Corrigendum and addendum to “Modules whose \(p\)-submodules are direct summands”

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Abstract: This paper is written to correct the proof of Lemma 2.1(i) in [1] and to add some decomposition results for the class of PD-modules defined in [1].

Key words: PD-modules

In line 14 of the Introduction replace “[...] a sublattice of the lattice of submodules of \(M\)” with “closed under intersections.”

In the proof of Lemma 2.1(i) replace “Observe that \((M/N_1) \oplus (M/N_2) = M/(N_1 \cap N_2)\)” with “Define the homomorphism \(\alpha : M \to M/(N_1 \cap N_2)\) by \(\alpha(m) = (m + N_1, m + N_2)\). Observe that \(M/(N_1 \cap N_2) \cong \alpha(M) \leq (M/N_1) \oplus (M/N_2)\). Hence \(\alpha(M)\) is nonsingular, as \((M/N_1) \oplus (M/N_2)\) is nonsingular. Thus [...]”

In the statement of Proposition 2.4, insert “\(M_1\) is a \(p\)-submodule of \(M\) such that” before “and [...]”.

In the proof of Proposition 3.6 replace “[3, Lemma 4.11]” with “[3, Lemma 4.13]”.

Moreover, we obtain the following decomposition results with respect to the second singular submodule \(Z_2(M)\) of \(M\) for the class of PD-modules.

Proposition 1 Let \(M\) be a PD-module and \(K\) a \(p\)-submodule of \(M\). Then \(M = Z_2(M) \oplus T \oplus Y\), where \(K = Z_2(M) \oplus T\) and \(Y\) are PD-modules.

Proof Let \(M\) be a PD-module and \(K\) a \(p\)-submodule of \(M\). Then \(M = K \oplus K'\) for some \(K' \leq M\). Since \(K \subseteq_p M\), \(K\) and \(K'\) are PD-modules by [1, Proposition 3.6]. Recall that \(Z_2(M) \subseteq K\), as \(M/K\) is nonsingular. Since \(Z_2(M) \subseteq_p M\) and \(Z_2(M) \subseteq K\), \(Z_2(M) \subseteq_p K\). Moreover, \(Z(K/Z_2(M)) = 0\) yields that \(Z_2(M)\) is a \(p\)-submodule of \(K\). It follows that \(K = Z_2(M) \oplus T\) for some \(T \subseteq K\). Therefore, \(M = K \oplus K' = Z_2(M) \oplus T \oplus K'\). Hence, \(K' = Y\); \(Y\) is the desired direct summand. \(\Box\)

Corollary 2 \(M\) is a PD-module if and only if \(M = Z_2(M) \oplus Y\), where \(Z_2(M)\) and \(Y\) are PD-modules.

Proof It is clear from Proposition 1. \(\Box\)

References


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