direct one-step process, and the values of the cross-sections could be theoretically predicted using DWBA-theory. Our theoretical calculations for the cross-section are conducted in the semi-microscopic formulation [33], where a cluster spectroscopic amplitude is obtained from the microscopic shell-model transitions amplitude [16]. In addition, the triton-cluster approximation is used for the transferred three-particles state wave-function in the calculations of the form factor [33]. The form-factor is calculated with the help of the transferred-cluster wave function in its bound state. This bound-state is assumed to have a Woods-Saxon potential with a binding energy equals the experimental separation energy of the cluster in it, with quantum numbers  $N$ ,  $L$  and  $J$ . Where, the values of  $N$  and  $L$  are controlled by the oscillator-quanta conservation law

$$2N + L = Q;$$

$Q$  equals 3 for the  $(1p)^3$ -transferred triton-cluster and  $J$  is its total angular momentum, given by  $J = L \pm S$  where  $S = 1/2$ .

According to Kurath-Millener [16], the spectroscopic factor amplitude required for the triton-cluster transfer was calculated in the SU(3) approximation under some assumptions [6, 32] and have the form

$$S_{SU3}^{1/2} = \left(\frac{A}{A-3}\right)^{Q/2} \cdot G(q_i) \cdot \langle ITK | \chi^{2L_J(QO)} | I_o T_o K_o \rangle. \quad (8)$$

In these calculations, the direct-transferred three nucleons considered to be represented by a triton-cluster, has a spatial symmetric function under nucleons exchange with spin = 1/2. If the nuclear states for this cluster are described by a shell model using harmonic oscillator radial wave functions, then the wave function of the transferred nucleons can be projected onto an internal state 0s, which has no oscillator excitation quanta, times a centre-of-mass (c.m.) function containing all the quanta  $Q$  and orbital angular momentum  $L$  of the transferred cluster. Then the transfer of a 0s cluster is the chief source of strong direct cluster transfer. The product of the radial wave function and the c.m. function for the transferred cluster gives the first two factors in equation (8), for the  $(1p)^3$ -transferred triton,  $Q = 3$  and  $G(q_i) = (2/9)^{1/2}$ . The third factor,  $\langle ITK | \chi^{2L_J(QO)} | I_o T_o K_o \rangle$  (the parentage amplitude), means the matrix element of the three-nucleons SU(3) creation operator evaluated between nuclear states and reduced both in ordinary and isospin spaces. This last factor is equal to the square root of the binomial coefficient  $\binom{A}{3}$  times the three-particle coefficient of fractional parentage for separating the three nucleons in the  ${}^2L_J(Q0)$  state. In Table 2, the experimental and calculated excitation energies for the first three  ${}^6\text{Li}$ -states are listed, together with the total angular momentum, isospin factor and the SU(3) spectroscopic factors amplitudes and the last row contains the total spectroscopic factors  $S_{JT}^{(p, \alpha)}$ .

**Table 2.** The experimental and calculated excitation energies for the first three excited states for  ${}^6\text{Li}$ , together with the total angular momentum, isospin factor, SU(3) spectroscopic factors amplitudes (Sp. Fa. Amp.) and total spectroscopic factors.

$E_x(\text{MeV})$ <sup>[37]</sup>	<b>Ground state</b>	<b>2.186</b>	<b>3.563</b>
$E_{cal}(\text{MeV})$ <sup>[16]</sup>	0.0	2.1	2.5
$J^\pi$ <sup>[37]</sup>	1 <sup>+</sup>	3 <sup>+</sup>	0 <sup>+</sup>
<b>Isospin Factor</b>	1	1	1/3
<b>Spectroscopic Factors Amplitudes (Sp. Fa. Amp.)**</b> <sup>[16]</sup>			
$P_{1/2}$	0.13166	—	—
$P_{3/2}$	0.07490	0.26675	0.0825
$F_{5/2}$	-0.01200	0.0555	—
$F_{7/2}$	—	0.1938	—
<b>Total Spectroscopic Factors, S</b>	0.057986	0.60361	0.02723

\*\*The values of the spectroscopic factor amplitudes (Sp. Fa. Amp.) are normalized to the transferred angular momentum  $j$  and final isospin.

## 6. Discussion

According to nuclear reaction theories, patterns of experimental angular distributions and the incident-energy dependence of the integrated experimental cross-sections  $\sigma_{exp}(0^\circ-90^\circ)$ , and  $\sigma_{exp}(0^\circ-180^\circ)$ , are affected by the dominant mechanism responsible for the reaction. On the other hand, under the dominant of direct nuclear reactions, the forward integrated experimental cross-section  $\sigma_{exp}(0^\circ-90^\circ)$  for a transition is usually directly proportional to the summation of the spectroscopic factors for the corresponding components [3–8, 10–14, 33]. We discuss these concepts in the following sections.

## 6.1. Angular Distributions

According to the above given assumption for the transition of the reaction mechanism from compound-nucleus formation to the direct one-step process, the obtained experimental angular distributions for  $E_p \geq 20$  MeV indicate that the  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction, indeed, proceeds via direct nuclear reactions. These angular distributions, as shown in Figures 1(a–c), have diffraction patterns with forward peaks. For example, the experimental angular distributions for the ground  $(1^+; 0)$  state have a fixed form for incident energies  $E_p \geq 26.7$  MeV with two clear maxima at  $\Theta_{cm} \sim 50^\circ$  and  $100^\circ$  and two clear minima at  $\Theta_{cm} \sim 75^\circ$  and  $125^\circ$ . Those data associated with the 2.186 MeV  $(3^+; 0)$  state have a fixed form for  $E_p \geq 22.5$  MeV with a maximum at  $\Theta_{cm} \sim 25^\circ$ ; but there are three odd minima in the measurements at  $\Theta_{cm} \sim 60^\circ$ – $70^\circ$  for  $E_p = 31$  MeV, at  $\Theta_{cm} \sim 40^\circ$  for  $E_p = 41$  MeV and at  $\Theta_{cm} \sim 125^\circ$  for  $E_p = 50$  MeV. Those associated with the 3.563 MeV  $(0^+; 1)$  state have a fixed form for  $E_p \geq 25$  MeV and two clear maxima at  $\Theta_{cm} \sim 40^\circ$ – $45^\circ$  and at  $\Theta_{cm} \sim 85^\circ$ – $100^\circ$ ; but the form of the angular distribution for  $E_p = 41$  MeV is not suitable to proceed via direct mechanism.

The angular distributions for the  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction could be analysed with the help of zero-range DWBA-theory [15] using the optical potential parameters given in Table 1 and the spectroscopic factor amplitudes presented in Table 2. They are characterized by a transfer angular momentum of  $L = 1$  or 3 (or a combination of these two values of angular momentum). A good fit for the experimental angular distributions of the ground  $(1^+; 0)$  state could be obtained for  $E_p \geq 22.5$  MeV (see Figure 2a). Good fits are also obtained at  $E_p \geq 15.6$  and  $E_p \geq 25$  for the two  ${}^6\text{Li}$ -states [2.186 MeV  $(3^+; 0)$ ] and [3.563 MeV  $(0^+; 1)$ ], respectively. At lower incident energies, the fit process is not suitable for the experimental data of the two  ${}^6\text{Li}$ -states ground  $(1^+; 0)$  and 3.563 MeV  $(0^+; 1)$  (see Figures 2a and 2c). This may be attributed to the existence of other reaction mechanisms at such low energies.

Such features for the forms of the experimental angular distributions and the predictive power of DWBA theory, i.e. its ability to fit data, are good tools to distinguish the reaction mechanisms. It is clear that, the beginning of the direct component for the reaction mechanism itself changes from state to another (as explained above for the investigated three excited states).

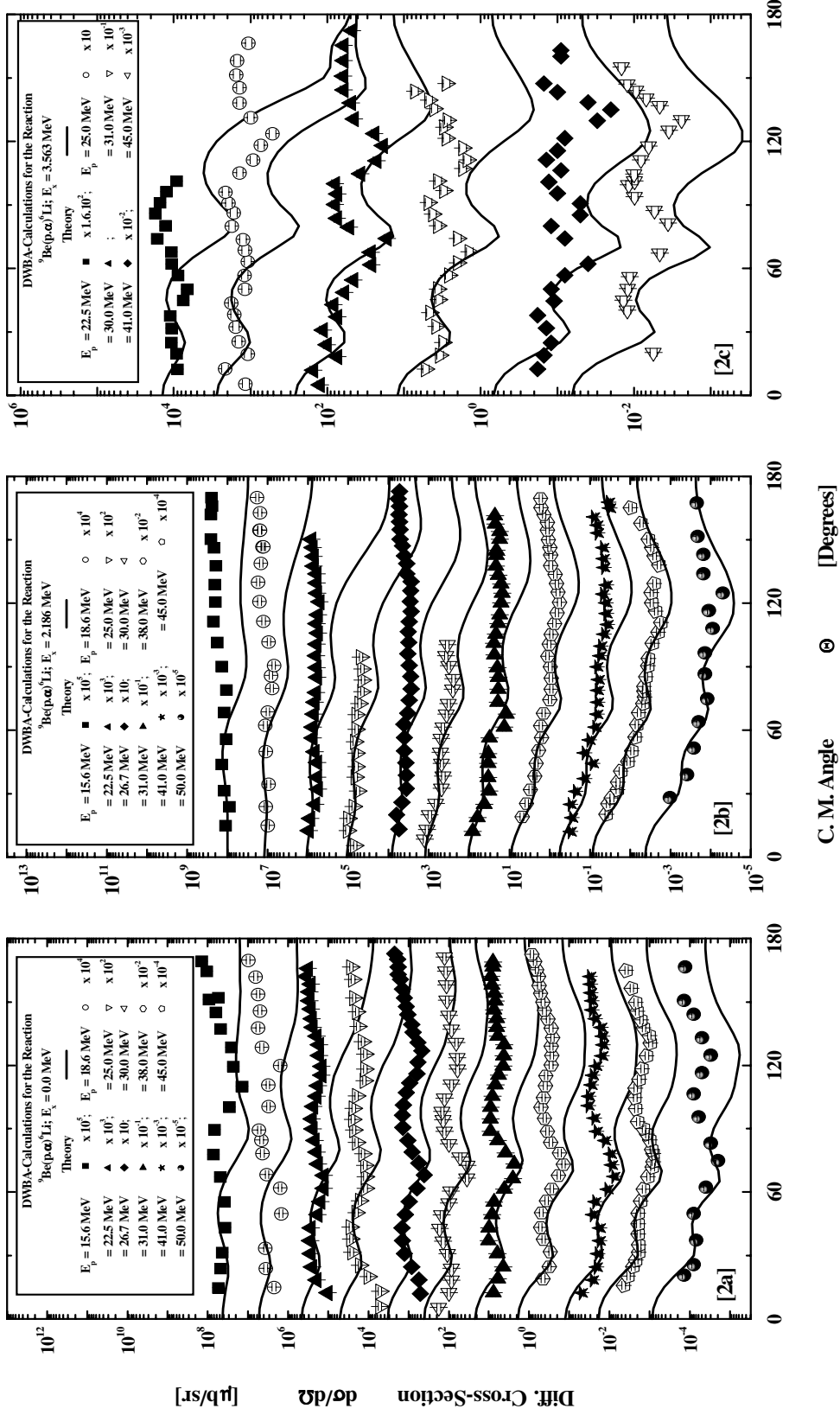
## 6.2. Spectroscopic Factors, Total Cross-sections and Energy Levels

It is known that, for transitions in direct nuclear reactions, the values of the forward integrated experimental cross-sections  $\sigma_{exp}(0^\circ\text{--}90^\circ)$  are directly proportional to the summation of the spectroscopic factors for the corresponding components for each transition [3–8, 10–14, 33–35]. As shown in Figures 3(a–c), a direct comparison of the bare SU(3) spectroscopic factors given in Table 2 with both the experimental  $\sigma_{exp}(0^\circ\text{--}90^\circ)$  and theoretical integrated cross-sections  $\sigma_{DW4}(0^\circ\text{--}90^\circ)$  is given for the ground, 2.186 and 3.563 MeV  ${}^6\text{Li}$ -states at the three incident energies  $E_p = 22.5$ , 31 and 45 MeV, respectively. The fit obtained in these figures, especially at higher incident energies, is due to the fact that, at such incident energies, the reaction runs either with a high component of the direct mechanism or as a pure direct reaction. This concept is actually established previously for different types of nuclear reactions on other different targets [6–8, 10–11, 13–14, 33–35].

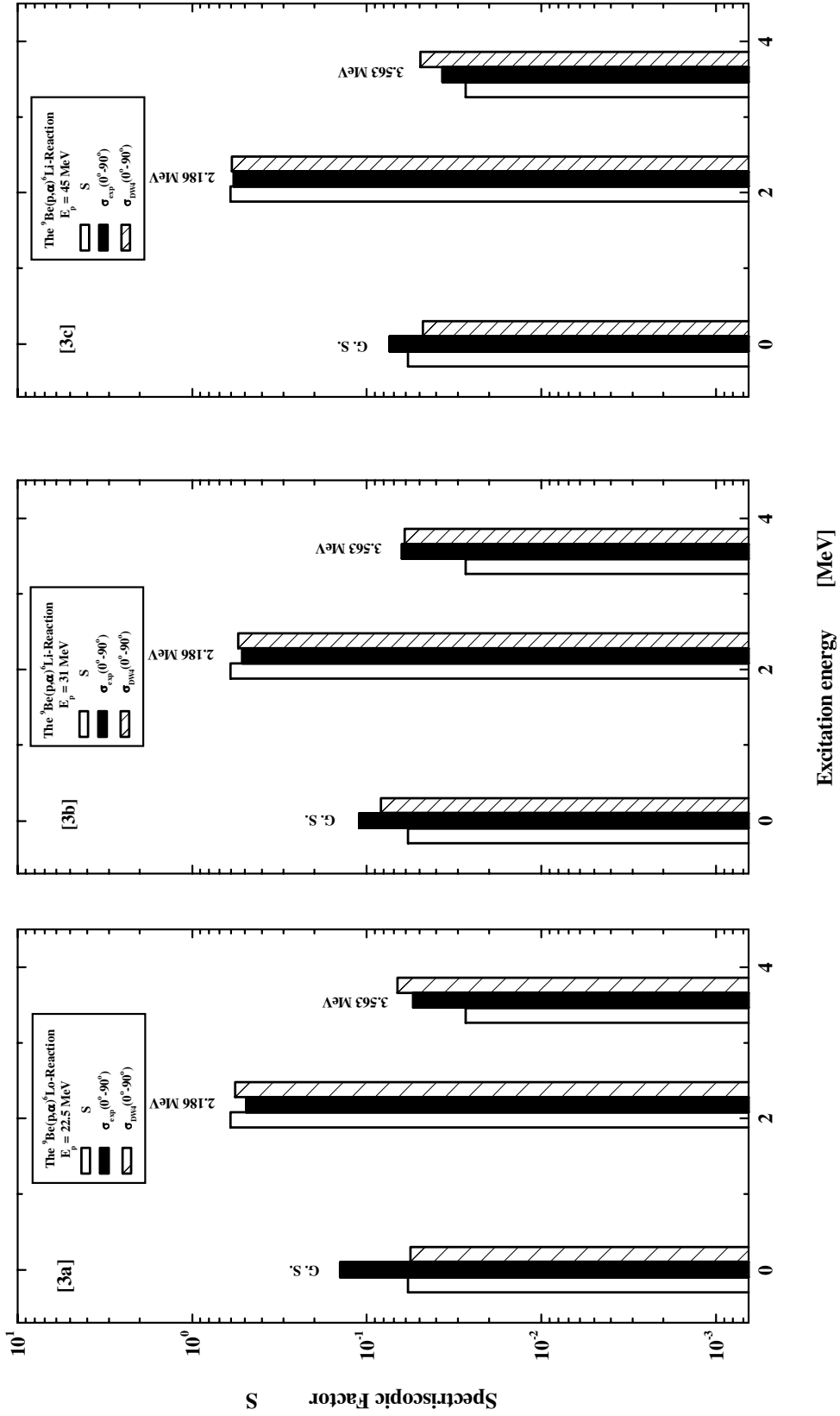
The effect of direct reaction mechanism is reflected on both the forms of the obtained angular distributions (see section 6.1) and the incident-energy dependence of the total experimental cross-sections  $\sigma_{exp}(0^\circ\text{--}90^\circ)$  and  $\sigma_{exp}(0^\circ\text{--}180^\circ)$  (see section 6.3).

The comparison found in Figures 3(a–c), leads to two different important final results. First, the calculated shell-model's spectroscopic factors for the three-particle transfer [16] are enough to reproduce the experimental total cross-sections for the transitions in the direct reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$ ; and second, it shows the minor importance of the reaction-dynamics in this reaction. Actually, this had already been established for the one-nucleon transfer reactions on 1p-shell nuclei [34], for the two-nucleon transfer reactions on 1p-shell nuclei [33, 35], for the  $(p, \alpha)$  reactions on 1p-shell [4, 5, 10–14, 36] and on 2s-1d shell nuclei [6–8, 32]. In





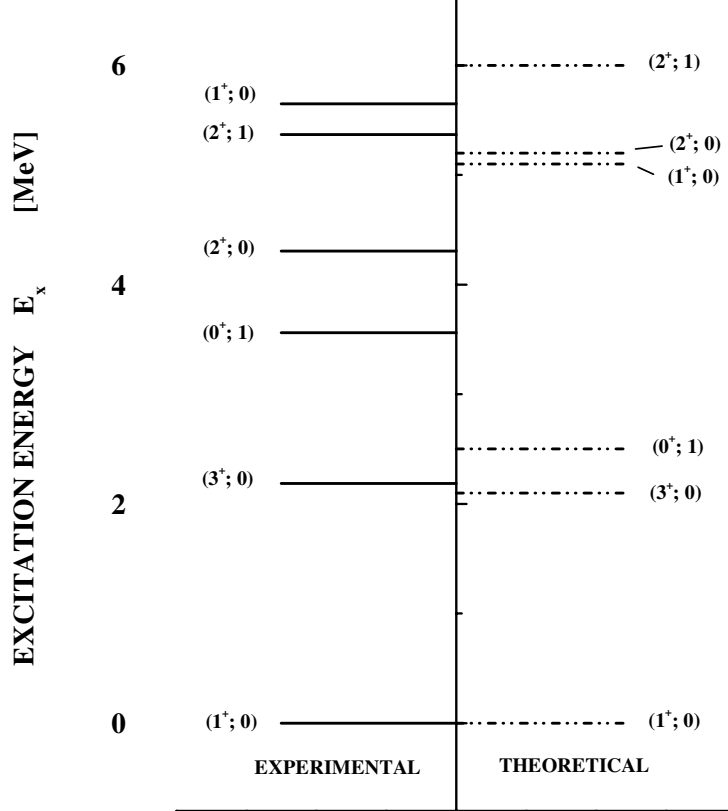
**Figure 2.** DWBA-calculations for the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  at the corresponding incident energies for: (a) the  ${}^6\text{Li}$  ground state; (b) the  ${}^6\text{Li}$  2.186 MeV state; (c) the  ${}^6\text{Li}$  3.563 MeV state. (The errors for the experimental cross-section values are taken to be  $\pm 10\%$ .)



**Figure 3.** Comparison of the spectroscopic factors  $S_{JT}^{(p, \alpha)}$  with both the experimental  $\sigma_{exp}(0^\circ-90^\circ)$  and theoretical total cross-section  $\sigma_{DW4}(0^\circ-90^\circ)$  for the three  ${}^6\text{Li}$  states 0.0, 2.186 and 3.563 MeV in the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  at (a)  $E_p = 22.5$  MeV; (b)  $E_p = 31$  MeV; (c)  $E_p = 45$  MeV.

these Figures the open, black and dashed bars represent the spectroscopic factors ( $S_{JT}^{(p,\alpha)}$ ), the experimental  $\sigma_{exp}(0^\circ-90^\circ)$  and theoretical  $\sigma_{DW4}(0^\circ-90^\circ)$  integrated cross-section for a certain transition, respectively.

Tables 3 presents the calculated excitation energies for the lower six  ${}^6\text{Li}$ -states, obtained as eigenvalues of the Hamiltonian operator in Schrödinger-equation for these states [16], with their corresponding experimental values [37]. The comparison (Figure 4) illustrates that the experimental excitation energies for the  ${}^6\text{Li}$ -states and those calculated by Kurath-Millener are exactly fitted in both values and orders.

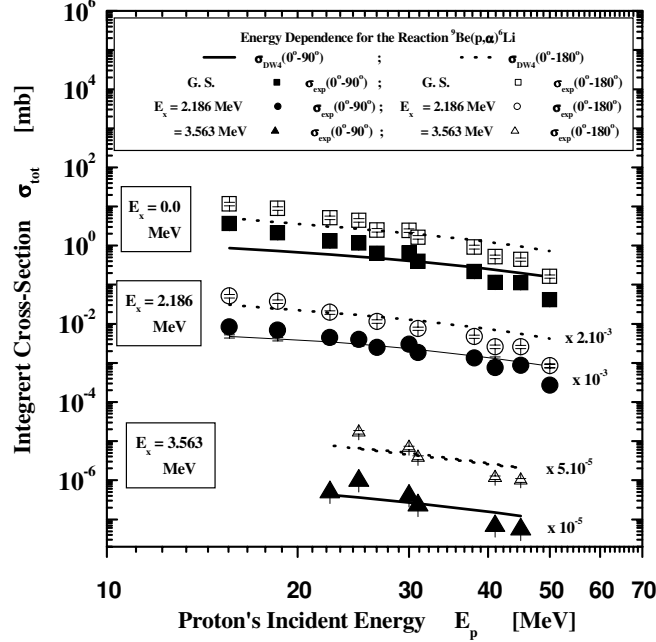


**Figure 4.** Experimental and theoretical excitation energies for the nucleus  ${}^6\text{Li}$ , the first three states are studied and analysed in the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$ .

### 6.3. Energy dependence for the integrated cross-sections

In Figure 5 the experimental [ $\sigma_{exp}(0^\circ-90^\circ)$  and  $\sigma_{exp}(0^\circ-180^\circ)$ ] and theoretical [ $\sigma_{DW4}(0^\circ-90^\circ)$  and  $\sigma_{DW4}(0^\circ-180^\circ)$ ] total cross-sections are plotted together, for incident energies from  $E_p \sim 18.6$  to  $\sim 50$  MeV for the three  ${}^6\text{Li}$ -states: ground, 2.186 and 3.563 MeV. The theoretical curves for both the two angular ranges ( $0^\circ-90^\circ$ ) and ( $0^\circ-180^\circ$ ) for the investigated three  ${}^6\text{Li}$ -states agree well with the corresponding experimental points, especially at higher incident energies, i.e.  $E_p \geq 22.5$  MeV. One interpretation could be that, at such higher incident energies, the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  runs with greater involvement of the direct mechanism, or it could be the only mechanism. This is actually reflected in the fit between the forms of the experimental angular distributions with the corresponding theoretical DWBA predictions for these states at such higher incident energies (see Figures 2(a-c)). In the case of the 2.186 MeV excited state, the direct-mechanism component starts at  $E_p \approx 18.6$  MeV, which is at an energy lower than in the two other states. For this reason the experimental integrated cross-section value  $\sigma_{exp}(0^\circ-180^\circ)$  for the state at  $E_p = 18.6$  MeV is a

factor  $\sim 3$  greater than the corresponding theoretical value; while that of the ground state is a factor  $\sim 5$  greater (at the same incident energy).



**Figure 5.** The energy dependence of the integrated cross-sections for the three  ${}^6\text{Li}$ -states: ground, 2.186 and 3.563 MeV in the reaction  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$ . The dotted curves represent the integrated cross sections  $\sigma_{DW4}(0^\circ-90^\circ)$  for the angular range  $(0^\circ-90^\circ)$  and the solid curves represent the integrated cross sections  $\sigma_{DW4}(0^\circ-180^\circ)$  for the angular range  $(0^\circ-180^\circ)$ . (The error in this Figure is taken to be  $\pm 10\%$ .)

**Table 3.** Experimental and theoretical energy levels for the  ${}^6\text{Li}$  nucleus; the first three levels are identified in the  ${}^9\text{Be}(p, \alpha){}^6\text{Li}$  reaction.

$J^\pi ; T$	$E_{exp}(\text{MeV})$ [37]	$E_{cal}(\text{MeV})$ [16]
$1^+ ; 0$	(ground state)	0.0
$3^+ ; 0$	2.186	2.1
$0^+ ; 1$	3.563	2.5
$2^+ ; 0$	4.310	5.2
$2^+ ; 1$	5.366	6.0
$1^+ ; 0$	5.650	5.1

## 7. Conclusion

Kurath and Millener [16] calculated the coefficients of fractional parentage (cfp) and the excitation energies for the three-particle pickup reaction on  ${}^9\text{Be}$  nucleus by using the shell-model wave functions and the intermediate coupling model [17] for the 1p-shell nuclei. The spectroscopic factors amplitudes obtained in these calculations could be used in zero-range DWBA-theory to predict exactly the forms of the diffraction patterns for the experimental angular distributions, and the transition strengths for the first lower three  ${}^6\text{Li}$ -states. In the present work, both the experimental angular distributions and transition strengths were in agreement with the corresponding predictions of DWBA-theory for these states (see Figures 2 and 3). Also, there is good agreement in values and positions between the calculated and experimental excitation energies (see Tables 2 and 3, and Figure 4). On the other hand, we found that the incident energy at which

the reactions begin to proceed via the direct-component mechanism (i.e., the critical incident energy) varies from one state to another.

Thus the present work shows that the Kurath-Millener calculation for three-particle pick-up reactions on the 1p-shell nuclei can be used to describe and explain the  $(p, \alpha)$  reaction on  ${}^9\text{Be}$ . This observation is not in isolation, for it has been previously observed in other  $(p, \alpha)$  reactions on 1p-shell nuclei [10–14, 33].

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