Tr. J. of Mathematics
21 (1997) , 69 – 72.

THE PAPER ON THE MATRIX VALUED TIME SERIES

M. Tuncay Gençoğlu

Abstract

In this paper, random series oscillatory were examine in matrix valued time series.
This series' calculated correlation coefficient between elements.
Positive serial correlation coefficient between terms examined relation.

Definition 1. *Total function of elements nook of square matrix is trace function (1). If variance is elements of matrix, trace of the matrix' II average variance.*

Definition 2. *The k-dimensional time series* $\{x_t : t = 0 \pm 1, \pm 2, ...\}$ *is defined by*

$$x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \\ \cdot \\ \cdot \\ x_{kt} \end{bmatrix}$$

where $\{x_t : t = 0, \pm 1, \pm 2, ...\}, 1, 2, ..., k$ are scalar time series. The expected value of x_t is

$$E\{x_t\} = \begin{bmatrix} E\{x_{1t}\} \\ E\{x_{2t}\} \\ \cdot \\ \cdot \\ \cdot \\ E\{x_{kt}\} \end{bmatrix}$$

Assuming the mean is zero, we define the covariance matrix of x_t and $\mathrm{x}t + h$ by

$$E\{x_t x'_{t+h}\} = \begin{bmatrix} E\{x_{1t}x_{1,t+h}\} & E\{x_{1t}x_{2,t+h}\} & \cdots & E\{x_{1t}x_{k,t+h}\} \\ E\{x_{2t}x_{1,t+h}\} & E\{x_{2t}x_{2,t+h}\} & \cdots & E\{x_{2t}x_{k,t+h}\} \\ \\ E\{x_{kt}x_{1,t+h}\} & E\{x_{kt}x_{2,t+h}\} & \cdots & E\{x_{kt}x_{k,t+h}\} \\ \end{bmatrix}$$

As with scalar time series, we define x_t to be covariance stationary if:

1) The expected value of x_t is a constant function of time.

2) The convariance matrix of x_t and x_{t+h} is the same as the convariance matrix of $x_,$ and x_{j+h} for all $t, t+h, j, j+h$ in the index set (2).

1. Application of Random Series Oscillatory

Let; matrixes

$$x_1 = (a_{ij}) X_2 = (b_{ij}), ..., x_n = (\alpha_{ij})$$

Let the random series be

$$x_1, x_2, ..., x_n$$

where we assume $E(\gamma_{ij}) = E(\gamma_{ij})^2 = \sigma^2$. Take an MA(3)

$$X'_i = \frac{1}{3}(X_i + X_{i+1} + X_{i+2})$$

Then the variance of x'_i is

$$\begin{aligned} E(x'_i)^2 &= E\{(1/3^2(x_i + x_{i+1} + x_{i+2})^2)\} \\ &= 1/3^2 E\{x_1^2 + x_{i+1}^2 + x_{i+2}^2 + 2(x_i x_{i+1} + x_i x_{i+2} + x_{i+1} x_{i+2})\} \end{aligned}$$

where

$$\begin{aligned} E(\gamma_{ij}\gamma_{ij}) &= 0, i \neq j \\ E(x'_i)^2 &= 1/3^2\{E(x'_i)^2 + E(x'_{i+1}) + E(x'_{i+2})^2\} \end{aligned}$$

where $E(x'_i)^2, E(x'_{i+1})^2, E(x'_{i+2})^2$ matrixes are diagonal matrix. The matrixes

$$\text{Trace } E(x'_i)^2 = \sigma^2 + \sigma^2 + \sigma^2 + \cdots + \sigma^2 = k\sigma^2$$

so

$$\begin{aligned} \text{Trace } E(x_{i+1})^2 = k\sigma^2 \\ \text{Trace } E(x_{i+2})^2 = k\sigma^2 \end{aligned}$$

Therefore

$$E(x_i)^2 = (1/3)(k\sigma^2) = k\sigma^2/3$$

for $n = 3$;

$$E(x_i)^2 = k\sigma^2/n$$

The covariance is

$$\begin{aligned} \text{Cov } (x'_i, x'_{i+r}) &= E(x'_i x'_{i+r}) \\ &= (1/3^3) E((x_i + x_{i+1} + x_{i+2})(x_{i+r} + x_{i+r+1} + x_{i+r+2})) \end{aligned}$$

If $n = 3 > r$ (say, $r = 2$) then

$$\begin{aligned}\text{Cov }(x'_i x'_{i+r}) &= (1/3^2)E((x_i x_{i+r} + x_i x_{i+3} + x_i x_{i+4} + x_{i+1}x_{i+2} + x_{i+1}x_{i+3} \\ &\quad + x_{i+1}x_{i+4} + x_{i+2}x_{i+2} + x_{i+2}x_{i+3} + x_{i+2}x_{i+4})) \\ &= (1/3^2)(E(x_{i+2})^2 + E(x_i x_{i+2} + \cdots + x_{i+2}x_{i+4}))\end{aligned}$$

where

$$E(\gamma_{ij}\gamma_{ji}) = 0, \ i \neq j$$

$$\text{Cov }(x'_i x'_{i+r}) = (1/3^2)E(x_{i+2})^2$$

Where, $E(x_{i+2})^2$ matrix is a covariance matrix. It is diagonal.

$$\begin{aligned}\text{Trace } E(x_{i+2})^2 &= \sigma^2 + \sigma^2 + \sigma^2 + \cdots + \sigma^2 = k\sigma^2 \\ \text{Cov }(x'_i, x'_{i+r}) &= k\sigma^2/3^2 \\ &= (1/3^2)(3-2)k\sigma^2 \\ &= (1/n^2)(n-r)k\sigma^2\end{aligned}$$

If $n = 3 \leq r$ (Say, $r = 3$) then

$$\text{Cov }(x'_i, x'_{i+r}) = (1/3^2)(3-3)k\sigma^2 = 0$$

Hence, the serial Correlation Coefficient between x'_i and x'_{i+r} is, for $n > r$,

$$\begin{aligned}\rho_{i,i+r} &= \frac{\text{Trace Cov }(x'_i, x'_{i+r}}{\text{Trace Cov }(x'_i, x'_i) \text{ Trace Cov }(x'_{i+r}, x'_{i+r}} \\ &= \frac{\frac{k}{n^2}(n-r)\sigma^2}{\sqrt{(k\sigma^2)/n}\sqrt{(k\sigma^2)/n}} = (n-r)/n\end{aligned}$$

For $n \leq r$

$$\rho_{i,i+r} = 0/n = 0$$

We see that as the lag r becomes smaller, and the serial correlation coefficient between successive terms indicates that the terms are not independent, and the sequence of moving averages will generate an oscillatory series.

References

[1] Asil, V. Master Thesis, 1989, Elazığ

[2] Fuller, A.W. Inroduction to Statistical Time Series, 1976, London
Yamane, T. Statistics and Inroductory Analysis, 1964, Tokyo

MATRİS DEĞERLİ ZAMAN SERİLERİ ÜZERİNE BİR ÇALIŞMA

Özet

Bu çalışmada salınımlı tesadüfi seriler matris değerli zaman serileri içinde incelenmiştir. Bu serinin elemanları arasındaki korelasyon katsayısı hesaplanmıştır. Terimler ile pozitif korelasyon katsayısı arasındaki ilişki incelenmiştir.

M. Tuncay GENÇOĞLU
Department of Mathematics
University of Fırat
23119, Elazığ-TURKEY

Received 21.12.1995
Revised 04.12.1996