THE PAPER ON THE MATRIX VALUED TIME SERIES

M. Tuncay Gencoğlu

Abstract
In this paper, random series oscillatory were examine in matrix valued time series. This series’ calculated correlation coefficient between elements. Positive serial correlation coefficient between terms examined relation.

Definition 1. Total function of elements nook of square matrix is trace function (1). If variance is elements of matrix, trace of the matrix’ II average variance.

Definition 2. The k-dimensional time series \{x_t : t = 0 \pm 1, \pm 2, \ldots\} is defined by

\[
x_t = \begin{bmatrix}
x_{1t} \\
x_{2t} \\
\vdots \\
x_{kt}
\end{bmatrix}
\]

where \{x_t : t = 0, \pm 1, \pm 2, \ldots\}, 1, 2, \ldots, k are scalar time series. The expected value of \(x_t\) is

\[
E\{x_t\} = \begin{bmatrix}
E\{x_{1t}\} \\
E\{x_{2t}\} \\
\vdots \\
E\{x_{kt}\}
\end{bmatrix}
\]

Assuming the mean is zero, we define the covariance matrix of \(x_t\) and \(x_{t+h}\) by

\[
E\{x_t x_{t+h}'\} = \begin{bmatrix}
E\{x_{1t} x_{1,t+h}\} & E\{x_{1t} x_{2,t+h}\} & \cdots & E\{x_{1t} x_{k,t+h}\} \\
E\{x_{2t} x_{1,t+h}\} & E\{x_{2t} x_{2,t+h}\} & \cdots & E\{x_{2t} x_{k,t+h}\} \\
E\{x_{kt} x_{1,t+h}\} & E\{x_{kt} x_{2,t+h}\} & \cdots & E\{x_{kt} x_{k,t+h}\}
\end{bmatrix}
\]
GENÇOĞLU

As with scalar time series, we define \( x_t \) to be covariance stationary if:

1) The expected value of \( x_t \) is a constant function of time.

2) The covariance matrix of \( x_t \) and \( x_{t+h} \) is the same as the covariance matrix of \( x_1 \) and \( x_{j+h} \) for all \( t, t+h, j, j+h \) in the index set \( (2) \).

### 1. Application of Random Series Oscillatory

Let; matrices

\[
x_1 = (a_{ij}) x_2 = (b_{ij}), ..., x_n = (a_{ij})
\]

Let the random series be

\[
x_1, x_2, ..., x_n
\]

where we assume \( E(\gamma_{ij}) = E(\gamma_{ij})^2 = \sigma^2 \). Take an

MA(3)

\[
X_i' = \frac{1}{3}(X_i + X_{i+1} + X_{i+2})
\]

Then the variance of \( x_i' \) is

\[
E(x_i')^2 = E\{(1/3^2(x_i + x_{i+1} + x_{i+2})^2)\}
\]

\[
= 1/3^2E\{x_i^2 + x_{i+1}^2 + x_{i+2}^2 + 2(x_i x_{i+1} + x_i x_{i+2} + x_{i+1} x_{i+2})\}
\]

where

\[
E(\gamma_{ij}\gamma_{ij}) = 0, i \neq j
\]

\[
E(x_i')^2 = 1/3^2\{E(x_i')^2 + E(x_{i+1}')^2 + E(x_{i+2}')^2\}
\]

where \( E(x_i')^2, E(x_{i+1}')^2, E(x_{i+2}')^2 \) matrices are diagonal matrix. The matrices

\[
\text{Trace } E(x_i')^2 = \sigma^2 + \sigma^2 + \sigma^2 + \cdots + \sigma^2 = k\sigma^2
\]

so

\[
\text{Trace } E(x_{i+1})^2 = k\sigma^2
\]

\[
\text{Trace } E(x_{i+2})^2 = k\sigma^2
\]

Therefore

\[
E(x_i)^2 = (1/3)(k\sigma^2) = k\sigma^2 / 3
\]

for \( n = 3; \)

\[
E(x_i)^2 = k\sigma^2 / n
\]

The covariance is

\[
\text{Cov} (x_i', x_{i+r}') = E(x_i' x_{i+r}')
\]

\[
= (1/3^3)E((x_i + x_{i+1} + x_{i+2})(x_{i+r} + x_{i+r+1} + x_{i+r+2}))
\]
If \( n = 3 > r \) (say, \( r = 2 \)) then

\[
\text{Cov} \left( x'_i x'_{i+r} \right) = \left( 1/3^2 \right) E \left( (x_i x_{i+r} + x_i x_{i+4} + x_{i+1} x_{i+2} + x_{i+1} x_{i+3} + x_{i+1} x_{i+4} + x_{i+2} x_{i+3} + x_{i+2} x_{i+4}) \right)
= (1/3^2) (E(x_{i+2})^2 + E(x_i x_{i+2} + \cdots + x_{i+2} x_{i+4}))
\]

where

\[
E(\gamma_i \gamma_j) = 0, \ i \neq j
\]

\[
\text{Cov} \left( x'_i x'_{i+r} \right) = (1/3^2) E(x_{i+2})^2
\]

Where, \( E(x_{i+2})^2 \) matrix is a covariance matrix. It is diagonal.

\[
\text{Trace } E(x_{i+2})^2 = \sigma^2 + \sigma^2 + \cdots + \sigma^2 = k \sigma^2
\]

\[
\text{Cov} \left( x'_i, x'_{i+r} \right) = k \sigma^2 / 3^2
= (1/3^2) (3-2) k \sigma^2
= (1/n^2) (n-r) k \sigma^2
\]

If \( n = 3 \leq r \) (Say, \( r = 3 \)) then

\[
\text{Cov} \left( x'_i, x'_{i+r} \right) = (1/3^2) (3-3) k \sigma^2 = 0
\]

Hence, the serial Correlation Coefficient between \( x'_i \) and \( x'_{i+r} \) is, for \( n > r \),

\[
\rho_{i,i+r} = \frac{\text{Trace Cov} \left( x'_i, x'_{i+r} \right)}{\text{Trace Cov} \left( x'_i, x'_i \right) \text{ Trace Cov} \left( x'_{i+r}, x'_{i+r} \right)}
= \frac{k}{n^2} \frac{(n-r) \sigma^2}{\sqrt{(k \sigma^2)/n} \sqrt{(k \sigma^2)/n}} = (n-r)/n
\]

For \( n \leq r \)

\[
\rho_{i,i+r} = 0 = 0
\]

We see that as the lag \( r \) becomes smaller, and the serial correlation coefficient between successive terms indicates that the terms are not independent, and the sequence of moving averages will generate an oscillatory series.
GENÇOĞLU

References

    Yamane, T. Statistics and Introductory Analysis, 1964, Tokyo

MATRİS DEĞERLİ ZAMAN SERİLERİ ÜZERİNE BİR ÇALIŞMA

Özet

Bu çalışmada salıncı tesadüfi seriler matris değerli zaman serileri içinde incelenmiştir. Bu serinin elemanları arasındaki korelasyon katsayısı hesaplanmıştır. Terimler ile pozitif korelasyon katsayısı arasındaki ilişki incelenmiştir.

M. Tuncay GENÇOĞLU
Department of Mathematics
University of Fırat
23119, Elazığ, TURKEY

Received 21.12.1995
Revised 04.12.1996

72