Erratum to “Study on quasi-$\Gamma$-hyperideals in $\Gamma$-semihypergroups”

Niovi KEHAYOPULU∗
Nikomidias 18, 16122 Kesariani, Greece

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Abstract: We wrote this note to show that the definition of $\Gamma$-hypersemigroups in [2] should be corrected, and that it is not enough to replace the hyperoperation $\circ$ of the hypersemigroup by $\Gamma$ to pass from a hypersemigroup to a $\Gamma$-hypersemigroup. Care should be taken about the definitions of $(m, n)$-quasi-$\Gamma$-hyperideal, the $m$-left $\Gamma$-hyperideal and the $n$-right $\Gamma$-hyperideal as well.

Key words: $\Gamma$-semihypergroup, $(m, n)$-quasi-$\Gamma$-hyperideal

According to Definition 1.1 of the paper, “if $H$ and $\Gamma$ are two nonempty sets, any mapping $H \times \Gamma \times H \rightarrow \mathcal{P}(H)$ is called a $\Gamma$-hypermultiplication in $H$ and denoted by $(\cdot)_{\Gamma}$. The result of this hypermultiplication for $a, b \in H$ and $\alpha \in \Gamma$ is denoted by $a_{\alpha}b$. A $\Gamma$-semihypergroup $S$ is an ordered pair $(H, (\cdot)_{\Gamma})$ where $H$ and $\Gamma$ are nonempty sets and $(\cdot)_{\Gamma}$ is a $\Gamma$-hypermultiplication on $H$ which satisfies the following property: For all $(a, b, c, \alpha, \beta) \in H^3 \times \Gamma^2$, $(a_{\alpha}b_{\beta}c = a_{\alpha}(b_{\beta}c))$. If every $\gamma \in \Gamma$ is an operation, then $H$ is a $\Gamma$-semigroup. Let $A$ and $B$ are two nonempty subsets of $H$. Then, we define $A_{\Gamma}B = \bigcup_{\gamma \in \Gamma} A_{\gamma}B = \bigcup \{a_{\gamma}b \ | \ a \in A, b \in B \text{ and } \gamma \in \Gamma\}.”

So $a_{\alpha}b$ is defined as a nonempty subset of $S$ (as it was expected to be). That being so, the expression of the form $(a_{\alpha}b_{\beta}c$ should be corrected, there is no sense in the way is stated. According to this definition, “if every $\gamma \in \Gamma$ is an operation”, but we have to clarify here what “$\gamma$” is an operation means. Is it an operation between elements or an operation between sets? The $A_{\Gamma}B$ in (\ast) shows that $\gamma$ is an operation between sets while $a_{\gamma}b$ in (\ast) shows that it is an operation between elements. As a result, the definition of $\Gamma$-semihypergroup is wrong and a correct definition of $\Gamma$-hypersemigroups is needed.

A $\Gamma$-hypersemigroup $H$ is called “regular” [2] if for every $x \in H$ there exists $y \in H$ such that $x = x_{\alpha}y_{\beta}x$ without any explanation what the $x_{\alpha}y_{\beta}x$ means. What is the $x_{\alpha}y_{\beta}x$? What is the “$\Gamma$” in it? It seems like an operation between elements, but even in that case, what is the $x_{\alpha}y_{\beta}x$? According to [2], the most possibly case concerning the concept of regularity is the following: A $\Gamma$-hypersemigroup $H$ is called regular if for every $x \in S$, there exist $y \in H$ and $\alpha, \beta \in \Gamma$ such that $x = x_{\alpha}y_{\beta}x$; but as we already said expression of the form $x_{\alpha}y_{\beta}x$ should be corrected.

∗Correspondence: nkehayop@math.uoa.gr
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For the sake of simplicity, throughout the paper, $\underbrace{H \Gamma H \ldots \Gamma H}_n$ is denoted by $H^n$. But what the $\underbrace{H \Gamma H \ldots \Gamma H}_n$ means? First of all, do we have the right to write $H \Gamma H \ldots \Gamma H$ without using parentheses? There is nothing about it in the bibliography.

According to Definition 2.2 in [2], a nonempty subset $Q$ of a $\Gamma$-semihypergroup $H$ is called $(m, n)$-quasi-$\Gamma$-hyperideal of $H$ is $H^m \Gamma Q \cap Q \Gamma H^n \subseteq Q$. This is true for $m = n = 1$, but what about arbitrary $m, n$? Considering that this definition is a generalization of the concept of the $(m, n)$-quasi-ideals of semigroups introduced by Lajos (see, for example [4]), an $(m, n)$-quasi-$\Gamma$-hyperideal should be defined as $Q^m \Gamma H \cap H \Gamma Q^n \subseteq Q$. The definition of the $m$-left $\Gamma$-hyperideal $L$ is defined as $H^m \Gamma L \subseteq L$; the correct is $L^m \Gamma H \subseteq L$. The $n$-right $\Gamma$-hyperideal of $H$ is defined as $R \Gamma H^n \subseteq R$; the correct is $R^n \Gamma H \subseteq R$. But we have to keep in mind that, for an arbitrary nonempty subset $A$ of $S$, the $A^n$ is the set $\underbrace{((A \Gamma A) \Gamma A) \Gamma A \ldots \Gamma A}_n$. We are not in a semigroup or a $\Gamma$-semigroup where this is simple. It might be mentioned here that an element $q$ of a poe-groupoid $S$ is called an $(m, n)$-quasi-ideal element of $S$ if $q^m e \land eq^n$ exists in $S$ and $q^m e \land eq^n \leq q$ [3] and it is called $(0, n)$ (resp. $(m, 0)$)-ideal element of $S$ if $ea^n \leq a$ (resp. $a^me \leq a$) [3]. Every $(m, 0)$-ideal element is a $(m, 0)$-quasi-ideal and every $(0, n)$-ideal element is a $(0, n)$-quasi-ideal element.

In what follows, the aim is to show that is not enough to pass from a semigroup to a hypersemigroup by replacing the multiplication “·” of the semigroup by the hyperoperation “◦” and to pass from a hypersemigroup to a $\Gamma$-hypersemigroup replacing the “◦” by “$\Gamma$”.

The paper in [2] is the paper in [1] with the only difference that the hypeoperation ◦ in [1] has been replaced by $\Gamma$ in [2].

In fact,

Lemma 2.1 in [2] is the Lemma 2.10 in [1];
Proposition 2.1 in [2] is the Proposition 2.11 in [1];
Theorem 2.1 in [2] is the Proposition 2.10 in [1];
Theorem 2.2 in [2] is the Theorem 2.14 in [1];
Theorem 2.3 in [2] is the Theorem 2.16 in [1];
Theorem 2.4 in [2] is the Theorem 2.17 in [1];
Theorem 2.5 in [2] is the Theorem 2.18 in [1];
Theorem 2.6 in [2] is the Theorem 2.19 in [1];
Proposition 2.3 in [2] is the Proposition 2.20 in [1];
Theorem 2.7 in [2] is the Theorem 2.21 in [1];
Proposition 2.3 in [2] is the Proposition 2.22 in [1];
Lemma 3.1 in [2] is the Lemma 3.4 in [1];
Theorem 3.1 in [2] is the Theorem 3.5 in [1];
Corollary 3.1 in [2] is the Corollary 3.6 in [1];
Theorem 3.2 in [2] is the Theorem 3.7 in [1];
Theorem 3.3 in [2] is the Theorem 3.11 in [1];
Theorem 3.4 in [2] is the Theorem 3.12 in [1].
The definition of quasi-hyperideals and the definitions of \( m \)-left and \( n \)-right hyperideals of semihypergroups in [1] should be also corrected; in addition, except of Theorem 2.14, Theorem 2.17 (and the Examples), the results of section 2 in [1] duplicates, without citation, the paper “A note on \((m, n)\)-quasi-ideals in semigroup” by Moin A. Ansari, M. Rais Khan, J. P. Kaushik in International Journal of Mathematical Analysis 3 (2009), 1853-1858 (with the usual change), that is a further indication that is not enough to pass from a semigroup to a hypersemigroup just replacing the \( \cdot \) by \( \circ \); but this is out of the scope of the present note.

The definition of \( \Gamma \)-hypersemigroups, the regularity, and related information has been given by the author of the present note in the paper “Lattice ordered semigroups and \( \Gamma \)-hypersemigroups” submitted to Turkish Journal of Mathematics.

References


