

# IBA-1 Calculations of E2/M1 Mixing Ratios and Quadrupole Moments in $^{154}\text{Gd}$

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## Abstract

In this study the Interacting Boson Approximation is applied in its basic form (IBA-1) to the calculation of gamma-ray E2/M1 multipole mixing ratios and quadrupole moments  $[Q(2^+)]$  of electromagnetic transitions in  $^{154}\text{Gd}$ . The numerical results obtained for  $^{154}\text{Gd}$  are in reasonably good agreement with previous experimental and theoretical values.

**Key Words:** IBA-1, deformation parameter, quadrupole moment, mixing ratios.

## 1. Introduction

The  $\gamma$ -ray multipole mixing ratios  $\delta(\text{E2/M1})$  or the energy-independent quantities  $\Delta(\text{E2/M1})$ , provide a sensitive test of collective nuclear models. Arima and Iachello have given for their original interacting boson approximation (IBA-1) the M1 operator in the restricted case of U(5) dynamic symmetry [1] and as well as the general case [2]. However, even when starting with the general operator, they derive the E2/M1 mixing ratio by neglecting the terms which break SU(3) symmetry [2]. It follows that the reduced mixing ratio is given by the same simple formula for both U(5) and SU(3) symmetries. The formula contains only one parameter and the initial and final spins. Warner [3] has developed an IBA description of the E2/M1 mixing ratio whose point of departure is essentially the same as that of Scholten et al [4]. To the present time, several systematic studies [5,6,7] have been performed within the framework of the IBA. The most spectacular difference is that IBA-2 predicts collective M1 excitations [8] absent from IBA-1 and they have been observed in both deformed and spherical nuclei; see refs. [9]. Less spectacular, yet important is the difference regarding E2/M1 multipole mixtures in gamma transitions connecting low-lying states predicted equally by both versions of the IBA [10]. In deformed nuclei such states group into the familiar ground  $\beta$  and  $\gamma$  bands

and in transitional and spherical nuclei into their quasi counterparts [11].

The interacting boson approximation has been rather successful at describing the collective properties of several medium and heavy nuclei. The aim of the present study is to explore the description of E2/M1 mixing ratios by IBA-1. Our results for the E2/M1 mixing ratios are compared with previous theoretical and experimental values.

Kumar and Gupta [12] studied the quadrupole shape characteristics of even-even gadolinium isotopes by the use of the dynamic deformation theory.

## 2. Model

The elementary excitations of the basic interacting boson approximation are s (L=0) and d (L=2) bosons. In the IBA-1 study of Gd isotopes, extra core nuclei are treated as active bosons. In terms of the s and d boson operators, with  $\bar{d}_m = (-1)^m d_{-m}$ , the IBA-1 Hamiltonian consists of single-boson terms and all possible boson-boson interaction [13]:

$$\begin{aligned}
 H = & \varepsilon_s s^+ s + \varepsilon_d d^+ \bar{d} + \sum_{L=0,2,4} \frac{1}{2} C_L [d^+ d^+]_L \cdot [\bar{d} \bar{d}]_L + \frac{\bar{v}_2}{\sqrt{10}} [(d^+ d^+)_2 \cdot \bar{d} s \\
 & + s^+ d^+ \cdot (\bar{d} \bar{d})_2] + \frac{\bar{v}_0}{2\sqrt{5}} (d^+ d^+ s s + s^+ s^+ \bar{d} \cdot \bar{d}) + \frac{u_2}{\sqrt{5}} d^+ s^+ \cdot \bar{d} s + \frac{u_0}{2} s^+ s^+ s s, \quad (1)
 \end{aligned}$$

where,  $\varepsilon_s$  and  $\varepsilon_d$  are the binding energies of the s and d boson; the operators ( $s^+ s$ ) and ( $d^+ \bar{d}$ ) count the number of s and d bosons, respectively. The interaction of the s-bosons with the d-bosons is given by  $v_0$ ,  $u_2$  and  $v_2$ .

The basic rule of the IBA is to construct the electromagnetic operators as linear functions of the U(6) generators in the sd-boson realization. Thus the E2 tensor operator is written as [14]

$$T(E2) = \alpha_2 (d^+ s + s^+ d) + \beta_2 [d^+ \bar{d}]_2. \quad (2)$$

Similarly, the M1 operator would be just  $\beta_1 [d^+ \bar{d}]_1$ . To have M1 transitions the IBA rule must be extended to second order in the U(6) generators [1,4]. Angular momentum recoupling shows that the most general second order M1 operator can then be written as [10]

$$T(M1) = (g_b + AN)L + B_1 [Q_1 L]_1 + B_2 [Q_2 L]_1 + C n_d L, \quad (3)$$

where  $Q_1 \equiv d^+ s + s^+ d$  and  $Q_2 \equiv [d^+ \bar{d}]_2$ . Rather than attempting to evaluate the E2 and M1 matrix elements for  $^{154}\text{Gd}$  essential in theoretical mixing ratio calculations, it is possible to obtain these ratios in an analytic form as the matrix elements have a simple structure in the SU(5) and SU(3) limits.

The theoretical reduced mixing ratios,

$$\Delta(E2/M1) = \frac{\langle \chi I_f \| T(E2) \| \chi' I_i \rangle}{\langle \chi I_f \| T(M1) \| \chi' I_i \rangle} \quad (4)$$

are related to mixing ratios  $\delta(E2/M1)$  by

$$\delta(E2/M1) = 0832 \cdot E_\gamma \cdot \Delta(E2/M1), \quad (5)$$

where  $E_\gamma$  is in MeV and  $\Delta(E2/M1)$  is eb/ $\mu_N$  [15].

The quadropole moments for the  $L^+$  state are defined as [16]

$$Q_{L^+} = \left[ \frac{16\pi}{5} (2L+1) \right]^{1/2} \cdot (LL20|LL)(L \| T^{(2)} \| L). \quad (6)$$

For  $L = 2$ , we have

$$Q_{2^+} = \left[ \frac{32\pi}{175} \right]^{1/2} (2 \| T^{(2)} \| 2). \quad (7)$$

### 3. Results and Discussion

The  $\delta(E2/M1)$  multipole mixing ratios of the electromagnetic transitions between the energy states of  $^{154}Gd$  nucleus was calculated by using equation (5). The calculated values are given in Table 1. The IBA-1 model, particularly in its extended consistent- $\delta$ formalism, has been applied to the calculation of E2/M1 multipole mixing ratios over a wide range of nuclei. Overall, fair agreement is obtained with experiment. This agreement is hardly fortuitous because of the great variety of data and because we have insisted on smooth systematics of the parameters. In general, it can be seen from the table that calculated results are in better agreement with the previous experimental and theoretical data.

It can be seen from Table 1 that the mixing ratio found for the 873 keV transition is  $-9,3 \pm 0,3$  and this value is in agreement with the experimental values of  $-9,7 \pm 1,0$  of Hamilton et al [17] and  $-9,5 (+0,8; -0,6)$  of Krane [18]. The result obtained for the 1005 keV is  $-8,1 \pm 0,4$ . This value is in agreement with the experimental values. However, it can be seen from the Table 1 that our results are in better agreement with the previous experimental data. The overall agreement is surprisingly good in view of the Interacting Boson Approximation.

We also calculated the quadrupole moments for  $2_1^+$  state in  $^{154}Gd$  isotope and the values obtained for these nuclei are listed together with the previous experimental and theoretical values in Table 2. The agreement of our calculated and measured  $Q(2_1^+)$  quadrupole moment of  $^{154}Gd$  is good. However,  $Q(2_1^+)$  value of  $^{154}Gd$  in previous works are not exactly same as from experimental data nor our result.

**Table 1.** Experimental and various theoretical E2/M1 multipole mixing ratios for  $^{154}\text{Gd}$ .

Spin Parity $I_i \rightarrow I_f$	Transition Energy ( keV )	Mixing Ratios [ $\delta$ ( E2/M1 )]		
		This Work	Experimental	Previous Work
$2_{\gamma^+} \rightarrow 2_{g^+}$	873	$-9.3 \pm 0.3$	$-13.3 \pm 0.7^a$	$-32.6^d$
			$-9.7 \pm 1.0^b$	$-13.44^e$
$3_{\gamma^+} \rightarrow 2_{g^+}$	1005	$-8.1 \pm 0.4$	$-9.5(+0.8;-0.6)^c$	$-12.5^d$
			$-8.9 \pm 0.5^a$	$-153^g$
			$-7.5 \pm 0.6^c$	
$3_{\gamma^+} \rightarrow 4_{g^+}$	757	$-7.8 \pm 0.2$	$-7.1 \pm 0.4^f$	$-13.1^d$
			$-7.7 \pm 0.9^f$	$-127^g$
			$-8.9 \pm 0.3^a$	
$4_{\gamma^+} \rightarrow 4_{g^+}$	893	$-4.9 \pm 0.3$	$-5.8 \pm 0.2^c$	$-4.3^d$
			$-5.5 \pm 0.5^a$	$-16.6^g$
$5_{\gamma^+} \rightarrow 4_{g^+}$	1061	$-4.7 \pm 0.5$	$-3.6 \pm 2.2^f$	
			$-4.9(+1.4;-2.9)^a$	$-3.0^d$
$5_{\gamma^+} \rightarrow 6_{g^+}$	715	$-7.6 \pm 0.8$	$-4.3(+1.2;-2.6)^h$	$-53^g$
			$-11.6(+7.1;-\infty)^a$	$14.6^d$
			$-2.2(+6.0;-2.2)^h$	$-55^g$

<sup>a</sup>Ref.[14], <sup>b</sup>Ref.[17], <sup>c</sup>Ref.[18], <sup>d</sup>Ref.[10], <sup>e</sup>Ref.[20], <sup>f</sup>Ref.[15], <sup>g</sup>Ref.[21], <sup>h</sup>Ref.[19]

**Table 2.** Quadrupole moments Q(  $2_1^+$  ), (in eb) for the  $^{154}\text{Gd}$  isotope.

Nucleus	This Work	Experiment	Previous Work
$^{154}\text{Gd}$	- 1.84	- 1.82 <sup>a</sup>	- 2.19 <sup>b</sup>

<sup>a</sup>Ref.[22], <sup>b</sup>Ref.[16].

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