The Zero Position of the Forward-Backward Asymmetry in the $B \to K\ell^+\ell^-$ Decay with New Physics Effects

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Abstract

Using the most general model-independent effective Hamiltonian comprising local four-Fermi operators (scalar, vector, and tensor operators), we study the sensitivity of the zero position of the lepton asymmetry to the new operators beyond the Standard Model (SM) in the $B \to K\ell^+\ell^-$ decay. It is found that among all operators, only the scalar and tensor operators contribute to the forward-backward asymmetry, in which case the forward-backward asymmetry has a non-vanishing value.

Key Words: B-decays, Standard Model, forward-backward asymmetry, zero position of the lepton asymmetry.

1. Introduction

The flavour-changing neutral current (FCNC) processes provide an excellent testing ground for the Standard Model (SM), and are possibly the most sensitive to the various extensions to the SM, because these transitions occur at the loop level in the SM. Among all the FCNC phenomena, the rare B decays are especially important [1], since one can both test the SM and search for possible new physics effects. Rare B meson decays induced by $b \to s\ell^+\ell^-$ transitions has been studied in the framework of the SM and its various extensions [2, 3, 4, 5, 6, 7, 8, 9, 10].

Concerning the semi-leptonic B decays, $B \to X_s\ell^+\ell^-$ ($X_s = K^*, K, \ell = e, \mu, \tau$) decay is an example having both theoretical and experimental importance. This work is a study of the zero of the forward-backward asymmetry ($A_{FB}$) in the $B \to K\ell^+\ell^-$ decay using the most general form of the effective Hamiltonian. The symmetry of those decays is a particularly interesting quantity, since it vanishes at the specific value of the dilepton invariant mass [11, 12]. In the recent literature, the dilepton invariant mass spectra, and the forward-backward asymmetry in $B \to X_s\ell^+\ell^-$ decays has been analyzed in detail [12, 13, 14].

It has been found that $A_{FB}$ may become zero for the certain value of the dilepton invariant mass for the exclusive $B \to K^*\ell^+\ell^-$ decay. On the other hand the forward-backward asymmetry is zero for the exclusive $B \to K\ell^+\ell^-$ decay within the SM [12]. In addition, the zero position of the forward-backward asymmetry has been analyzed in the most general model in the $B \to K^*\ell^+\ell^-$ decay and found that the zero of the $A_{FB}$ is sensitive to the new Wilson coefficients [15].

The organization of the present work is as follows. In Section 2, starting from the most general effective Hamiltonian, we compute the differential decay width of the exclusive $B \to K\ell^+\ell^-$ decay and the numerator.
of the forward-backward asymmetry. Its intersectional value with the zero axes will determine the zero position of the forward-backward asymmetry. In Section 3, we carry out the numerical analysis to study the dependence of the zero position on the new Wilson coefficients. We conclude in Section 4.

2. The Model

The matrix element of the $b \rightarrow s\ell^+\ell^-$ decay can be written as the sum of the SM contribution and the contribution from the model-independent part [16, 17]

$$\mathcal{M} = \mathcal{M}_{SM} + \mathcal{M}_{MI},$$

where $\mathcal{M}_{SM}$ is given by

$$\mathcal{M}_{SM} = \frac{G_F^2}{\sqrt{2}} V_{ts} V_{tb}^* \left\{ (C_9^s + C_10) \bar{s} L \gamma_\mu b_L \ell L \gamma^\mu \ell_L + (C_9^b + C_10) \bar{b}_R \gamma_\mu L \ell_R \gamma^\mu \ell_R - 2C_7 \bar{s} \sigma_{\mu\nu} \frac{\not{b}}{s} (\hat{m}_L + \hat{m}_R)b \ell \gamma^\mu \ell \right\}.$$ (2)

The model-independent part has ten independent local four-Fermi operators and is defined as

$$\mathcal{M}_{MI} = \frac{G_F^2}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_{LL} \bar{s} L \gamma_\mu b_L \ell L \gamma^\mu \ell_L + C_{LR} \bar{s} L \gamma_\mu b_L \ell_R \gamma^\mu \ell_R + C_{RL} \bar{s} R \gamma_\mu b_R \ell_L \gamma^\mu \ell_R + C_{RR} \bar{s} R \gamma_\mu b_R \ell_R \gamma^\mu \ell_R + C_{LRLL} b_L \ell_R \ell_R + C_{LRLR} b_R \ell_L \ell_L + C_{RRLL} b_R \ell_R \ell_R + C_{RLRL} b_R \ell_L \ell_R + C_{LLRR} b_L \ell_R \ell_R + C_{RRRR} b_R \ell_R \ell_R \right\} + iC_T \bar{s} \sigma_{\mu\nu} \ell \ell \right\}.$$ (3)

Among ten Wilson coefficients, there are four vector type interactions ($C_{LL}$, $C_{LR}, C_{RL}, C_{RR}$), four scalar type ($C_{LRLL}, C_{LRLR}, C_{RRLL}, C_{RLRL}$) and two tensor type ($C_T, C_{TE}$ interactions. Here $L$ and $R$ denote $(1 \pm \gamma_5)/2$ and $b_{L,R} = [(1 \mp \gamma_5)/2]$, $\hat{m}_b = m_b/m_B$, $\hat{m}_s = m_s/m_B$, and $q = p_B - p_K$. With these definitions the matrix element can be written as

$$\mathcal{M} = \frac{G_F^2}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ (2C_9^s + C_{LL} + C_{LR}) \bar{s} \gamma_\mu (1 - \gamma_5) \right\} + \left\{ (2C_9^b + C_{RR}) \bar{s} \gamma_\mu (1 + \gamma_5) \right\} + \left\{ \hat{m}_b \bar{s} \sigma_{\mu\nu} \gamma^\nu (1 + \gamma_5) \right\} + \left\{ (2C_{10} - C_{LL} + C_{LR}) \bar{s} \gamma_\mu (1 - \gamma_5) \right\} + \left\{ \hat{m}_b \bar{s} \sigma_{\mu\nu} \gamma^\nu (1 - \gamma_5) \right\} + \left\{ (C_{LL} + C_{LR}) \bar{s} \gamma_\mu (1 + \gamma_5) \right\} + \left\{ (C_{RR} - C_{RL}) \bar{s} \gamma_\mu (1 + \gamma_5) \right\} + \left\{ (C_{LRLL} + C_{LRLR}) \bar{s} (1 + \gamma_5) b + (C_{RRLL} + C_{RLRL}) \bar{s} (1 - \gamma_5) b \right\} + \left\{ (C_{LRLL} - C_{LRLR}) \bar{s} (1 + \gamma_5) b + (C_{RRLL} - C_{RLRL}) \bar{s} (1 - \gamma_5) b \right\} + 4C_T \bar{s} \sigma_{\mu\nu} \ell \ell + 4iC_{TE} \bar{s} \sigma_{\mu\nu} \ell \ell \right\}.$$ (4)
The expression for $C_{9}^{eff}(\hat{s})$ in the above equation is given by

$$C_{9}^{eff}(\hat{s}) = C_3 + g(z, \hat{s})(3C_1 + C_2 + 3C_2 + C_4 + 3C_5 + C_6)$$

$$- \frac{1}{2} g(1, \hat{s})(4C_1 + 4C_4 + 3C_5 + C_6)$$

$$- \frac{1}{2} g(0, \hat{s})(3C_3 + 3C_4 + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6),$$

where $z = \frac{m_K}{m_B}$, and the values of $g(z, \hat{s}), g(1, \hat{s}), g(0, \hat{s})$ can be found in [18, 19], and the values of $C_i$ in the SM are given in the numerical analysis.

In Eq. (4) we have neglected the strange quark mass. In order to calculate the matrix element describing the exclusive $B \to K \ell^+ \ell^-$ decay, with the effective Hamiltonian over B and K meson states, we need the following expressions [17]:

$$\langle K(p_K) | \bar{\sigma}_\mu b | B(p_B) \rangle = \left[ (p_B + p_K)_\mu - \frac{1 - \hat{m}_K^2}{\hat{s}} (p_B - p_K)_\mu \right] f_+$$

$$+ \frac{1 - \hat{m}_K^2}{\hat{s}} (p_B - p_K)_\mu f_0$$

with $f_+(0) = f_0(0)$;

$$\langle K(p_K) | \bar{\sigma}_\mu \gamma_5 b | B(p_B) \rangle = -i \left[ (p_B + p_K)_\mu (p_B - p_K)_\nu - (p_B + p_K)_\nu (p_B - p_K)_\mu \right]$$

$$\times \frac{f_T}{m_B + m_K};$$

$$\langle K(p_K) | \bar{s}\sigma_{\mu\nu} q^\nu b | B(p_B) \rangle = \left[ (p_B + p_K)_\mu (p_B - p_K)^2 - (m_B^2 - m_K^2)(p_B - p_K)_\mu \right]$$

$$\times \frac{f_T}{m_B + m_K};$$

$$\langle K(p_K) | \bar{b} | B(p_B) \rangle = \frac{m_B(1 - \hat{m}_K^2)}{\hat{m}_b} f_0;$$

With the help of Eqs. (1–4) the matrix element of the $B \to K \ell^+ \ell^-$ decay is written as

$$\mathcal{M} = \frac{G_F}{4\sqrt{2}\pi} V_{ts}^* V_{tb} \left\{ M_1(p_B + p_K)_\mu (\bar{\tau}^\mu \ell) + M_2(p_B - p_K)_\mu (\bar{\tau}^\mu \ell) + M_3(p_B + p_K)_\mu (\bar{\tau}^\mu \gamma_5 \ell) + M_4(p_B - p_K)_\mu (\bar{\tau}^\mu \gamma_5 \ell) + M_5(\bar{\tau} \ell) + M_6(\bar{\tau} \gamma_5 \ell) + i M_7 \left[ (p_B + p_K)_\mu (p_B - p_K)_\nu - (p_B + p_K)_\nu (p_B - p_K)_\mu \right] (\bar{\tau}^\mu \gamma_5 \ell) + M_8 \left[ (p_B + p_K)_\mu (p_B - p_K)_\nu - (p_B + p_K)_\nu (p_B - p_K)_\mu \right] (\epsilon^{\mu\nu\alpha\beta} \bar{\sigma}_{\alpha\beta} \ell), \right\}$$

(10)
where
\[ f_- = \frac{1 - \tilde{m}_K^2}{s}(f_0 - f_+) \]
and \( \mathcal{M}_i (i = 1, \ldots, 8) \) are auxiliary functions given by the following:
\[
\begin{align*}
\mathcal{M}_1 &= (2C_9^{\ell\ell} + C_{LL} + C_{LR} + C_{RL} + C_{RR})f_+ - 4\tilde{m}_b C_7^{\ell\ell} \frac{f_T}{1 + \tilde{m}_K} \\
\mathcal{M}_2 &= (2C_9^{\ell\ell} + C_{LL} + C_{LR} + C_{RL} + C_{RR})f_- + 4\tilde{m}_b C_7^{\ell\ell} \frac{1 - \tilde{m}_K}{s}f_T \\
\mathcal{M}_3 &= 2C_{10} + C_{LR} + C_{RR} - (C_{LL} + C_{RL})f_+ \\
\mathcal{M}_4 &= \left[ 2C_{10} + C_{LR} + C_{RR} - (C_{LL} + C_{RL}) \right]f_- \\
\mathcal{M}_5 &= (C_{LRLR} + C_{LRLR} + C_{RLLR} + C_{RLRL}) \frac{m_B(1 - \tilde{m}_K^2)}{\tilde{m}_b}f_0 \\
\mathcal{M}_6 &= \left[ C_{LRLR} + C_{LRLR} - (C_{LRLR} + C_{RLRL}) \right] \frac{m_B(1 - \tilde{m}_K^2)}{\tilde{m}_b}f_0 \\
\mathcal{M}_7 &= -4C_T \frac{f_T}{m_B + m_K} \\
\mathcal{M}_8 &= 4C_{TE} \frac{f_T}{m_B + m_K}.
\end{align*}
\]

Using the matrix element of the \( B \to K\ell^+\ell^- \) decay (Eq. (10)) for the differential decay width, we get
\[
\frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dsdu} = \frac{v\lambda^{1/2}(1, \tilde{m}_K^2, \tilde{s})}{2^{11/2} m_B^3 G_F^2 \alpha^2 |V_{ub}|^2 \frac{1}{8}} \left\{ |\mathcal{M}_1|^2 \lambda(1 - v^2 \cos^2 \theta) \right. \\
+ |\mathcal{M}_3|^2 \left[ \lambda(1 - v^2 \cos^2 \theta) + 4\tilde{m}_m^2(2 + \tilde{m}_m^2 - \tilde{s}) \right] \\
+ |\mathcal{M}_4|^2 \left[ 4\tilde{m}_m^2 \tilde{s} \right] \\
+ 2\text{Re}(\mathcal{M}_3\mathcal{M}_4^*)[4\tilde{m}_m^2(1 - \tilde{m}_K^2)] \\
- 2\text{Re}(\mathcal{M}_1\mathcal{M}_4^*)[2v\lambda^{1/2} \cos \theta \tilde{m}_m \tilde{s}] \\
+ 2\text{Re}(\mathcal{M}_1\mathcal{M}_7^*)[4\tilde{m}_m \lambda] + 2\text{Re}(\mathcal{M}_3\mathcal{M}_8^*)[2\frac{\tilde{m}_m}{m_B}(1 - \tilde{m}_K^2)] \\
+ 2\text{Re}(\mathcal{M}_3\mathcal{M}_9^*)[8\tilde{m}_m v\lambda^{1/2} \cos \theta \tilde{m}_m^2] \\
+ 2\text{Re}(\mathcal{M}_4\mathcal{M}_9^*)[2\tilde{m}_m \tilde{s}] - 2\text{Re}(\mathcal{M}_4\mathcal{M}_8^*)[8v\lambda^{1/2} \cos \tilde{s}] \\
+ |\mathcal{M}_5|^2 \left[ \frac{v^2}{m_B^2} \tilde{s} \right] + |\mathcal{M}_6|^2 \left[ \frac{\tilde{s}}{m_B^2} \right] \\
- 2\text{Re}(\mathcal{M}_5\mathcal{M}_6^*)[\frac{1}{2} v\lambda^{1/2} \cos \tilde{s}] \\
- 2\text{Re}(\mathcal{M}_6\mathcal{M}_7^*)[4v\lambda^{1/2} \cos \tilde{s}] \\
+ |\mathcal{M}_7|^2 \left[ 4v^2 \lambda \tilde{s} \cos^2 \theta \tilde{m}_m^2 + 4\tilde{s} \tilde{m}_m^2 - 4\lambda \tilde{m}_m^2 v^2 \right] \\
+ |\mathcal{M}_8|^2 \left[ 16v^2 \lambda \tilde{s} \cos^2 \theta \tilde{m}_m^2 \right].
\]

In Eq. (12) the variables are defined as
\[
\lambda(1, \tilde{m}_K^2, \tilde{s}) = 1 + \tilde{m}_m^4 + \tilde{s} - 2\tilde{s} - 2\tilde{m}_K^2 - 2\tilde{m}_K \tilde{s}
\]
\[ \theta \] is the angle between the four-momentum of K-meson and that of \( \ell^- \) in the dilepton CMS-frame [20] and \( v \) lepton velocity. Next, we want to determine the zero position of the forward-backward asymmetry,

\[
\frac{d}{ds} A_{FB}(s) = \int_0^1 du \frac{d\Gamma}{d\cos \theta} - \int_{-1}^0 du \frac{d\Gamma}{d\cos \theta},
\]

To discuss the effects of the new Wilson coefficients on the zero position of the forward-backward asymmetry, we compute its numerator; thus we get

\[
\mathcal{R} = \frac{G_F^2 \alpha^2}{2 \pi^3} |V_{ts} V_{tb}|^2 m_B^3 \frac{1}{4} \mathcal{N},
\]

where

\[
\mathcal{N} = 4v\lambda^{1/2} m_B m_t \left( \frac{\bar{m}_K - 1}{\bar{m}_b} \right) f_{qT} f_0 \left[ 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) + 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) \right] + 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) + 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) \left[ 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) + 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) \right] - 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) - 2\text{Re}(C_{LL}^{eff} C_{LL}^{*}) \]
the zero position of the $A_{FB}$, we analyze the variation of function $N$ with the dilepton invariant mass. The intersectional value of $N$ with the zero axis will determine the zero position of the $A_{FB}$, which can be interesting as an alternative testing platform for the SM, and provide clues about the nature of the new operators beyond the SM.

3. Numerical Analysis

For the numerical analysis we used the following values of the input parameters: $|V_{ts}^* V_{tb}| = 0.0385, 1/\alpha = 129, G_F = 1.16639 \times 10^{-5}$ GeV$^2, m_B = 5.28$ GeV, $m_K = 0.495$ GeV, $m_t = 4.8$ GeV and the numerical values of the coefficients at $\mu = m_t$ within the SM given in Table 1.

Table 1. Values of the SM Wilson coefficients used in the numerical calculations.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.248$</td>
<td>$1.107$</td>
<td>$0.011$</td>
<td>$-0.026$</td>
<td>$0.007$</td>
<td>$-0.031$</td>
<td>$-0.313$</td>
<td>$4.344$</td>
<td>$-4.669$</td>
</tr>
</tbody>
</table>

We choose the light cone QCD sum rules method to compute the form factors [21]. Thus, using the results of Ref. [21] the $\hat{s}$-dependence of any of the form factors could be parametrized as

$$F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2}. \quad (19)$$

The parameters for $F_0, a_F$ and $b_F$ for each form factor are given as follows:

<table>
<thead>
<tr>
<th></th>
<th>$f_+$</th>
<th>$f_0$</th>
<th>$f_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(0)$</td>
<td>$0.341$</td>
<td>$0.341$</td>
<td>$0.374$</td>
</tr>
<tr>
<td>$a_F$</td>
<td>$1.41$</td>
<td>$0.410$</td>
<td>$1.42$</td>
</tr>
<tr>
<td>$b_F$</td>
<td>$0.406$</td>
<td>$-0.361$</td>
<td>$0.434$</td>
</tr>
</tbody>
</table>

With the help of Eqs. (16–17) we will analyze the variation of function $N$ with the dilepton invariant mass. In forming the scatter plots, we consider two cases where each new coefficients have the values of $C_{10}$ and $-C_{10}$ to analyze the zero position of $N$.

Figure 1. The dependence of $N$ on $\hat{s}$ for $B \to Ke^+e^-$ decay corresponding to the cases $C_{LRLR} = -C_{10}$ (top curve), $C_{LRLR} = C_{10}$ (bottom curve).
**Figure 2.** The dependence of $N$ on $\hat{s}$ for $B \to K e^+ e^-$ decay which corresponds to the cases: $C_{TE} = -C_{10}$ (top curve), $C_{TE} = C_{10}$ (bottom curve).

**Figure 3.** The dependence of $N$ on $\hat{s}$ for $B \to K \mu^+ \mu^-$ decay which corresponds to the cases: $C_{LR\bar{L}R} = -C_{10}$ (top curve), $C_{LR\bar{L}R} = C_{10}$ (bottom curve).

**Figure 4.** The dependence of $N$ on $\hat{s}$ for $B \to K \mu^+ \mu^-$ decay which corresponds to the cases: $C_{TE} = -C_{10}$ (top curve), $C_{TE} = C_{10}$ (bottom curve).
Figure 5. The dependence of $\mathcal{N}$ on $\hat{s}$ for $B \to K\tau^+\tau^-$ decay which corresponds to the cases: $C_{LRLR} = -C_{10}$ (top curve), $C_{LRLR} = C_{10}$ (bottom curve).

Figure 6. The dependence of $\mathcal{N}$ on $\hat{s}$ for $B \to K\tau^+\tau^-$ decay which corresponds to the cases: $C_{TE} = -C_{10}$ (bottom curve), $C_{TE} = C_{10}$ (top curve).

In all plots, the dependence of the zero position of the forward-backward asymmetry for three scalar operators ($C_{LRLR}, C_{RLLR}, C_{RLRL}$) is the same with the dependence for $C_{LRLR}$ in all lepton cases. In addition, the plot of the tensor type coefficient $C_T$ is the same as the plot of $C_{TE}$ in the $e, \mu, \tau$ lepton cases. In Figs. (1) and (2) we plot the dependence of $\mathcal{N}$ on $\hat{s}$ for the Wilson coefficients $C_{LRLR} = \pm C_{10}$ and $C_{TE} = \pm C_{10}$ in the $B \to Ke^+e^-$ decay. Similarly, Figs. (3) and (4) show the dependence of $\mathcal{N}$ on $\hat{s}$ for the Wilson coefficients $C_{LRLR} = \pm C_{10}$ and $C_{TE} = \pm C_{10}$ for $B \to K\mu^+\mu^-$ decay. In Figs. (5) and (6), we show the dependence of $\mathcal{N}$ on $\hat{s}$ for the Wilson coefficients $C_{LRLR} = \pm C_{10}$ and $C_{TE} = \pm C_{10}$ in the $B \to K\tau^+\tau^-$ decay.

A comparative analysis of Figs. (1–6) shows that the zero of the asymmetry are highly sensitive to the sign and size of the tensor operators. On the other hand its zero are less sensitive to scalar type coefficients for three cases. We would like to note that in the $B \to K\ell^+\ell^-$ decay, the forward-backward asymmetry is zero in the SM [12]. However, in the existence of scalar and tensor operators, it is seen that the forward-backward asymmetry has a non-vanishing value. Therefore, any non-zero measurement of the asymmetry in this system will certainly signals for new physics effects.

4. Conclusion

We have analyzed the sensitivity of the zero position of the forward-backward asymmetry to the new physics effects for the $B \to K\ell^+\ell^-(\ell = e, \mu, \tau)$ decay. It is found that the asymmetry is different from zero
although it vanishes in the SM. The numerator of the $A_{FB}$ depends on only six new Wilson coefficients. The dependence of the zero position of the $A_{FB}$ is sensitive to the scalar and tensor type coefficients and the asymmetry does not vanish anywhere in the kinematical region.

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References


