Properties of Cold and Hot Polarized Nuclear Matter with Realistic Nucleon Nucleon Interaction

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Abstract

The Thomas-Fermi (TF) model is used to calculate the equation of state of thermal polarized nuclear-matter (NM) within a nonrelativistic Hartree-Fock (HF) scheme. The potential employed is a new realistic version of the density-dependent M3Y effective nucleon-nucleon (NN) interaction where the Yukawa strengths are based on the G-matrix of the Paris interaction. To study the basic properties of asymmetric nuclear matter, this potential is generalized by introducing spin and spin-isospin components into the original M3Y effective NN interaction. The resulting equation of state (EOS) is soft. The results obtained are in reasonable agreement with previous theoretical estimates.

1. Introduction

In recent years, the study of the EOS of asymmetric nuclear matter has been a subject of growing interest. The EOS is particularly connected with astrophysical problems, such as supernova explosion [1], the evolution of neutron stars and in nuclear physics such as heavy ion (HI) collisions [2]. Microscopic calculations of the EOS at finite temperature are quite rare. Most of these calculations consider the symmetric case, while only few consider the asymmetric one.

In the temperature and density domains of the liquid-gas phase transition occurring in NM, the EOS is derived from the nucleon-nucleon (NN) interaction which is the most important input in the calculation of the properties of NM. The effective interaction can be either obtained from a sophisticated G-matrix calculation [3] or parameterized directly as a whole in a simple form convenient for numerical calculations such as Skyrme forces [4].

Among different kinds of effective interaction, the so-called M3Y interaction [5] has been widely used. However, due to the attractive character of the M3Y forces, the NM binding energy increases with the density; the saturation condition is, therefore, not fulfilled leading to NM collapse [6].

Khoa et al. [7] showed that an improved description of the NM properties can still be obtained on the level of non-relativistic HF calculations provided an appropriate density dependence is introduced into the original M3Y interaction.

To study asymmetric NM many authors have considered only the neutron excess parameter [7,8]. If the spin terms are usually but relatively unimportant for determining cross sections, they become important for polarized nuclear matter (PNM) calculations.

In the present work, we shall construct a general form of the density-dependent M3Y Paris interaction which is suitable for PNM. This potential will contain an additional two terms to account for the spin and spin-isospin asymmetries. This analysis is extended to the case of hot PNM by applying TF model in the $T^2$ approximation.
The aim of this paper is, therefore, to investigate whether this new potential predicts different symmetry energies and related quantities of PNM. Moreover, one would like to trace possible different trends in these properties arising from different types of the density dependence.

A brief description of the generalized density-dependent interaction and some details of the numerical calculations are given in section II. In section III, the results are presented and discussed.

2. Theory

The description of the ground state properties of NM in terms of a microscopic NN interaction has been one of the major challenges in modern nuclear physics.

We shall take an extended density dependent effective interaction based on the original M3Y Paris interaction, suggested in Ref. [7], with explicit form of the spin and spin-isospin dependent components.

The direct and exchange parts of the central NN forces for infinite NM has the following general form in the r space:

\[ V_{D(EX)}(r) = V_{oD}^{EX}(r) + V_{\sigma D}^{EX}(r)\sigma_1 \cdot \sigma_2 + V_{\tau D}^{EX}(r)\tau_1 \cdot \tau_2 + V_{\sigma\tau D}^{EX}(r)(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) \]  

(1)

where

\[ V_{oD}(r) = 11061.625 e^{-4r/4r} - 2537.5 e^{-2.5r/2.5r}; \]

\[ V_{\sigma D}^{EX}(r) = -1524.25 e^{-4r/4r} - 518.75 e^{-2.5r/2.5r} - 7.8474 e^{-0.7072r/0.7072r}; \]

\[ V_{\tau D}^{EX}(r) = 313.625 e^{-4r/4r} + 223.5 e^{-2.5r/2.5r}; \]

\[ V_{\sigma\tau D}^{EX}(r) = -4118.0 e^{-4r/4r} + 1054.75 e^{-2.5r/2.5r} + 2.6157 e^{-0.7072r/0.7072r}; \]

(2a)

\[ V_{oD}(r) = 3938.875 e^{-4r/4r} - 36 e^{-2.5r/2.5r}; \]

\[ V_{\sigma D}^{EX}(r) = -3492.75 e^{-4r/4r} + 795.25 e^{-2.5r/2.5r} + 2.615 e^{-0.7072r/0.7072r}; \]

\[ V_{\tau D}^{EX}(r) = -969.125 e^{-4r/4r} + 450 e^{-2.5r/2.5r}; \]

\[ V_{\sigma\tau D}^{EX}(r) = -2210 e^{-4r/4r} + 568.75 e^{-2.5r/2.5r} - 0.872 e^{-0.7072r/0.7072r}; \]

(2b)

These terms are determined from the singlet and triplet even and odd components of the M3Y-Paris two nucleon forces [9].

Since the original M3Y interaction gives a wrong description of the saturation properties of cold symmetric NM, a realistic density dependence has been introduced into Interaction (1):

\[ V_{D(EX)}(\rho, r) = f(\rho) V_{D(EX)}(r) \]  

(3)

and

\[ f(\rho) = \begin{cases} 
C(1 + \alpha \exp(-\beta \rho)) & \text{DDM3Y type} \\
C(1 - \alpha \rho^\beta) & \text{BDM3Y type}
\end{cases} \]  

(4)
where the parameters $C$, $\alpha$ and $\beta$ are adjusted to reproduce the saturation of symmetric cold NM at density $\rho = 0.17 \text{ fm}^{-3}$ with a binding energy $E/A \approx 16 \text{ MeV}$. One may notice that for the DDM3Y type $\beta$ is given in fm$^3$ and $\alpha$ is dimensionless, while for the BDM3Y type $\beta$ is dimensionless and $\alpha$ is given in fm$^3$.

In a HF calculation, the ground state energy of cold NM is given by:

$$E = E_{\text{kin}} + 1/2 \sum_{k_1 \sigma_1 \tau_1} \sum_{k_2 \sigma_2 \tau_2} \left[ <k_1 \sigma_1 \tau_1 k_2 \sigma_2 \tau_2 | V^D | k_1 \sigma_1 \tau_1 k_2 \sigma_2 \tau_2> + <k_1 \sigma_1 \tau_1 k_2 \sigma_2 \tau_2 | V^{EX} | k_2 \sigma_1 \tau_1 k_1 \sigma_2 \tau_2> \right],$$

(5)

where $|k_1 \sigma_1 \tau_1>$ are ordinary plane waves and $V^{D(EX)}$ are given by Equation (3).

PNM is composed of number $N^\uparrow$ ($N^\downarrow$) of spin up (spin down) neutrons and $P^\uparrow$ ($P^\downarrow$) spin up (spin down) protons, with corresponding $\rho_\uparrow$, $\rho_\downarrow$, $\rho_\uparrow$, $\rho_\downarrow$, respectively. Thus,

$$A = N^\uparrow + N^\downarrow + P^\uparrow + P^\downarrow$$

(6)

is the total number of particles and the total density $\rho$ is

$$\rho = \rho_\uparrow + \rho_\downarrow = \rho_\uparrow + \rho_\downarrow + \rho_\uparrow + \rho_\downarrow .$$

(7)

For PNM, the following parameter are usually defined [10,11].

The neutron excess parameter is

$$X = (\rho_\uparrow - \rho_\downarrow)/\rho ;$$

the neutron spin up excess parameter is

$$\alpha_n = (\rho_\uparrow - \rho_\downarrow)/\rho ;$$

and the proton spin up excess parameter is

$$\alpha_p = (\rho_\uparrow - \rho_\downarrow)/\rho .$$

In this general case, there are four Fermi momenta, namely $k_n(\lambda_n)$ for neutrons with spin up (spin down) and $k_p(\lambda_p)$ for protons with spin up (spin down).

These Fermi momenta are related to the excess parameters through

$$\lambda_n^3 = k_n^3 (1 + X \pm Y \pm Z)$$

(8)

and

$$\lambda_p^3 = k_p^3 (1 - X \pm Y \pm Z),$$

(9)

taking $k_f$ being the Fermi momentum of unpolarized symmetric NM (taken to be equal to 1.36 fm$^{-1}$),

$$Y = \alpha_n + \alpha_p$$

and

$$Z = \alpha_n - \alpha_p$$

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In the present calculations, one shall use the generalized form given by Equation (3) for the interaction. Using this form, the energy per nucleon of PNM at zero temperature can be written as:

\[ E_0 = E_v + X^2 E_x + Y^2 E_y + Z^2 E_z, \]  

where

\[ E_v = \frac{3h^2 k_f^2}{10m} + \frac{f(\rho)\rho}{2} (J_D^0 + \int J_x^2(k_f r) V_{EX}^E(r) dr) \]  

\[ E_{x,y,z} = \frac{h^2 k_f^2}{6m} + \frac{f(\rho)\rho}{2} \left\{ \int -j_1^2((k_f r)V_{EX}^E(r)dr + J_D^p \right\} \]

where \( J_D^i = J_i(V_d(r)) dr; i = 0, \tau, \sigma, \sigma \)

\( J_1(X) = \frac{j_1(X)}{X}, \)

\( j_n(x) \), is the nth order spherical Bessel function and m is the nucleon mass. Terms higher than quadratic in X, Y and Z are neglected in Equation (10).

The pressure of a nuclear system is given by

\[ P = \rho \frac{\partial E}{\partial \rho} \]  

and the compressibility is

\[ K = 9 \frac{\partial \rho}{\partial \rho}, \]  

which is valid also outside the P=0 saturation point.

Using Equation (10), one gets the pressure and the compressibility, at zero temperature as

\[ P_0 = P_v + X^2 P_x + Y^2 P_y + Z^2 P_z \]  

and

\[ K_0 = K_v + X^2 K_x + Y^2 K_y + Z^2 K_z. \]

Using Equations (11) and (12) for \( E_v, E_x, E_y \) and \( E_z \), one gets simple analytical equation for \( P_v(K_v), P_x(K_x), P_y(K_y) \) and \( P_z(K_z) \).

To study the PNM at finite temperature, it is well known that the thermodynamics properties of NM are determined completely if the free energy \( F \) is known in terms of the density \( \rho \) and the temperature \( T \), where

\[ F(\rho, T) = E(\rho, T) - TS(\rho, T), \]  

\( E \) being the total energy and \( S \) is the entropy. The relevant equations are derived in Ref. [12] where the reader is referred to for details. According to the Fermi liquid approximation of Landau, the entropy of the
system can be calculated in terms of the Fermi integrals which are temperature dependent. One gets the entropy of the PNM in the $T^2$ approximation as:

$$S(\rho, T) = \frac{T}{3} \left( \frac{2m^*_{s,n}}{\hbar^2} \right)^{\frac{3}{2}} \rho^{\frac{1}{2}} \rho^{-2\frac{3}{2}} \left[ 1 - \frac{1}{9} (X^2 + Y^2 + Z^2) \right],$$

where $m^*_{s,n}$ is the effective mass which can be readily obtained from the determination of the single nucleon potential. One notices that the only dependence of the thermal properties of the system on the effective interaction, is through the dependence of the entropy on the effective mass.

Therefore, the free energy can be written as

$$F(\rho, T) = E_\rho(\rho, T = 0) + E_T(\rho, T),$$

where $E_\rho$ is given by Equation (10) and $E_T$ is the temperature dependent part of the energy and is given by

$$E_T(\rho, T) = -\frac{T^2}{6} \left( \frac{2m^*_{s,n}}{\hbar^2} \right)^{\frac{3}{2}} \rho^{-2\frac{3}{2}} \left[ 1 - \frac{1}{9} (X^2 + Y^2 + Z^2) \right].$$

One now obtains the pressure and the compressibility at finite temperature as follows:

$$P(\rho, T) = P_\rho(\rho, T = 0) + P_T(\rho, T)$$

and

$$K(\rho, T) = K_\rho(\rho, T = 0) + K_T(\rho, T),$$

where $P_\rho$ and $K_\rho$ are given by Equations (15) and (16), $P_T$ and $K_T$ are directly derived from Equation (20).

3. Results and Discussion

In the present work, the PNM has been studied within the HF scheme, using a generalized density dependent M3Y Paris interaction, which is a more realistic interaction. In the calculations reported here, explicit form of the spin and spin-isospin dependent components of the NN Paris interaction have been considered. One emphasizes that the motivation of the present analysis is not to reconsider the symmetric unpolarized NM nor the asymmetric cold NM arising from the neutron excess parameter [7], but mainly to study the effect of the spin on the asymmetric cold NM as well as hot NM and the effect of the neutron excess parameter on PNM at finite temperature. The terms $E_x$, $E_y$, and $E_z$ for cold NM (with the corresponding derived quantities) are only pointed out for completeness and purposes of comparison. Moreover, since the interaction is density-dependent, the effect of the density dependent term has been also investigated. The set of parameters $C$, $\alpha$, and $\beta$ of $f(\rho)$ take the values given in Table 1 of Ref. [7] for the Paris interaction.

Figures 1 and 2 show the spin energy $E_y$ and the spin-isospin energy $E_z$ as a function of the relative density $(\rho/\rho_0)$ for the three different versions of the density dependence. While these values increased steadily with density for the DDM3Y1 type force, those obtained with BDM3Y type forces begin to decrease at densities $\rho > 1.5 \rho_0$. This may be attributed to the fact that the exponential dependence on $\rho$ does not change sign as $\rho$ increases, while for the power law dependence the sign can change for large $\rho$. An increasing behavior is observed for the dependence of $E_x$ on the density [7]. This result which is confirmed by Engvik et al. [8] where different types of potential have been used may favor the DDM3Y1 type of interaction. The three energies $E_x$, $E_y$ and $E_z$ at $\rho = \rho_0$ are 29.72 MeV, 20.53 MeV and 26.54 MeV, respectively, for the three types of interaction. These values are relatively smaller than those found by many authors [12,13]. However, Maheswari et al. [14], using the Seyler-Blanchard potential, found that the observed maximum neutron star masses and the surface magnetic field are best explained with $E_y = 15$ MeV corresponding to
$E_x = 33.4$ MeV and $E_z = 36.5$ MeV. In general, the values of $E_y$ and $E_z$ are uncertain to some extent. As an illustration, the four terms, namely $E_v$, $E_x$, $E_y$, and $E_z$, are displayed in Figure 3 for the density dependent type interaction DDM3Y1. The pressure density isotherm at $T = 0$, $P_y$ and $P_z$ are plotted in Figures 4 and 5. Here again, a similar behavior is observed concerning the effect of the different versions of the density dependence.

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1.** Spin symmetry energy as a function of the relative density $\rho/\rho_o$ for cold PNM, using different density dependent M3Y interaction.

**Figure 2.** The same as Figure 1 for the spin-isospin symmetry energy.

![Figure 3](image3.png)  ![Figure 4](image4.png)

**Figure 3.** Symmetry energy $E_x$, spin symmetry energy $E_y$, spin-isospin symmetry energy $E_z$, as a function of the relative density $\rho/\rho_o$ for cold PNM, together with the volume energy $E_v$, using the DDM3Y1 interaction.

**Figure 4.** The pressure-density isotherm of the spin symmetry pressure $P_y$ for cold PNM, using different density dependent M3Y interaction.

A plot of the four terms $P_v$, $P_x$, $P_y$ and $P_z$ is presented in Figure 6 corresponding to the DDM3Y1 type. Clearly the saturation condition is satisfied together with the binding energy (see Figure 3). The nuclear compressibility $K$ plays a crucial role in the determination of the EOS. However, the value of $K$ has been subject to intense debate. Values have been extracted from the excitation energies of the observed giant monopole resonance [15] and from supernova calculations [16]. The spin symmetry compressibility and the
spin-isospin symmetry compressibility versus the relative density $\rho/\rho_0$ are shown in Figures 7 and 8. The values of $K_x$, $K_y$, $K_z$ and $K_v$ at equilibrium, corresponding to different types of force are listed in Table 1.

![Figure 5](image1.png)

**Figure 5.** The same as Figure 4 for the spin-isospin symmetry pressure.

![Figure 6](image2.png)

**Figure 6.** The symmetry pressure $P_x$, the spin symmetry pressure $P_y$ and the spin-isospin symmetry pressure $P_z$ as a function of the relative density $\rho/\rho_0$ for cold PNM, together with the volume pressure $P_v$, using the DDM3Y1 interaction.

![Figure 7](image3.png)

**Figure 7.** Spin symmetry compressibility as a function of the relative density $\rho/\rho_0$ for cold PNM using a different density dependent M3Y interaction.

![Figure 8](image4.png)

**Figure 8.** The same as Figure 7 for the spin-isospin symmetry compressibility.

<table>
<thead>
<tr>
<th>Type of force</th>
<th>$K_x$ (MeV)</th>
<th>$K_y$ (MeV)</th>
<th>$K_z$ (MeV)</th>
<th>$K_v$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDM3Y1</td>
<td>169.69</td>
<td>146.05</td>
<td>188.47</td>
<td>176.23</td>
</tr>
<tr>
<td>BDM3Y0</td>
<td>150.24</td>
<td>137.16</td>
<td>171.78</td>
<td>218.12</td>
</tr>
<tr>
<td>BDM3Y1</td>
<td>128.79</td>
<td>127.35</td>
<td>154.34</td>
<td>270.05</td>
</tr>
</tbody>
</table>

The present model of calculations predicts a soft EOS (K ranging from 176-270 MeV). These values are in agreement with those given by Myers and Swiatecki: $K = 234 \text{ MeV}$ [17] which are close to that found by Blaizot et al.: $K = 210 \pm 30 \text{ MeV}$ [18].
Two conclusions can be deduced from the above table. First, the compressibility generally decreases with asymmetry (the only exception is found for $K_z$ corresponding to the DDM3Y1 interaction). The softening of the EOS with asymmetry was predicted by many authors [8,19,20]. Second, if for $K_v$ it was found that, when employing the BDM3Y type of force, the larger the parameter $\beta$ the harder the EOS [6], the situation reversing for the symmetry terms of the compressibility. Therefore, for the BDM3Y type of interaction, a strong softening occurs when going from symmetric to asymmetric NM. The dependence of the compressibilities $K_v$, $K_x$, $K_y$ and $K_z$ on the density is illustrated in Figure 9, using the DDM3Y1 interaction. From the above analysis of the asymmetric cold NM, one concludes that the compressibility $K$ is very sensitive to the different types of interaction. This result reflects the importance of the accurate determination of the nuclear compressibility for the study of properties of nuclei (radii, masses, ...) supernova collapses and HI collisions.

Microscopic calculations of the nuclear EOS at finite temperature are quite few. One of these first few semi-microscopic investigation of the finite temperature EOS was that carried out by Friedman and Pandharipande [21] in a variational calculation.

The free energy $F_v$ at temperature $T = 5$ MeV is presented in Figure 10 as a function of $\rho/\rho_0$ for different $f(\rho)$. The results obtained are very close to that found by FP [21] and by Mansour [22] who used the Seyler-Blanchard potential, particularly for the BDM3Y type of interaction with a little bit higher values for large $\rho$. The influence of the density on the symmetry free energy $F_x$, the spin symmetry free energy $F_y$ and the spin-isospin symmetry free energy $F_z$ at $T = 5$ MeV for the DDM3Y1 interaction is shown in Figure 11. The results reported here for the free energies $F_v$, $F_x$, $F_y$ and $F_z$ as a function of temperature are displayed in Figure 12 using DDM3Y1 interaction. The free energy $F_v$ decreases with increasing temperature. This result is in agreement with that obtained in Refs. [1,21,22,23]. The symmetry terms $F_x$, $F_y$ and $F_z$ almost increase with temperature. Abd-Alla et al. [24] obtained the same result using a generalized Skyrme interaction. The different dependence of the free energy function on temperature arises most probably from the opposite sign of the entropy. The pressure-density isotherm at $T = 5$ MeV is plotted in Figure 13 for different types of force. The results obtained for the BDM3Y interaction are in agreement with those predicted in Refs. [21,22]. While the pressure-density isotherms obtained with the DDM3Y1 interaction is comparable to that found in Ref. [24], particularly for large $\rho$. An increasing behavior of the pressures $P_x$, $P_y$ and $P_z$ with density is observed in Figure 14, at $T = 5$ MeV and using DDM3Y1 interaction. The pressures $P_v$, $P_x$, $P_y$ and $P_z$ as a function of temperature are displayed in Figure 15 for the DDM3Y1 interaction. The increase of $P_v$ with temperature is consistent with the calculations carried out in Refs. [1,22,24]. The compressibility $K_v$.

![Figure 9](image9.png)  
**Figure 9.** The symmetry compressibility $K_x$, the spin symmetry compressibility $K_y$ and the spin-isospin symmetry compressibility $K_z$ as a function of the relative density $\rho/\rho_0$ for cold PNM, together with the volume term $K_v$ using the DDM3Y1 interaction.

![Figure 10](image10.png)  
**Figure 10.** Free energy as a function of the relative density $\rho/\rho_0$ at $T = 5$ MeV for unpolarized symmetric NM, using different density-dependent M3Y interaction.
as a function of the relative density, at $T = 5$ MeV and for different $f(\rho)$ is presented in Figure 16. One may notice the difference in the order of magnitude of the $K$ values corresponding to the two different types of interaction, namely DDM3Y and BDM3Y. Figure 17 shows a plot of the symmetry compressibilities, $K_x$, $K_y$, and $K_z$ as a function of $\rho/\rho_0$ at $T = 5$ MeV and using DDM3Y1 interaction. Figure 18 reflects the effect of increasing the temperature on the compressibilities $K_v$, $K_x$, $K_y$, and $K_z$ for the BDM3Y0 interaction. The temperature has a little decreasing effect on $K_x$, $K_y$ and $K_z$. The volume compressibility $K_v$ drops more rapidly with increasing temperature. This result is in agreement with previous works [25,26]. A similar behavior, for the dependence of the compressibilities on temperature using DDM3Y1 and BDM3Y1 is observed. But one has chosen the BDM3Y0 type to show also the dependence of the compressibilities on the asymmetry (see Table 1).

**Figure 11.** The symmetry free energy $F_x$, the spin symmetry free energy $F_y$, and the spin-isospin symmetry free energy $F_z$ as a function of the relative density $\rho/\rho_0$ at $T = 5$ MeV using the DDM3Y1 interaction.

**Figure 12.** The free energies $F_x$, $F_y$, and $F_z$ for PNM together with $F_v$ as a function of temperature using the DDM3Y1 interaction.

**Figure 13.** The pressure-density isotherm at $T = 5$ MeV for unpolarized NM, using different density-dependent M3Y interaction.

**Figure 14.** The symmetry pressure $P_x$, the spin-symmetry pressure $P_y$, and the spin-isospin symmetry pressure $P_z$ as a function of the relative density $\rho/\rho_0$ at $T = 5$ MeV using the DDM3Y1 interaction.
Figure 15. The pressures $P_x$, $P_y$ and $P_z$ of PNM, together with $P_v$ as a function of temperature, using the DDM3Y1 interaction.

Figure 16. The compressibility as a function of the relative density $\rho/\rho_o$ at $T = 5$ MeV for unpolarized symmetric NM using different density-dependent M3Y interaction.

Figure 17. The compressibilities $K_x$, $K_y$ and $K_z$ as a function of the relative density $\rho/\rho_o$ at $T = 5$ MeV using the DDM3Y1 interaction.

Figure 18. The compressibilities $K_x$, $K_y$ and $K_z$ of PNM, together with $K_v$ as a function of temperature, using the BDM3Y0 interaction.

4. Summary

We have constructed a generalized and realistic explicit spin and spin-isospin dependence with density dependences into the original M3Y effective NN interaction that were based upon the G-matrix elements of Paris NN potentials. The values of the parameters that describe the density dependence have been fixed so as to reproduce the saturation properties of normal NM within a non-relativistic HF scheme. This has been applied to investigate several properties of polarized cold and hot NM with particular attention to the influence of the different types of interaction on these properties. The results obtained are compared with the available data.

The present calculations predict a soft EOS and a decrease of the nuclear compressibility coming from both the temperature and the asymmetry dependence. This result is in agreement with a realistic supernova evolution. One can say that the method and potential proposed here are quite satisfactory for the purpose.
of the present work. However, one can argue that there are still missing effects, such as three-body forces and/or relativistic effects which are not taken into account in the present calculations and can change the results.

References