Turk J Phys
25 (2001) , 499 – 511.

A Theoretical Analysis of Transverse Mode Control in Waveguide Gas Lasers

Erol TAŞAL, M. Selami KILIÇKAYA
Department of Physics, Osmangazi University, Eskisehir-TURKEY

Received 17.11.2000

Abstract

Theoretical of multiple transverse mode laser oscillation involving spatially varying gain and loss are investigated. The effect of gain and loss distribution on mode competition is analyzed. A theoretical analysis of laser transverse mode competition is investigated from the perspective of the spetial overlap of modes with a transverse gain-loss distribution. The dominant mode of laser oscillation is the mode that is stable under small perturbations.

Key Words: Waveguide gas lasers, Two – mode system, Transverse mode, Gain and loss.

1. Introduction

In this study, we investigated a general analytical method to understand the interaction between the modes and the medium and its effect on mode competition. In section 2.1 saturation of population inversion by multiple transverse modes is investigated [1-5]. Section 2.2 considers the spatial variation of the cavity loss and gain, and gain saturation. Differential equations for laser mode development in time are derived. Section 2.3 studies the stability of single-mode solutions in a two-mode system. The conditions on gain and loss distributions for competing modes to oscillate are derived [6].

2. Theory

2.1. Saturation of Population Inversion by Multiple Transverse Modes

This analysis is based on a small-signal approximation, which is not valid in our situation. Thus, we derive a new relation by starting from Yariv's analysis and expand it to mutli mode case. We investigated density matrix elements given in Yariv [1] of the form

$$\frac{d}{dt}\rho_{21} = -i\omega_0\rho_{21} + i\frac{\mu}{\hbar}E(t)(\rho_{11} - \rho_{22}) - \frac{\rho_{21}}{T_2} \tag{1}$$

$$\frac{d}{dt}(\rho_{11} - \rho_{22}) = i2\frac{\mu}{\hbar}E(t)(\rho_{21} - \rho_{21}^*) - \frac{(\rho_{11} - \rho_{22}) - (\rho_{11} - \rho_{22})_0}{\tau}. \tag{2}$$

Assume a single longitudinal mode that can oscillate in any of N transverse modes. The frequency differences among these modes are very small compared to the gain line width. Define the linearly polarized electric field as follows:

$$E(t) = \frac{1}{2}\sum_{j}^{N} e_{j0}(t)e^{-i\omega_j t}\phi_j(r) + c.c., \tag{3}$$

where $e_{j0}(t)$is a slowly varying function of time.

We can write the off-diagonal density matrix element ρ_{21} as follows:

$$\rho_{21}(t) = \sigma_{21}(t)e^{i\omega_a t} \tag{4}$$

$$\omega_a = \frac{\sum_{j}^{N}\omega_j}{N}. \tag{5}$$

Thus for σ_{21} we obtain an equation different from Equation (1):

$$\dot{\sigma}_{21} = i(\omega_a - \omega_0)\sigma_{21} + i\frac{\mu}{2\hbar}(\rho_{11} - \rho_{22}) \cdot \sum_{j}^{N} e_{j0}\phi_j e^{i\Delta_{aj}t} - \frac{\sigma_{21}}{T_2}, \tag{6}$$

where Δ_{aj} is the frequency difference:

$$\Delta_{aj} = \omega_a - \omega_j, \tag{7}$$

and ω_a, ω_j are high-frequency oscillatory terms. High frequency Equations (6) are ignored because their contributions to the integration average to zero over the time scale for variations in σ_{21}. Multiply both sides of Equation (6) by the factor $e^{[i(\omega_0-\omega_a)+(1/T_2)]t}$ and with some manipulation, Equation (6) becomes

$$\frac{\partial}{\partial t}\left(\sigma_{21}e^{\left[i(\omega_0-\omega_a)+\left({}^{1}\!/\!{T_2}\right)\right]t}\right) = i\frac{\mu}{2\hbar}(\rho_{11} - \rho_{22}) \cdot \sum_{j}^{N} e_{j0}\phi_j e^{\left[i(\omega_0-\omega_j)+\left({}^{1}\!/\!{T_2}\right)\right]t}. \tag{8}$$

Assuming the rate of change of $(\rho_{11} - \rho_{22})$ and e_{j0} are much slower than ${}^{1}\!/\!{T_2}$, we can pull them out of the integration. Then we obtain by integrating Equation (8)

$$\sigma_{21} = i\frac{\mu}{2\hbar}\left(\rho_{11}-\rho_{22}\right)\sum_{j}^{N} e_{j0}\phi_j e^{i\Delta_{aj}t} D\left(T_2,\omega_0-\omega_j\right), \tag{9}$$

where $D\left(T_2,\omega_0-\omega_j\right)$ is defined as

$$D\left(T_2,\omega_0-\omega_j\right) = \frac{1}{{}^{1}\!/_{T_2} + i\left(\omega_0-\omega_j\right)}. \tag{10}$$

The term $\rho_{21}-\rho_{21}^*$ in equation (2) becomes

$$\rho_{21}-\rho_{21}^* = \sigma_{21}(t)e^{-i\omega_a t} - \sigma_{21}^*(t)e^{i\omega_a t} = i\frac{\mu}{2\hbar}\left(\rho_{11}-\rho_{22}\right) \left[\sum_{j}^{N} e_{j0}\phi_j e^{i\omega_j t} D\left(T_2,\omega_0-\omega_j\right) + c.c.\right]. \tag{11}$$

Thus, we obtain

$$\frac{d}{dt}\left(\rho_{11}-\rho_{22}\right) = i\frac{\mu}{2\hbar}E(t)\left(\rho_{21}-\rho_{21}^*\right) - \frac{\left(\rho_{11}-\rho_{22}\right)-\left(\rho_{11}-\rho_{22}\right)_0}{\tau}$$

$$= -\frac{1}{2}\left(\frac{\mu}{\hbar}\right)^2\left(\rho_{11}-\rho_{22}\right)\left[\sum_{j}^{N} e_{j0}e^{-i\omega_j t}\phi_j(r) + c.c.\right] \times \left[\sum_{j}^{N} e_{j0}\phi_j(r)e^{i\omega_j t} D\left(T_2,\omega_0-\omega_j\right) + c.c.\right]. - \frac{\left(\rho_{11}-\rho_{22}\right)-\left(\rho_{11}-\rho_{22}\right)_0}{\tau} \tag{12}$$

If the frequency spread of the transverse modes is small compared with the line width, we can approximate $D(T_2,\omega_0\text{-}\omega_j)$ with $D(T_2,\omega_0\text{-}\omega_a)$. And if the rate of change of ρ_{11}-ρ_{22} is much slower than $e^{i(\omega_i-\omega_k)t}$ for any $k \neq j$, we can keep only the terms without exponential time dependence on the right side of Equation (12):

$$\frac{d}{dt}\left(\rho_{11}-\rho_{22}\right) = -\frac{1}{2}\left(\frac{\mu}{\hbar}\right)^2\left(\rho_{11}-\rho_{22}\right)\cdot\left[D\left(T_2,\omega_0-\omega_a\right)+c.c.\right]\sum_{j}^{N} e_{j0}e_{jo}^*\phi_j^2(r) - \frac{\left(\rho_{11}-\rho_{22}\right)-\left(\rho_{11}-\rho_{22}\right)_0}{\tau} = -\frac{1}{2}\left(\frac{\mu}{\hbar}\right)^2\left(\rho_{11}-\rho_{22}\right)\frac{2T_2}{1+T_2^2\left(\omega_0-\omega_a\right)^2}\cdot\sum_{j}^{N} e_{j0}e_{jo}^*\phi_j^2(r) - \frac{\left(\rho_{11}-\rho_{22}\right)-\left(\rho_{11}-\rho_{22}\right)_0}{\tau}. \tag{13}$$

We make the standard definitions of the normalized line shape function as follows:

$$g(\omega) = \frac{2T_2}{1+T_2^2\left(\omega_0-\omega_a\right)^2}; \tag{14}$$

the saturation intensity:

$$I_s = cn\varepsilon_0 \frac{\hbar^2}{\mu^2 \tau g(\omega_a)} = \frac{1}{\eta} \frac{\hbar^2}{\mu^2 \tau g(\omega_a)}; \tag{15}$$

and the laser intensity as:

$$I = \frac{1}{2\eta} \sum_j^N e_{j0} e_{j0}^* \phi_j^2(r) = \sum_j^N I_j. \tag{16}$$

Now we can write the steady-state expression for $(\rho_{11} - \rho_{22})$ as

$$\rho_{11} - \rho_{22} = \frac{(\rho_{11} - \rho_{22})_0}{1 + \frac{I}{I_s}} \tag{17}$$

From [4], the susceptibility satisfies the following:

$$\chi = \rho_{11} - \rho_{22} \tag{18}$$

and

$$\chi = -\frac{n^2 c}{\omega} \gamma, \tag{19}$$

where γ is the gain coefficient of lasing medium in units of $[\mathrm{m}^{-1}]$ [2].

2.2. Model and Theory

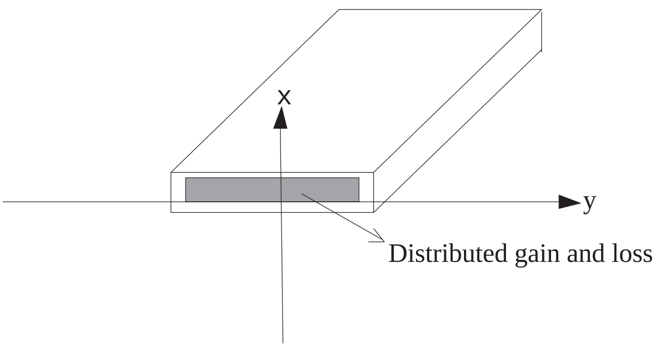

Figure 1. Schematic showing the general structure of the waveguide array of two channels with distributed gain and loss [3].

We concentrate our attention onto the coupled waveguide laser array, the general form of which is shown Figure 1 [3]. For this model, Maxwell's equations are given by

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial}{\partial t} P_{laser} \tag{20}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}, \tag{21}$$

where the current density J is introduced to account for the loss of energy that is not in resonance with the laser, for example, the loss aassociated with mirror transmission or the loss introduced by the waveguide boundaries; and P_{laser} is the complex polarization of the medium that resonates with the laser [7].

Using the normal mode expansion of the resonator field, the total electric and magnetic field are given as follows:

$$E(r,t) = \sum_{j}^{N} e_j(t)\phi_j(r) \tag{22}$$

$$H(r,t) = \sum_{j} h_j(t)\chi_j(r), \tag{23}$$

where the modal field $\phi_j(r)$ and $\chi_j(r)$ are dimensionless and are normalized to the volume of the cavity V:

$$\frac{1}{V}\int_V \phi_j(r)\cdot\phi_k(r)dV = \delta_{jk} \tag{24}$$

$$\frac{1}{V}\int_V \chi_j(r)\cdot\chi_k(r)dV = \delta_{jk} \tag{25}$$

The loss and the resonant polarization become

$$J = \sigma(r)\sum_{i} e_j(t)\phi_j(r) \tag{26}$$

$$P_{laser} = \chi(r,\omega_j)\sum_{j} e_j(t)\phi_j(r). \tag{27}$$

The formal conductivity $\sigma(r)$ and the complex dielectric susceptibility $\chi(r)$ are functions of space and are treated as constants [8]. The first of Maxwell's equations (Eqn. 20) can be written as

$$\sum_{j} h_j(t)\nabla\times\chi_j(r) = \sigma(r)\sum_{j} e_j(t)\phi_j(r) + [\varepsilon+\varepsilon_0\chi(r)]\sum_{j}\dot{e}_j(t)\phi_j(r). \tag{28}$$

Taking the time derivative of Equation (28), we obtain:

$$\sum_{j} \dot{h}_j(t)\nabla\times\chi_j(r) = \sigma(r)\sum_{j} \dot{e}_j(t)\phi_j(r) + [\varepsilon+\varepsilon_0\chi(r)]\sum_{j}\ddot{e}_j(t)\phi_j(r). \tag{29}$$

The normal mode fields satisfy Maxwell's equations for the empty, unperturbed resonator, which is charge free, uniform, passive and lossless. Then

$$\nabla \times \phi_j(r) = -i\omega_0\mu\chi_j(r) \tag{30}$$

$$\nabla \times \chi_j(r) = i\omega_{0j}\varepsilon\phi_j(r), \tag{31}$$

where ω_{0j} is the normal frequency for the jth mode in the passive cavity. Substituting the first of equations Equation (30) in the second of Maxwell's equations (Eqn. 21), we have

$$e_j(t) = -\frac{i}{\omega_{0j}}\dot{h}_j(t). \tag{32}$$

Using Equations (30)-(31) and (32) in Equation (29), we get

$$\sum_j -\omega_{0j}^2\varepsilon e_j(t)\phi_j(r) = \sigma(r)\sum_j \dot{e}_j(t)\phi_j(r) + [\varepsilon + \varepsilon_0\chi(r)]\sum_j \ddot{e}_j(t)\phi_j(r), \tag{33}$$

and rearranging the above, we obtain:

$$-\varepsilon\sum_j \left(\omega_{0j}^2 e_j + \ddot{e}_j\right)\phi_j(r) = \sum_j [\sigma(r)\dot{e}_j + \varepsilon_0\chi(r)\ddot{e}]\phi_j(r). \tag{34}$$

Multiplying both sides of the above equation with $V^{-1}\phi_k(r)$ and integrating over the volume V, we have:

$$-\varepsilon\left(\omega_{0k}^2 e_k + \ddot{e}_k\right) = \sum_j (\sigma_{jk}\dot{e}_j + \varepsilon_0\chi_{jk}\ddot{e}_j), \tag{35}$$

where

$$\sigma_{jk} = \frac{1}{V}\int_V \sigma(r)\phi_j(r)\cdot\phi_k(r)dV \tag{36}$$

$$\chi_{jk} = \frac{1}{V}\int_V \chi(r)\phi_j(r)\cdot\phi_k(r)dV. \tag{37}$$

We may write $e_j(t)$ as

$$e_j(t) = \frac{1}{2}e_{j0}(t)e^{i\omega_j t} + c.c., \tag{38}$$

where ω_j is the laser oscillation frequency and $e_{j0}$is the slowly varying part of the time dependent $e_j(t)$. Because we assume e_{j0} varies slowly, $|\ddot{e}_{j0}| << \omega_j|\dot{e}_{j0}|$ and the second derivation of $e_j(t)$ can be approximated as

$$\ddot{e}_j(t) = \frac{1}{2}\left[-\omega_j^2 e_{j0}(t) + i2\omega_j \dot{e}_{j0}(t)\right] e^{i\omega_j t} + c.c. \tag{39}$$

and equation (35) can be written as

$$-\varepsilon\left[\left(\omega_{0j}^2 - \omega_j^2\right) e_{j0} + i2\omega_j \dot{e}_{j0}\right] = \sum_k \left[\left(i\omega_k e_{k0} + \dot{e}_{k0}\right)\sigma_{kj} + \left(-\omega_k^2 e_{k0} + i2\omega_k \dot{e}_{k0}\right)\varepsilon_0 \chi_{kj}\right]. \tag{40}$$

We see from this equation that the spatial variation of the loss and gain causes a direct coupling between different modes. Only under the conditions that the orthogonality between the modes is not violated by the presence of χ and σ Equations (36)-(37), so that the following inequalities are true, can the direct coupling be omitted:

$$\begin{aligned} \left|\int_V \phi_k(r).\phi_j(r)\chi(r,\omega_j)dV\right| &<< \left|\int_V \phi_j(r).\phi_j(r)\chi(r,\omega_j)dV\right| \\ \left|\int_V \phi_k(r).\phi_j(r)\sigma(r)dV\right| &<< \left|\int_V \phi_j(r).\phi_j(r)\sigma(r)dV\right| \end{aligned} \qquad k \neq j. \tag{41}$$

This is particularly valid in the cases we are treating here, where $\sigma(r)$ and $\chi(r)$ are nearly symmetrical functions, and the kth and jth modes are of the opposite spatial symmetry. If $\sigma(r)$ and $\chi(r)$ are exactly symmetrical, then the left-hand side of equation (41) becomes exactly zero. With this approximation, Equation (40) becomes

$$\left[i2\omega_j\left(1 + \frac{\chi_j}{n^2}\right) + \frac{\sigma_j}{\varepsilon}\right]\dot{e}_{j0} = \left(-\omega_{0j}^2 + \omega_j^2 + \frac{\chi_j}{n^2}\omega_j^2 - i\frac{\omega_j\sigma_j}{\varepsilon}\right)e_{j0}, \tag{42}$$

where n is the index of refraction of the medium in the absence of gain and σ_j and χ_j are shorthand for σ_{ij} and χ_{ij}, respectively. The modal susceptibility χ_j can be further written in its real and imaginary parts:

$$\chi_j = \frac{1}{V}\int_V \phi_j(r).\phi_j(r)\chi(r,\omega_j)dV = \frac{1}{V}\int_V \phi_j(r).\phi_j(r)\left(\chi' - i\chi\right)dV. \tag{43}$$

When we require $\dot{e}_{j0} = 0$ in Equation (42), we obtain the steady-state solution, which turns into two conditions for steady-state laser oscillation, the first for phase and the second for amplitude

$$\omega_j = \frac{\omega_{0j}}{\sqrt{1 + \frac{Re(\chi_j)}{n^2}}}; \tag{44}$$

$$Im(\chi_j) = \frac{\sigma_j}{\varepsilon_0\omega_j}. \tag{45}$$

We consider only homogeneously-broadened media thus the imaginary part of χ_j can be written as

$$Im(\chi_j) = -\frac{1}{V}\int\limits_V |\phi_j(r)|^2 \chi(r) dV = \frac{1}{V}\frac{nc}{\omega_j}\int\limits_V \frac{\gamma_0(r)|\phi_j(r)|^2 dV}{1+\frac{|E(r,t)|^2}{2\eta I_s}}, \tag{46}$$

where the total electric field $E(r,t)$ is given by Equations (22)-(23), and the wave impedance of space filled with a dielectric material of permitivity $\varepsilon = \varepsilon_r\varepsilon_0$ is defined as

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}}. \tag{47}$$

The relationship between $\chi(r)$ and the medium gain, the definitions of small signal gain $\gamma_0\left[m^{-1}\right]$ and homogeneous saturation intensity $I_s\left[{}^{W}\!/_{m^2}\right]$, and their relations to the laser atomic parameters for waveguide gas laser media are given in Equation (15). Solving Equation (42) for $\dot{e}_j$and substituting laser frequency ω_j as given in the first of Equation (45), we obtain

$$\dot{e}_{j0} = \frac{\left(\frac{Im(\chi_j)}{n^2}\omega_j^2 - i\frac{\omega_j\sigma_j}{\varepsilon}\right)}{i2\omega_j\left(1+\frac{\chi_j}{n^2}\right)+\frac{\sigma_j}{\varepsilon}} e_{j0}. \tag{48}$$

For most gas laser media, it is true that $|\chi| << 1$, thus $\left({}^{|\chi_j|}\!/_{n^2}\right) << 1$. If the cavity loss is also small $\left({}^{\sigma_j}\!/_{\varepsilon}\right) << 2\omega_j$, the denominator above is effectively $i2\omega_j$ and Equation (48) can be further simplified. Using these simplifications and expanding the electric field using Equations (22)-(23) in Equation (46), finally Equation (48) becomes

$$\dot{e}_{j0} = \frac{1}{2}\left[\frac{Im\,(\chi_j)}{n^2}\omega_j - \frac{\sigma_j}{\varepsilon}\right] e_{j0} = \left[\frac{c}{2nV}\int\limits_V \frac{\gamma_0(r)|\phi_j(r)|^2 dV}{1+\frac{\frac{1}{2\eta}\sum\limits_{\ell} e_{j0}e_{jo}^*\phi_{\ell(r)}\phi_{\ell(r)}}{I_s}} - \frac{\sigma_j}{2\varepsilon}\right] e_{j0}. \tag{49}$$

Equation (49) is a differential equation for laser coefficients, each including phase and amplitude. The intensity coefficient for the jth mode can be defined as

$$b_j = \frac{e_{jo}e_{jo}^*}{2\eta}. \tag{50}$$

It represents the power density [W/m^2] for the jth mode, and its differential equations is:

$$\dot{b}_j = \frac{d}{dt}\frac{e_{jo}e_{jo}^*}{2\eta} = \frac{1}{2\eta}\left(\dot{e}_{jo}e_{jo}^* + e_{jo}\dot{e}_{jo}^*\right) = 2\left[\frac{c}{2nV}\int\limits_V \frac{\gamma_0(r)|\phi_j(r)|^2 dV}{1+\frac{\sum\limits_{\ell} b_j\phi_\ell(r)\phi_\ell(r)}{I_s}} - L_j\right] b_j. \tag{51}$$

In deriving Equation (51), we used the fact that the square bracket terms is real [8]. We also introduced a modal loss coefficient [9]

$$L_j = \frac{\sigma_j}{2\varepsilon}. \quad (52)$$

2.3. Mode Stability Analysis

We may now use the results of the previous section to investigate the stability of a given mode for when it is possible for a second mode, one with a different transverse field distribution $\phi_j(r)$, to oscillate. We make the following assumptions [10].

There are only two modes that are sufficiently near the threshold to be appreciably excited. The total field is then:

$$E(r,t) = e_{10}(t)e^{i\omega_1 t}\phi_1(r) + e_{20}(t)e^{i\omega_2 t}\phi_2(r). \quad (53)$$

The z-dependence of the modal field is in the form of a standing wave and is the same for all modes considered, since they have the same longitudinal mode number. Thus, the spatial hole burning resulted from this fine standing wave pattern in the z-direction does not favor one mode over the other and can be ignored.

The population inversion and gain are dependent not only on the transverse coordinates but also on the longitudinal coordinate. Therefore, the gain does not depend on z. Thus the integral in Equation (49) is only affected by the variations in the transverse direction (x,y).

The loss $\sigma(r)$ is divided in to a spatially varying part and a constant part:

$$\sigma(r) = \sigma'(r) + \sigma_0. \quad (54)$$

With these assumptions, the effects of the localized loss and the distributed loss on laser mode competition can be separated. The modal loss is then:

$$L_j = \frac{1}{2\varepsilon V}\int\limits_V \sigma'(r)|\phi_j(r)|^2 dV + \frac{\sigma_0}{2\varepsilon} = L'_j + L_0. \quad (55)$$

To obtain steady-state solutions for Equation (51), we require $\dot{b}_j = 0$ for j=1,2. Then on the right-hand side either the term in square brackets is zero or b_j=0. All possibilities considered, we will have three nontrivial steady-state solutions as follows:

$$(b_1, b_2) = (f, 0)\,, (0, g)\; or\; (p_1, p_2)\,, \quad (56)$$

where $(f, 0)$ and $(g, 0)$ are single-mode solutions. First, consider the stability of the state $(f,\, 0)$. Under perturbation (δ_1, δ_2), this state becomes $(f + \delta_1, \delta_2)$. We use Equation (51) to find equations for the perturbation δ_1 and δ_2:

$$\dot{\delta}_1 = \left[\frac{c}{2nS} \int\limits_S \frac{\gamma_0(x,y)|\phi_1(x,y)|^2 dxdy}{1 + \frac{(f+\delta_1)|\phi_1|^2 + \delta_2|\phi_2|^2}{I_s}} - L_1 \right] (f + \delta_1)$$

$$\dot{\delta}_2 = \left[\frac{c}{2nS} \int\limits_S \frac{\gamma_0(x,y)|\phi_2(x,y)|^2 dxdy}{1 + \frac{(f+\delta_1)|\phi_1|^2 + \delta_2|\phi_2|^2}{I_s}} - L_2 \right] \delta_2. \tag{57}$$

Here ϕ_1 and ϕ_2 are functions of (x,y), and S is the total cross-section area of the laser cavity. Because the perturbations are very small $|f| >> |\delta_1|, |\delta_2|$, we can ignore all the second order δ term in Equations (57.1)-(57.2):

$$\dot{\delta}_1 = \left\{ \frac{c}{2nS} \int\limits_S \frac{\gamma_0(x,y)|\phi_1(x,y)|^2}{1 + \frac{f|\phi_1(x,y)|^2}{I_s}} . \left[1 - 2f . \frac{\delta_1 |\phi_1(x,y)|^2}{I_s \left(1 + \frac{f|\phi_1(x,y)|^2}{I_s} \right)} \right] dxdy - L_1 \right\} (f + \delta_1) . \tag{58}$$

The fact that (f,0) is a steady-state solution implies that the saturated gain equals the total loss:

$$\frac{c}{2nS} \int\limits_S \frac{\gamma_0(x,y)|\phi_1(x,y)|^2 dxdy}{1 + \frac{f|\phi_1(x,y)|^2}{I_S}} - L_1 = 0. \tag{59}$$

So, the equations in (57.1)-(57.2) evolve into the following form:

$$\dot{\delta}_1 = -\frac{cf}{nI_S S} \int\limits_S \gamma_0(x,y)|\phi_1(x,y)|^2 \frac{\delta_1 |\phi_1(x,y)|^2}{\left(1 + \frac{f|\phi_1(x,y)|^2}{I_S} \right)^2} dxdy$$

$$\dot{\delta}_2 = \left[\frac{c}{2nS} \int\limits_S \frac{\gamma_0(x,y)|\phi_2(x,y)|^2 dxdy}{1 + \frac{f|\phi_1(x,y)|^2}{I_S}} - L_2 \right] \delta_2. \tag{60}$$

We can write Equations (60.1)-(60.2) in a matrix form by defining a vector δ:

$$\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \tag{61}$$

The time derivative of δ is [11]:

$$\dot{\delta} = \begin{pmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \tag{62}$$

We see from Equation (58) that the matrix elements B and C are zero. Thus, the eigenvalues for the operating matrix are A and D. If both of them are negative, then the vector δ will not grow, and the steady-state (f,0) is stable; if either A or D is positive, (f,0) will be unstable. We also see from Equations (60.1)-(60.2):

$$A = -\frac{cf}{nI_S S}\int_S \frac{\gamma_0(x,y)|\phi_1(x,y)|^4 dxdy}{\left(1+\frac{f|\phi_1(x,y)|^2}{I_S}\right)^2} < 0 \tag{63}$$

$$D = \frac{c}{2nS}\int_S \frac{\gamma_0(x,y)|\phi_2(x,y)|^2 dxdy}{1+\frac{f|\phi_1(x,y)|^2}{I_S}} - L_2. \tag{64}$$

The sign of D is not known until $\gamma_0(x,y)$, $\phi_1(x,y)$, $\phi_2(x,y)$ and $\sigma'(x,y)$ are specified. We see that because A is negative, the sign of D determines the stability of the solution (f,0): if it is positive or negative, this state is unstable or stable [11,12], respectively.

Similar arguments obviously apply to the other single-mode steady-state (0,g), which becomes $(\delta_1, g+\delta_2)$ with perturbation. The equations are

$$\dot{\delta}_1 = \left[\frac{c}{2nS}\int_S \frac{\gamma_0(x,y)|\phi_1(x,y)|^2 dxdy}{1+\frac{g|\phi_2(x,y)|^2}{I_S}} - L_1\right]\delta_1, \tag{65}$$

$$\dot{\delta}_2 = -\frac{cg}{nI_S S}\int_S \gamma_0(x,y)|\phi_2(x,y)|^2 \frac{\delta_2|\phi_2(x,y)|^2}{1+\frac{g|\phi_2(x,y)|^2}{I_S}} dxdy. \tag{66}$$

We define the self-saturated modal gain S_k and cross-saturated modal gain C_{kj}, both in unit of s^{-1}, as follows:

$$S_k = \frac{c}{2nS}\int_S \frac{\gamma_0(x,y)|\phi_k(x,y)|^2 dxdy}{1+\frac{\alpha_k|\phi_k(x,y)|^2}{I_S}}; \tag{67}$$

$$C_{kj} = \frac{c}{2nS}\int_S \frac{\gamma_0(x,y)|\phi_j(x,y)|^2 dxdy}{1+\frac{\alpha_k|\phi_k(x,y)|^2}{I_S}}; \tag{68}$$

$$k, j = 1, 2 (k \neq j)(\alpha_1 = f)(\alpha_2 = g).$$

Then we may summarize the conditions for the existence and stability of the jth and kth modes: for the jth mode to exist,

$$S_j - L_j' - L_0 = 0. \tag{69}$$

For the kth mode to exist,

$$S_k - L_k' - L_0 = 0. \tag{70}$$

The stability of the jth mode is determined by

$$C_{jk} - L'_k - L_0 = 0\{ \begin{array}{ll} <0 & \text{stable} \\ >0 & \text{unstable.} \end{array} \quad (71)$$

The stability of the kth mode is determined by [12]:

$$C_{jk} - L'_k - L_0 = 0\{ \begin{array}{ll} < 0 & \text{stable} \\ > 0 & \text{unstable.} \end{array} \quad (72)$$

Thus for the laser to have single jth mode Equation (71) must be <0 and Equation (72) must be >0, and vice versa for a single kth mode to exist.

3. Results and Discussions

A theoretical analysis of laser transverse mode competition is investigated from the perspective of the spatial overlap of modes with a transverse gain-loss distribution. The dominant mode of laser oscillation is the mode that is stable under small perturbations. The conditions that this mode must satisfy were derived. In this study property of the coupled array may be important in the understanding active super mode control. Numerical calculations applied to waveguide lasers, with the result that a change in gain or loss in the small coupling region between the channels of the array was capable of switching the laser oscillation from one mode to the other. This property of the coupled array may be important in the understanding active super-mode control.

References

[1] A. Yariv, Quantum Electronics, 3^{rd} ed. 150-152, New York: Wiley (1989).

[2] J. H, Wilson, Introduction to Optoelectronics, *PracticeHall*, **232** (1983).

[3] B. E. Cherrington, Gaseous Electronics and Gas Lasers, Oxford: Pergamon, Press, (1966).

[4] A. Yariv, Introduction to Optical Electronics, 2^{nd} ed. Chap.7, New York (1971).

[5] S. A. Losev, Gasdynamic Laser, New York: *Springer, Verlay,* **99** (1981).

[6] R. E. Jensen and M. S. Tobin, CO_2 Waveguide gas Laser, *Appl. Phys. Lett.* **20.** 508-510 (1972).

[7] K. D. Laakman, Waveguides, *Appl. Optics*, **5**, 5, 1334-1340, (1976)

[8] A. A. Golubentsev, Use of a Spatial Filter for Phase Locking a Laser Array, *Sov. J., Quan. Elec.*, **2**, 8, 934-938, (1990).

[9] Y. Zhang and W. B. Bridges, Stable in-phase Locked Arrays of CO_2 Waveguide Lasers, *Optics Society of USA*, Nov. **3-8**, (1991).

[10] C. L. Tang, A. Schremer, Bistability in two-mode Semiconductor Lasers via Gain Saturation, *App. Phy. Lett,* **51**, 18, 1392-1394 (1987).

[11] R. Abrams and B. Bridges, Characteristics of Sealed-off Waveguide CO_2 Lasers, IEEE *J. Quantum Elec.*, **QE-9**, 940-946, (1973).

[12] W. B. Bridges and Y. Zhang, Coupled Waveguide Gas Laser Research, *Appl. Phy. Lett*, **44**, 4, 365-367, (1984).