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Electron-Deuteron Tensor Polarization and D-State Probability

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Abstract

Electron-deuteron tensor polarization $T_{21}(q)$ is calculated for thirty-three N-N local potential models. New relations have been found between the peak value of $T_{21}(q)$ and some of the deuteron properties. An experimental value of deuteron D-state probability $P_D = 5.9861\% \pm 0.2687\%$ is deduced.

Key Words: ed scattering, tensor polarization, deuteron D-state probability, deuteron quadrupole moment, deuteron form factors.

1. Introduction

The deuteron is the lightest of all composite nuclei and has the unique property of having an abnormally low binding energy per nucleon, with no excited bound states. The binding energy E_B of the deuteron is 2.22 MeV, that is 1.11 MeV/nucleon, which is roughly a factor of eight lower than what is typical for stable nuclei (between 7–9 MeV/nucleon [1]). The D-state probability P_D of the deuteron is an important quantity for nuclear forces. It is of great interest to know the amount of the D-state probability P_D. Although P_D is not an observable, typical values inferred from measurements are in the range of 3–7% [2]. The D-state probability P_D of the deuteron is related to the radial wave function $w(r)$ by the integral

$$P_D = \int_0^\infty w^2(r)dr \tag{1}$$

The mean goal of this work is to investigate the relation between the electron-deuteron tensor polarization $T_{21}(q)$ and deuteron D-state probability P_D and extract an experimental value of P_D.

2. Elastic Electron-Deuteron Scattering Observables

As an estimation of a quantitative understanding of the structure of the deuteron, the S and D bound states have long been considered an important testing ground for models of the nucleon-nucleon potential. Nevertheless, the charge distribution of the deuteron is not well known experimentally, because it is only through the use of both polarization measurements and unpolarized elastic scattering cross sections that it can be unambiguously determined [3]. The differential cross section for elastic scattering of unpolarized electrons on unpolarized deuterons without measuring the polarization of the outgoing electrons and deuterons is [4]

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT} S \tag{2}$$

where

$$S = A(q) + B(q)\tan^2\frac{\vartheta}{2}$$

and ϑ is the scattering angle in the laboratory frame and q, in units of fm^{-1}, is the momentum transfer to the deuteron. $A(q)$ and $B(q)$ are the electric and magnetic structure functions [4]:

$$A(q) = F_C^2(q) + \frac{8}{9}\eta^2 F_Q^2(q) + \frac{2}{3}\eta F_M^2(q), \tag{3}$$

$$B(q) = \frac{4}{3}\eta(1+\eta)F_M^2(q), \tag{4}$$

where $\eta = q^2/4M_D^2$. The charge $F_C(q)$, quadrupole $F_Q(q)$, and magnetic $F_M(q)$ form factors, which contain the whole information about the electromagnetic properties of the deuteron, are given by the following equations [4]:

$$F_C(q) = \left[G_{E_p}(q) + G_{E_n}(q)\right]\int_0^\infty \left[u^2(r) + w^2(r)\right] j_\circ\left(\frac{qr}{2}\right) dr, \tag{5}$$

$$\sqrt{\frac{8}{9}}\eta F_Q(q) = 2\left[G_{E_p}(q) + G_{E_n}(q)\right]\int_0^\infty \left[u(r)w(r) - \frac{w^2(r)}{\sqrt{8}} j_2\left(\frac{qr}{2}\right)\right] dr, \tag{6}$$

$$F_M(q) = 2\left[G_{M_p}(q) + G_{M_n}(q)\right]\int_0^\infty \left[\left(u^2(r) - \frac{1}{2}w^2(r)\right) j_0(\frac{qr}{2}) + \left((\frac{1}{\sqrt{2}})u(r)w(r) + \frac{1}{2}w^2(r)\right) j_2(\frac{qr}{2})\right] dr$$

$$+\frac{3}{2}\left[G_{E_p}(q) + G_{E_n}(q)\right]\int_0^\infty w^2\left[j_0(\frac{qr}{2}) + j_2(\frac{qr}{2})\right] dr, \tag{7}$$

where $u = \psi_0 r$ and $w = \psi_2 r$ are the s- and d-radial wave functions of the deuteron. G_{E_p}, G_{E_n} are the proton and neutron electric form factors and G_{M_p}, G_{M_n} are the proton and neutron magnetic form factors. They are given by the relations [4]

$$\begin{aligned} G_{Ep}(q) &= (1 + q^2/18.235 fm^{-2})^{-2}, \\ G_{En}(q) &= 0, \\ G_{Mp}(q) &= \mu_p G_{Ep}(q), \\ G_{Mn}(q) &= \mu_n G_{Ep}(q), \end{aligned} \tag{8}$$

where μ_p and μ_n are the proton and neutron magnetic moments in nuclear magnetons.

In unpolarized elastic scattering experiments the structure functions can only be measured by determining $B(q)$ directly from the backward scattering cross section. Equation (4) yields the magnetic form factor, but $F_C(q)$ and $F_Q(q)$ cannot be separated in Eq. (3). Of course, one needs a third observable to get all three-form factors. Therefore, in addition to unpolarized scattering, polarization observables must also be considered.

Since the deuteron is a $s = 1$ system, tensor and vector polarizations are also observables that could be measured and can be considered. The tensor polarization of the outgoing deuterons is given by the following relations [3]:

$$T_{20} = -\frac{1}{\sqrt{2}S}\left(\frac{8}{3}\eta F_C(q)\cdot F_Q(q) + \frac{8}{9}\eta^2 F_Q^2(q) + \frac{1}{3}\eta\left[1 + 2(1+\eta)\tan^2\frac{\vartheta}{2}\right]F_M^2(q)\right) \tag{9}$$

$$T_{21} = \frac{2}{\sqrt{3}S\cos\frac{\vartheta}{2}}\eta\left[\eta + \eta^2\sin^2\frac{\vartheta}{2}\right]^{1/2} F_M(q)F_Q(q), \tag{10}$$

$$T_{22} = -\frac{1}{2\sqrt{3}S}\eta F_M^2(q) \tag{11}$$

Since T_{22} only depends on $F_M(q)$, which is already known from $B(q)$, there is no new information in Eq. (11). To determine $F_C(q)$ and $F_Q(q)$, we could in principle choose one of the other two observables together with $A(q)$ and $B(q)$.

3. Deuteron Tensor Polarization

One of the main hopes of electron-deuteron scattering experiments have been to measure certain features of the deuteron wave function and to use these properties to determine unknown properties of the nucleon-nucleon force. In particular, one would like to employ electron-deuteron scattering to describe the nature of the short-range repulsion and the tensor force strength, which is related to the deuteron D-state probability P_D. The main purpose of this part is to indicate how electron-deuteron tensor polarization $T_{21}(q)$ measurements can be used to distinguish different nuclear force potential models, especially with respect to the short-range behavior and tensor force strength. So, in our investigation of deuteron tensor polarization we employ thirty-three local potential models of the nucleon-nucleon force. The thirty-three local potentials are denoted by the following notation: GK1, GK3 and GK8 of Glendenning and Kramer [5]; PARIS of Lacomb et al. [6]; RHC, RSC, RSCA of Reid [7]; TSB and TSC of de Tourreil and Sprung [8]; HJ of Hamada and Johnston [9]; TRS of de Tourreil et al. [10]; L1, L2, 2, 4, . . . , 6 of Mustafa [11]; r1, r3, . . . , r7 of Mustafa et al. [12]; MHKZ of Mustafa et al. [13]; and a, b, c, . . . , i of Mustafa [14]. These thirty-three potential models have different deuteron properties, such as deuteron quadrupole moment Q_D, D-state probability P_D, asymptotic D-state amplitude A_D and asymptotic ratio ξ. The values of these properties are not equal, but have wide range of values in all potential models.

To discuss the properties of various wave functions of these potential models, the deuteron radial wave functions u and w of fourteen selected local potential models among the above mentioned 33 potential models (i, c, f, GK3, TSC, r6, RSC, RHC, r7, HJ, PARIS, MHKZ, 2 and 4) are chosen. Comparison between these functions is shown in Figure 1 and Figure 2. For the fourteen local potential models, the difference between them appears most dramatically in S- and D-waves at distances $r < 3$ fm. For large values of r there is a good agreement between these models.

Haftel et al. [4] suggest that measurements of $T_{21}(q)$ in the range $3.0\ \text{fm}^{-1} \leq q \leq 5.0\ \text{fm}^{-1}$ can be used to distinguish between interactions with different deuteron D-state probabilities P_D. The effect of using potential models with different properties on the behavior of the tensor polarization $T_{21}(q)$ at the scattering angle $\theta = 90°$ will be discussed. It is found that, for the thirty-three local potential models, $T_{21}(q)$ distinguishes between competing models when $q > 2.0$ fm^{-1} and a peak is found for each potential model at 3.8 fm^{-1} $< q <$ 4.4 fm^{-1}. A node occurs at 4.8 fm^{-1} $< q <$ 8.2 fm^{-1}. Figure 3 and Figure 4 illustrate $T_{21}(q)$ for the fourteen selected local potential models.

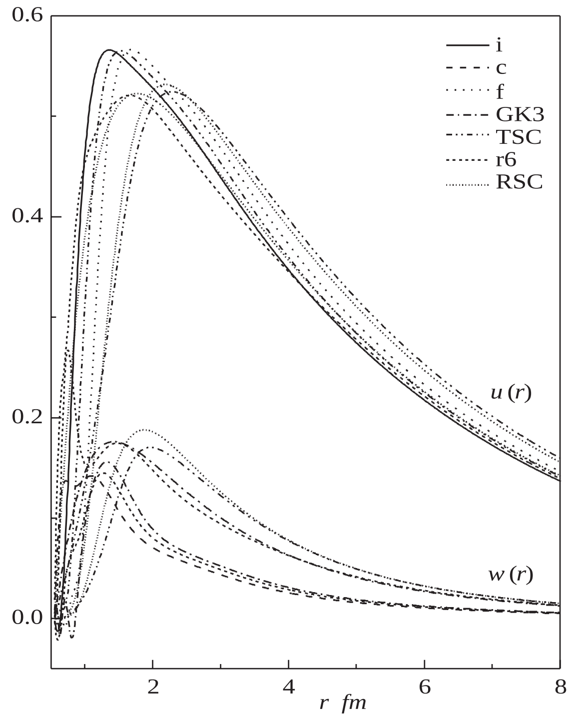

Figure 1. The variation of $u(r)$ and $w(r)$ against the distance r (in units of fm) for some potential models.

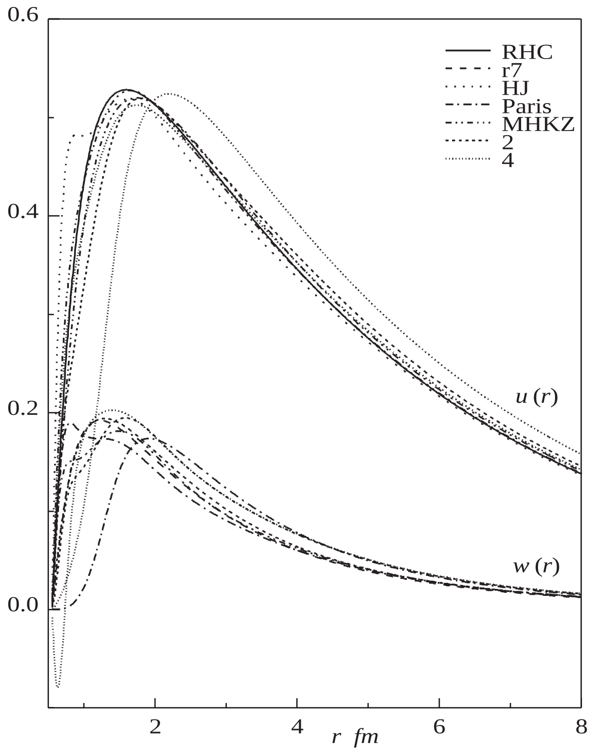

Figure 2. The variation of $u(r)$ and $w(r)$ against the distance r (in units of fm) for some potential models.

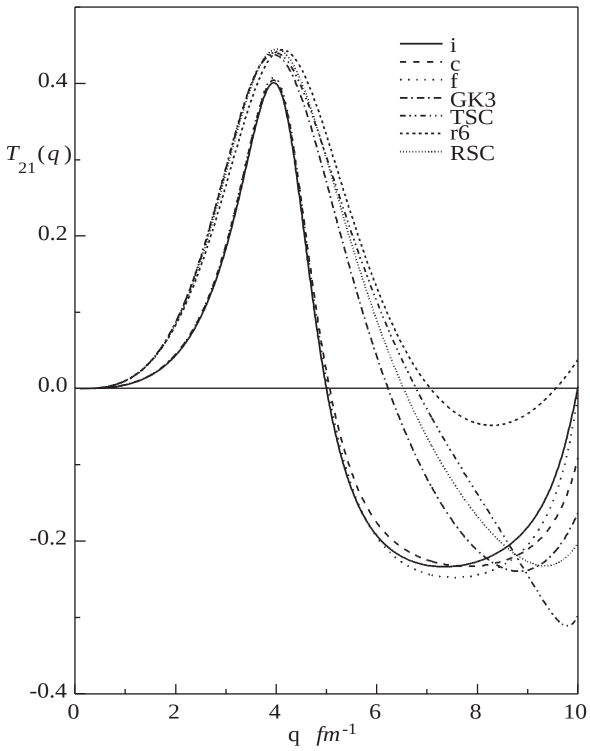

Figure 3. The variation of tensor moment $T_{21}(q)$ versus the momentum transfer q(in units of fm^{-1}) for some potentials.

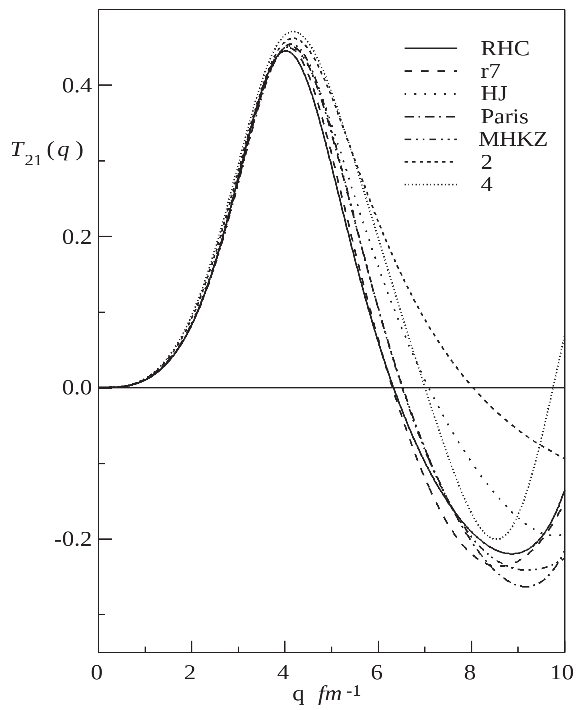

Figure 4. The variation of tensor moment $T_{21}(q)$ versus the momentum transfer q(in units of fm^{-1}) for some potentials.

4. $T_{21}^{\max}(q)$ and Deuteron Properties

In this section, we investigate relations between the peak of the tensor polarization $T_{21}(q)$, which is denoted by $T_{21}^{\max}(q)$, and deuteron properties.

Figures 3 and 4 suggest that every potential model exhibits a peak value in the relation between the tensor polarization $T_{21}(q)$ and the momentum transfer q. In fact, the peak value and the value of q associated with the peak have been extracted for all the models under examination. The smallest value of $T_{21}^{\max}(q)$ for all these thirty-three local potential models is 0.400883 for potential i, whereas as the largest value is 0.47072 for potential 4. The smallest and largest values of q are 3.94 and 4.22 fm^{-1} for the potential models d and GK1 respectively.

New relations have been found between the peak value $T_{21}^{\max}(q)$ and deuteron properties such as deuteron D-state probability P_D, quadrupole moment Q_D, asymptotic D-state amplitude A_D and asymptotic D/S ratio ξ.

Deuteron D-state probability P_D varied from 2.09571% for potential h to 8.17081% for potential 6. The relation between P_D and $T_{21}^{\max}(q)$ is shown in Figure 5 for the thirty-three local potential models. It is obvious that the relation tends to be a straight-line.

Deuteron quadrupole moment Q_D varied from 0.125 fm^2 for potential e to 0.340764 fm^2 for potential 6. Figure 6 shows that there is a new relation between Q_D and $T_{21}^{\max}(q)$ for the thirty-three local potential models. The relation is approximately a straight-line.

In a similar manner, we found the same behavior in the relation between deuteron asymptotic D-state amplitude A_D and $T_{21}^{\max}(q)$, as shown in Figure 7, where, A_D is varied from 0.00934 to 0.02972 for potentials e and 5, respectively. Also, the same behavior is found in the relation between deuteron asymptotic ratio ξ and $T_{21}^{\max}(q)$ as shown in Figure 8, where ξ is varied from 0.0107 to 0.033097 for potentials e and 5, respectively.

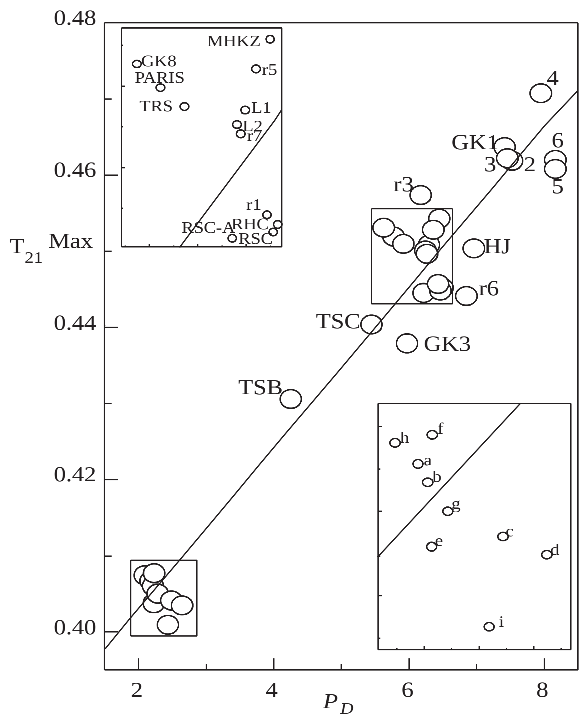

Figure 5. The relation between $T_{21}^{\max}(q)$ and deuteron D-state probability P_D for the thirty-three potential models.

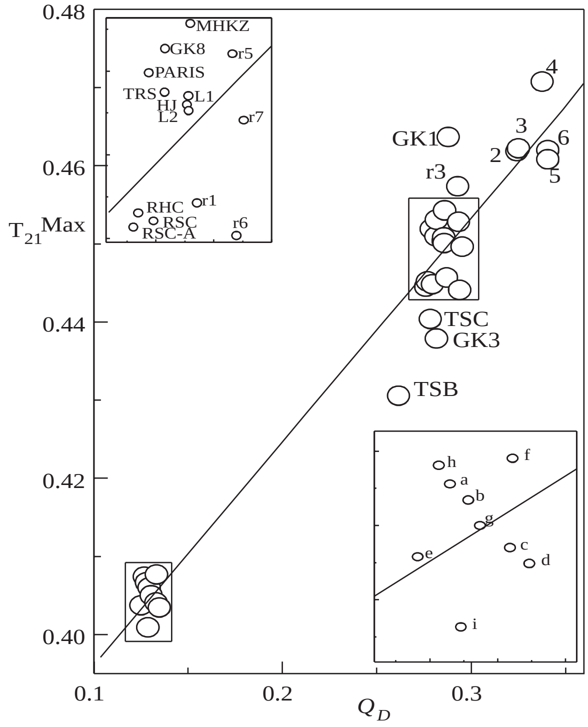

Figure 6. The relation between $T_{21}^{\max}(q)$ and deuteron quadrupole moment Q_D for the thirty-three potential models.

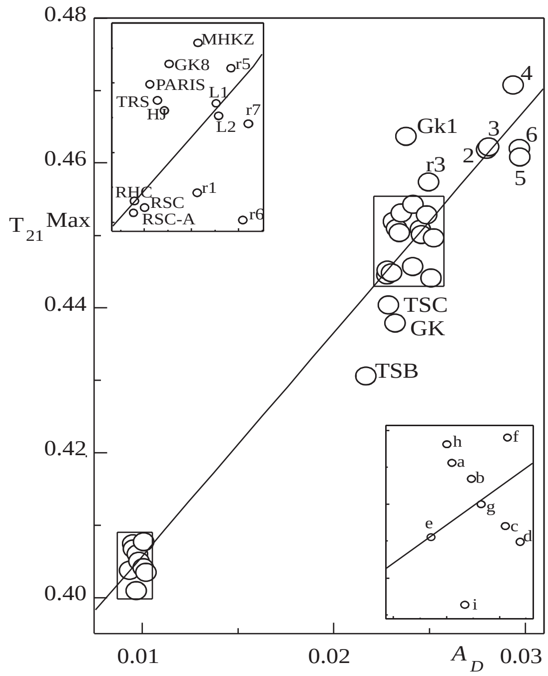

Figure 7. The relation between $T_{21}^{\max}(q)$ and deuteron D-state amplitude A_D for the thirty-three potential models.

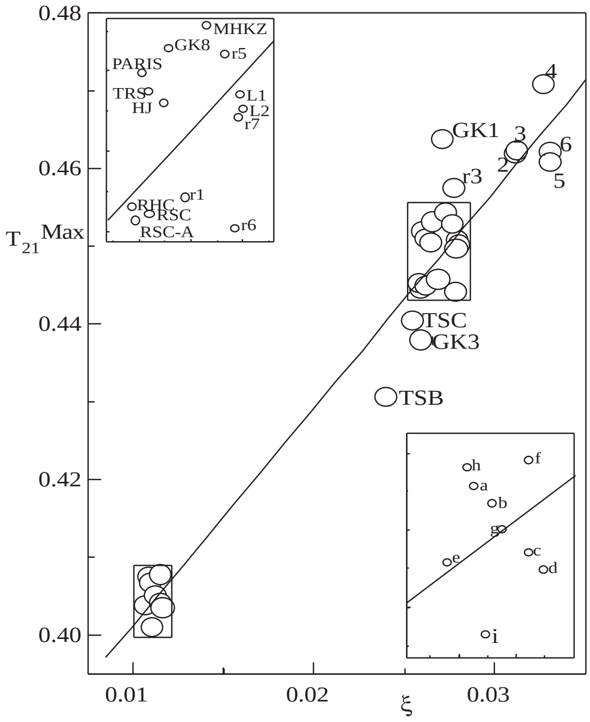

Figure 8. The relation between $T_{21}^{\max}(q)$ and deuteron asymptotic ratio ξ for the thirty-three potential models.

5. Extracting an Experimental Value of P_D

Least squares method is used to fit the relation between $T_{21}^{\max}(q)$ and both the deuteron D-state probability P_D, the quadrupole moment Q_D and the asymptotic ratio ξ. The best fit is found to be a straight-line. An experimental value of $T_{21}^{\max}(q)$ is obtained by substituting the experimental value of the deuteron asymptotic ratio $\xi = 0.0256 \pm 0.0004$ [15] in the relation between ξ and $T_{21}^{\max}(q)$. The deduced experimental value in this case is

$$T_{21}^{\max}(q) = 0.44513 \pm 0.00478 \tag{12}$$

The main purpose of this work was to find an experimental value of the deuteron D-state probability P_D. This is done by fitting the relation between the peak value of $T_{21}^{\max}(q)$ and the deuteron D-state probability P_D of the thirty-three local potential models. The appropriate experimental value of the deuteron D-state probabilityP_D has been taken as the value corresponding to the experimental value of $T_{21}^{\max}(q)$ in eq. (12).

Experimental value of the deuteron D-state probability P_D is deduced by substituting the experimental value of $T_{21}^{\max}(q)$ in the relation between $T_{21}^{\max}(q)$ and P_D. It is found to be:

$$P_D = 5.9861 \pm 0.2687\% \tag{13}$$

6. Conclusion

The deuteron tensor polarization $T_{21}(q)$ can be used to distinguish between potential models with different values of some deuteron properties such as deuteron quadrupole moment Q_D, D-state probability P_D, asymptotic D-state amplitude A_D and asymptotic ratio ξ. New relations between the peak values of $T_{21}(q)$, for the N-N potential models used, and these properties are found. A new experimental value for $T_{21}^{\max}(q)$ is deduced and an experimental value for deuteron D-state probability P_D is deduced from this experimental value of $T_{21}^{\max}(q)$.

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