Is There an Age of the Universe Problem after the 

*Hipparcos* Data?

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**Abstract**

We have reanalyzed the age of the universe problem under the assumption that the lower limit on the age of the globular clusters is \( \approx 11 \) Gyr, as predicted by the recent *Hipparcos* data. We find that the globular cluster and the expansion ages in a standard \( \lambda = 0 \) universe are consistent only if the present value \( H_0 \) of the Hubble constant is \( \leq 60 \text{km s}^{-1} \text{Mpc}^{-1} \). If \( H_0 > 60 \text{km s}^{-1} \text{Mpc}^{-1} \) some kind of modification of the standard \( \lambda = 0 \) model is required. Invoking a (time-independent) cosmological term \( \lambda \) in the Einstein field equations, as has been done frequently before, we have found that due to the gravitational lensing restrictions a flat universe with the present matter density parameter \( \Omega_M \) < 0.5 is not problem-free. A nonflat universe with \( \Omega_M \leq 1 \) does not suffer from the age problem if \( H_0 \leq 75 \text{km s}^{-1} \text{Mpc}^{-1} \).

1. **Introduction**

A lower limit on the present age \( t_0 \) of the universe is determined by estimating the age of the oldest objects in our galaxy, the globular clusters (hereafter GC). These are stellar systems that contain about \( 10^5 \) stars in the halo surrounding the galactic disk. The key element in estimating the age of a typical GC is the determination of its distance from us. To this end, the primary observational technique is main-sequence fitting against subdwarfs with well known parallaxes. The distance obtained this way or otherwise is used to convert the measured apparent magnitude of a GC to the absolute magnitude. The

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1The age of the universe is actually the GC age plus the time it took for the formation of GCs. The formation time is estimated to be between 0.1-2 Gyr [1]. The lower value of 0.1 Gyr is chosen for the lower limit on the age of the universe [1]. Since this difference of 0.1 Gyr in the lower limits on the ages is not significant, we will use the ‘age of the universe’ and the ‘GC age’ interchangeably, as is usually done in the literature.
age is then estimated by applying a stellar evolution model. The estimates obtained by
different astronomers agree rather well. For example, Bolte and Hogan [2] find 15.8 ± 2.1
These time scales are to be compared with the expansion age of the universe predicted
by the standard model of cosmology (hereafter SM) which requires the knowledge of
the present value \( H_0 \) of the Hubble constant. Even though estimates of \( t_0 \) from GCs are
based on the stellar evolution models, which are essentially the same, the situation is not
the same for the \( H_0 \) estimates. There are a number of different techniques (see the review
by Trimble [8]) which give values that differ substantially from each other. We present
the most quoted estimates: \( H_0 = 50 - 55 \text{km s}^{-1} \text{Mpc}^{-1} \) [9] and \( H_0 = 73 ± 10 \text{km s}^{-1} \text{Mpc}^{-1} \)
[10]. In a SM flat universe \( t_0 \) would be 13 Gyr and 8.2 Gyr if \( H_0 \) were 50\( \text{km s}^{-1} \text{Mpc}^{-1} \) and
80\( \text{km s}^{-1} \text{Mpc}^{-1} \), respectively, whereas in a SM open universe with \( \Omega_M = 0.1 \), \( \Omega_M \)
being the present nonrelativistic matter density parameter, the ages would be 17.6 Gyr
and 11 Gyr for the same \( H_0 \) values as above. Thus researchers were rightfully led to
think that, if \( H_0 \) has as large a value as determined by Freedman et al. [10] then, the
expansion age and the GC age of the universe are in conflict with each other.

An immediate solution to this so called age of the universe problem was suggested
by including a time-independent cosmological constant \( \Lambda \) in the Einstein field equations
[11-13]. The gravitational lensing studies, however, have shown that the cosmological
constant cannot be as large as one desires to increase the expansion age to the level of GC
age lest too many lensing events are predicted [14-16]. Recently, the supernova magnitude-
redshift approach of Perlmutter et al. [17] has given \( \Omega_{\Lambda} < 0.51 \) (95% confidence level) for
a flat universe which is significantly lower than the gravitational upper limit \( \Omega_{\Lambda} < 0.66 \)
of Kochanek [15]. Thus it had been concluded that the apparent contradiction between
the GC age and the expansion age could not be reconciled in a flat universe by invoking
a time-independent cosmological constant. This was the status of the age of the universe
problem before \( \text{Hipparcos} \). The lower limit on the age of the oldest GCs implied by the
\text{Hipparcos} data is \( \sim 11 \) Gyr [18, 19]. The purpose of this paper is to reexamine the age
problem in the light of this lower limit of 11 Gyr put by the \text{Hipparcos} data [18, 19].

2. The Age of the Universe Problem

The relation between the present value \( H_0 \) of the Hubble constant \( H = \dot{a}/a \), where \( a \)
is the scale factor of the universe and \( \dot{a} = da/dt \), and the present age \( t_0 \) is given by [20]

2Felten and Isaacman[5] call the models with \( \Lambda = 0 \) ‘standard models’. However, we follow the general
trend in the literature and call the totality of them the ’standard model’ and refer to each case by its \( k \)
value (see, for example, Misner, Thorne and Wheeler [6]; Weinberg [7].) The SM with \( k = 0 \) is called the
Einstein-de Sitter model [5].

Reid [18] has argued that this Freedman et al. [10] value of \( H_0 \) is reduces to \( H_0 = 68 ± 9 \text{km s}^{-1} \text{Mpc}^{-1} \)
because the \text{Hipparcos} data reveal a 7% increase in the distances inferred from previous ground-based
data.

Equations (1a) and (1c) agree numerically with those given in Weinberg [7] where a different but
equivalent functional form is used.
Here, $\Omega_M$ is the present value of the nonrelativistic matter density parameter defined as the ratio of the present nonrelativistic matter density to the present critical energy density

$$\Omega_M = \rho M c = \frac{\rho M}{3H_0^2/8\pi G}.$$  \hspace{1cm} (2)$$

Expressing the Hubble constant as $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, the age in billion years is given by $t_0$ (Gyr) = 9.78($H_0t_0$)/$h$, where ($H_0t_0$) is given in Eq.(1) and $h$ is a parameter assumed to be between 0.5 and 1. In Figure 1, we depict $t_0$ against $\Omega_M$ and $h$ in the SM. It is seen that $t_0$ is below the Hipparcos lower limit of 11 Gyr for large values of $h$. Thus it can be stated safely that the age of the universe problem still survives if $h$ is large.

**Figure 1.** The age of the universe in the SM for $k = -1$ (solid lines), $k = 0$ (dots) and $k = 1$ (dashed lines) versus the present value of the matter density parameter $\Omega_M$.

In Table 1, we display the maximum values of $h$ for which $t_0 = 11$ Gyr against $\Omega_M$. Note that the maximum $h$ values in Table 1 almost fall in the lower and upper limits of Freedman et al. [9]. Thus for each $\Omega_M$, if $h$ is greater than those given in Table 1, there is
an age problem. For example, if $\Omega_M = 0.5$ and $h > 0.67$ or $\Omega_M = 1$ and $h > 0.593$ the age problem survives. Now the problem is, however, milder in the sense that before \textit{Hipparcos} the age problem was thought to exist even for moderate values of $h$ whereas it now exists for large values of $h$. Emphatically, the SM has no age problem if $h < 0.593 \approx 0.6$.

### Table 1. Maximum values of $h$ in the SM for which $t_0 = 11$ Gyr

<table>
<thead>
<tr>
<th>$\Omega_M$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{\text{max}}$</td>
<td>0.799</td>
<td>0.753</td>
<td>0.719</td>
<td>0.692</td>
<td>0.67</td>
<td>0.651</td>
<td>0.634</td>
<td>0.619</td>
<td>0.605</td>
<td>0.593</td>
</tr>
</tbody>
</table>

*Note that if $h = h_{\text{max}}$ then $t_0 = 11$ Gyr, if $h > h_{\text{max}}$ then $t_0 < 11$ Gyr and if $h < h_{\text{max}}$ then $t_0 > 11$ Gyr.

Supposing that there is an age problem, one line of attack, as in the pre \textit{Hipparcos} era, is to invoke a (time-independent) cosmological constant $\Lambda$ in the Einstein field equations\textsuperscript{[11-13]}

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \quad (3)$$

where $R = R^\mu_\mu$ and $T_{\mu\nu}$ is the energy-momentum tensor. For a homogeneous and isotropic universe described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (4)$$

the energy-momentum tensor is assumed to have the perfect fluid form

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p), \quad (5)$$

where $p$ is the pressure of the matter described by $\rho$. Equations (3) and (4) give (with $c$, the speed of light, set to 1)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho(t) + \frac{\lambda}{3} \frac{k}{a^2}, \quad (6)$$

where $k = -1, 0, 1$ for a spatially open, flat and closed universe, respectively. At present, the universe is believed to be dominated by nonrelativistic massive matter rather than relativistic matter (radiation). It proves to be very useful to define the current cosmological constant density parameter

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\lambda/8\pi G}{\rho_c} = \frac{\lambda}{3H_0^2}, \quad (7)$$

and the current curvature density parameter

$$\Omega_k = -\frac{\rho_k}{\rho_c} = -\frac{k}{H_0^2 a_0^2}, \quad (8)$$
where \(a_0\) is the current value of the scale factor \(a\) of the universe. When written in terms of the present values, Equation (6) gives the constraint
\[
\Omega_M + \Omega_\Lambda + \Omega_k = 1, \quad (9)
\]
Equations (3) and (5) under (4) give the energy conservation equation in the matter dominated era as
\[
d\left[\rho_M(t)a^3 + \frac{\lambda}{8\pi G}a^3\right] + \left[p_M(t) - \frac{\lambda}{8\pi G}\right]da^3 = 0, \quad (10)
\]
where the pressure \(p_M\) of nonrelativistic matter is negligible. Thus it follows from Eq.(10) that \(\rho_M(t) = \rho_M a_0^3/a^3\) and the relation between \(H_0\) and \(t_0\) is
\[
H_0t_0 = \int_0^1 y^{1/2}[\Omega_M(1 - y) + \Omega_\Lambda(y^3 - y) + y]^{-1/2}dy, \quad (11)
\]
where \(\Omega_k\) has been eliminated by using Eq.(9) and \(y = a/a_0\). Now, a flat universe with \(\Omega_M < 1\) is rendered possible by postulating the existence of the cosmological term \(\lambda\) such that \(\Omega_M + \Omega_\Lambda = 1\). With \(t_0\) value of \(k\) not fixed a priori, a numerical investigation of Eq.(11) reveals that it is always possible to find a set of three parameters \((\Omega_M, \Omega_\Lambda, h_{\text{max}})\) for which \(t_0 = 11\)Gyr.

However, the achievement of a cosmological constant to solve the age problem and to have a flat universe with \(\Omega_M < 1\) may be illusory. The magnitude of \(\Omega_\Lambda\) required to solve the age problem may turn out to be too large to predict plausible number of gravitational lensing events. Therefore, each such set of parameters \((\Omega_M, \Omega_\Lambda, h_{\text{max}})\) need to be confronted with the gravitational lensing statistics, which we address ourselves next.

3. The Gravitational Lensing Statistics

The integrated probability, the so-called optical depth, for lensing by a population of singular isothermal spheres of constant comoving density relative to the Einstein-de Sitter model, is [21]
\[
P_{\text{lens}} = \frac{15}{4} \left[1 - \frac{1}{(1 + z_s)^{1/2}}\right]^{-3} \int_0^{z_s} (1 + z)^2 \frac{d(0, z)d(z, z_s)}{E(z)} d\Omega, \quad (12)
\]
where
\[
E(z) = (1 + z)^2(1 + z\Omega_M) - z(z + 2)\Omega_\Lambda \quad (13)
\]
and is defined by [22]
\[
\left(\frac{\dot{a}}{a}\right)^2 = H_0^2E(z)^2. \quad (14)
\]
Note that \(P_{\text{lens}} = 1\) for the Einstein-de-Sitter model (in which \(\Omega_k = 0\), \(\Omega_M = 1\) and \(\Omega_\Lambda = 0\)). \(z = (a_0/a) - 1\) is the redshift and \(z_s\) is the redshift of the source (quasar). The
angular diameter distance from redshift \( z_1 \) to redshift \( z_2 \) is 

\[
d(z_1, z_2) = \frac{1}{(1 + z_2) \left| \Omega_k \right|^{1/2}} \sin n \left[ \left| \Omega_k \right|^{1/2} \int_{z_1}^{z_2} \frac{dz}{E(z)} \right] \tag{15}
\]

where

\[
\sin n = \sinh, \quad if \quad \Omega_k > 0, \\
= 1, \quad if \quad \Omega_k = 0, \\
= \sin, \quad if \quad \Omega_k < 0. \tag{16}
\]

To determine how much of \( P_{\text{lens}} \) is permissible, we refer to the work of the Supernova Cosmology Project [17]. Using the initial seven of more than 28 supernovae discovered, Perlmutter et al. [17] have recently measured \( \Omega_M \) and \( \Omega_{\Lambda} \). For \( \Omega_M < 1 \), they find \( \Omega_{\Lambda} < 0.51 \) at the 95% confidence level for a flat universe, and \( \Omega_{\Lambda} < 1.1 \) for the more general case \( \Omega_M + \Omega_{\Lambda} \) unconstrained\(^5\). In Table 2 we present \( P_{\text{lens}} \) against \( \Omega_M \) and \( \Omega_{\Lambda} \) for a typical source redshift of \( z_s = 2 \). Table 2 helps us to determine the maximum allowed value of \( P_{\text{lens}} \). It is seen that for \( \Omega_{\Lambda} = 0.5 \), which is the maximum allowed value according to Perlmutter et al. [17], the corresponding \( P_{\text{lens}} \) is 1.92. Thus we shall assume that \( P_{\text{lens}} \) cannot be much larger than 2. Having determined the upper limit on \( P_{\text{lens}} \), we depict in Table 3 the three parameters \( \Omega_M, \Omega_{\Lambda} \) and \( h_{\text{max}} \) in a flat universe and the corresponding gravitational lensing prediction for \( t_0 = 11 \) Gyr. In preparing Table 3, we have first fixed \( \Omega_M \) and calculated \( H_0 t_0 \) from Eq.(11) with \( \Omega_{\Lambda} = 1 - \Omega_M \), and finally obtained the maximum value of \( h \) from \( h_{\text{max}} = 9.78(\dot{H}_0 t_0)/11 \).

Table 2. Normalized optical depths.

<table>
<thead>
<tr>
<th>( \Omega_M )</th>
<th>( \Omega_{\Lambda} )</th>
<th>( P_{\text{lens}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>13.25</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>5.98</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>3.94</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>2.93</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>2.33</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.92</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>1.63</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>1.42</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>1.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>1.11</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1</td>
<td>1.61</td>
</tr>
<tr>
<td>0.8</td>
<td>1.1</td>
<td>1.99</td>
</tr>
<tr>
<td>0.6</td>
<td>1.1</td>
<td>2.57</td>
</tr>
<tr>
<td>0.4</td>
<td>1.1</td>
<td>3.61</td>
</tr>
<tr>
<td>0.2</td>
<td>1.1</td>
<td>6.05</td>
</tr>
</tbody>
</table>

\(^5\)But of course \( \Omega_M + \Omega_{\Lambda} + \Omega_k = 1 \)

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Discarding those set of parameters which yield $P_{\text{lens}} > 2$ or have $\Omega_A > 0.5$, first we confirm, from Table 3, the previous conclusions that a cosmological constant cannot solve the age problem in a flat universe with $\Omega_M < 0.5$ due to too many lensing predictions. Next, we see that the maximum allowed value of $h$ in a flat universe is about 0.74-0.75. This is to be compared with the pre-Hipparcos lower limits for the age. For $t_0 = 13$ and 14 Gyr the $h_{\max}$ values are 0.64 and 0.60 in a flat universe, respectively.

Table 3. Maximum values of $h$ in a flat universe for which $t_0 = 11$ Gyr.

<table>
<thead>
<tr>
<th>$\Omega_M$</th>
<th>$\Omega_A$</th>
<th>$h_{\max}$</th>
<th>$P_{\text{lens}}^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>1.14</td>
<td>5.98</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>0.96</td>
<td>3.94</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>0.86</td>
<td>2.93</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>0.79</td>
<td>2.33</td>
</tr>
<tr>
<td>0.45</td>
<td>0.55</td>
<td>0.76</td>
<td>2.11</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.74</td>
<td>1.92</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>0.70</td>
<td>1.63</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>0.67</td>
<td>1.42</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>0.64</td>
<td>1.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>0.61</td>
<td>1.11</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>0.59</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$a$ Recall that $P_{\text{lens}}$ is independent of $h$ (see equations (12)-(15)).

As for a nonflat universe, one may either fix $\Omega_M$ and $\Omega_A$ first and then calculate $h_{\max}$ to give $t_0 = 11$ Gyr, or one may fix $\Omega_M$ and $h_{\max}$ first and then calculate the $\Omega_A$ value from Eq.(11) by trial and error to give again $t_0 = 11$ Gyr. We have chosen the second option and constructed Figures 2 and 3, which are contour diagrams of $h_{\max}$ (for $t_0 = 11$ Gyr) in the $(\Omega_M, \Omega_A)$ and $(\Omega_M, P_{\text{lens}})$ planes.

![Figure 2](image-url)  

**Figure 2.** Contours of $h_{\max}$ for which $t_0 = 11$ Gyr in the $(\Omega_M, \Omega_A)$ plane.
It is seen that for each contour there is a minimum value of $\Omega_M$ before which the age is greater or equal to 11 Gyr for $\Omega_\Lambda = 0$. In drawing Figures 2 and 3, we have assumed that the maximum allowed value of $\Omega_\Lambda$ is about 1.1, in accordance with the findings of Perlmutter et al. [17]. The age problem is seen to survive for $\Omega_M \geq 0.3$ only if $h$ is as large as 0.8 for which lensing predictions are larger than 2. There is no age problem in a non-flat universe provided $h \leq 0.75$ for all $\Omega_M \leq 1$.

**Figure 3.** Contours of $h_{\text{max}}$ for which $t_0 = 11$ Gyr in the ($\Omega_M, P_{\text{lens}}$) plane.

**Conclusions**

That the *Hipparcos* data [18, 19] implies that GCs may be as young as $\sim 11$ Gyr has raised hopes to reconcile the age of GCs and the expansion age of the universe. We have studied this matter in this work. As is well known, and as born out by our results, the realization of this hope depends solely on the value of $H_0$. If $H_0$ is as large as the upper limit of the Freedman et al. [10] value, the age of the universe problem continues to exist in the SM. The problem, however, is now milder than it had been before *Hipparcos*. Previously, it was thought to exist even for moderate values of $h$, whereas it seems to exist for large values of $h$ now. If, however, $H_0$ is as low as favored by Tammann and Sandage [9] then the GC and the expansion ages of the universe are consistent with each other in the SM.

Assuming that $H_0$ is high and hence modifying the SM by invoking a (time-independent) cosmological term in the Einstein field equations, as has been done before [11-13], we have confirmed the conclusion of previous workers that, due to lensing restrictions, the age problem still survives in a flat universe for $\Omega_M < 0.5$, and at the same time conclude that $h$ cannot be larger than about 0.75. As for a nonflat universe, we have shown that the age problem does not exist for all $\Omega_M \leq 1$ provided $h \leq 0.75$. 

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The above mentioned hope is realized in the SM only if $h \leq 0.6$ (see Table 1). Otherwise, some kind of modification of the SM is called for. One such— and the most-studied— attempt is the inclusion of a cosmological term in the field equations. With such a term, the age problem has a better standing in a nonflat (open or closed) universe with $\Omega_M \leq 1$. It should be noted, in the light of recent works, that such a cosmological term need not be a pure time-independent constant. Scalar fields, cosmic strings or some kind of stable textures with an energy density varying as $a^{-2}$ lead to viable cosmological models that stand as alternatives to the SM [23-25].

References


