C–Closed Sets in L–Fuzzy Topological Spaces and Some of its Applications

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Abstract

We introduce and study the notion of C–closed sets in L–fuzzy topological spaces. Then, C–convergence theory for nets and ideals is established in terms of C–closedness. Finally, we give a new concept of C–continuity on L–fuzzy topological space by means of L–fuzzy C–closedness and investigate some of its properties and its relationships with other L–fuzzy mappings introduced previously. Then we systematically study the characterizations of this notion with the aid of the C–convergence of L–fuzzy nets and L–fuzzy ideals.


1. Introduction

Continuity and its weaker forms constitute an important and intensely investigated area in the field of general topological spaces. For example, the notions of almost continuous, N–continuous, H–continuous, C–continuous, weakly continuous and semi–continuous have been introduced by different authors, and their inter–relationships with other topological notions have been established. Most of these notions turn out to be local properties; hence the pointwise approach is generally preferred in their studies and definitions. The concept of C–continuity in general topology was introduced by Gentry and Hoyle [5] in 1970. The class of C–closed sets (compact and closed) was defined by Garg and Kumar [4] in 1989. Then several characterizations of C–continuous mappings in terms of C–closed sets are given. Recently, Dang, Behera and Nanda [3] extended the concept to fuzzy topology, and introduced the notion of fuzzy C–continuous function using the fuzzy compactness given by Mukherjee and Sinha [8]. However, the fuzzy compactness has some shortcomings, such as the Tychonoff product theorem does not

2. Preliminaries

Throughout this paper, L denotes a complete, completely distributive lattice; M(L) denotes the set of all nonzero irreducible elements of L; and 0 and 1 denote the least and greatest element in L, respectively. L X and L Y denote the set of all L–fuzzy sets on crisp sets X and Y, respectively. Write M(L X) = \{x α ∈ L X : x ∈ X, α ∈ M(L)\}, and call the elements in M(L X) molecules or L–fuzzy points on X. For ϕ ⊆ L X, put ϕ' = \{μ' : μ ∈ ϕ\}.

Let (L X, τ) be an L–fuzzy topological space, briefly L–fts. For each μ ∈ L X, cl(μ), int(μ) and μ' will denote the closure, interior and the complement of μ, respectively. 0 X and 1 X denote, respectively, the least and the greatest element of L X. If μ ∈ L X and μ = int(cl(μ)), then it is called regular open. The complement of regular open is called regular closed. The class of all L–fuzzy regular open (resp. regular closed) sets will be denoted by RO(L X, τ) (resp. RC(L X, τ)). Let (X, T) be a crisp topological space and μ ∈ L X, if ∀ α ∈ L, \{x ∈ X : μ(x) ≤ α\} ∈ T', then we call μ a lower semi–continuous function. The set of all these functions is denoted by ω L(T) and is an L–fuzzy topology on X generated by T.

**Definition 2.1** [10]: Let (L X, τ) be an L–fts and x α ∈ M(L X). λ ∈ τ' is called a remoted neighbourhood (R–nbd, for short) of x α if x α ≤ λ. The set of all R–nbds of x α is denoted by R x α.

**Definition 2.2** [1,10]: Let (L X, τ) be an L–fts and μ ∈ L X. Ψ ⊆ τ' (resp. Ψ ⊆ RC(L X, τ)) is called an α–remoted (resp. α–regular closed remoted) neighbourhood family of μ, briefly α–RF (resp. α–rcRF) of μ, if for each L–fuzzy point x α ≤ μ, there is η ∈ Ψ such that η ∈ R x α.
Definition 2.3 [9,10]: Let \((L^X, \tau)\) be an L-fts. Then \(\mu \in L^X\) is called:

(i) \(Q\alpha\)-compact (resp. nearly \(Q\alpha\)-compact) if for any \(\alpha \in M(L)\) and every \(\alpha\)-RF (resp. \(\alpha\)-rcRF) \(\Psi\) of \(\mu\) there exists a finite subfamily \(\Psi_0\) of \(\Psi\) such that \(\Psi_0\) is an \(\alpha\)-RF of \(\mu\).

(ii) Strong \(Q\)-compact (resp. Strong nearly \(Q\)-compact) if it is \(Q\alpha\)-compact (resp. nearly \(Q\)-compact) for all \(\alpha \in M(L)\).

If \(1_X\) is \(Q\alpha\)-compact (resp. nearly \(Q\alpha\)-compact, strong \(Q\)-compact, strong nearly \(Q\)-compact), then we say that \((L^X, \tau)\) is a \(Q\alpha\)-compact (resp. nearly \(Q\alpha\)-compact, strong \(Q\)-compact, strong nearly \(Q\)-compact) space.

Definition 2.4 [7]: An \((L^X, \tau)\) is said to be:

(i) \(LFT_2\)-space (L-fuzzy Hausdorff space) iff (\(\exists x_\alpha, y_\gamma \in M(L^X), x \neq y\))
   \((\exists \eta \in R_{x_\alpha})(\exists \lambda \in R_{y_\gamma})(\eta \lor \lambda = 1_X)\).

(ii) \(LFR_2\)-space (L-fuzzy regular space) iff (\(\forall x_\alpha \in M(L^X)\)) (\(\forall \eta \in R_{x_\alpha}\))
   \((\exists \lambda \in R_{x_\alpha})(\exists \rho \in \tau')(\lambda \lor \rho = 1_X \text{ and } \eta \land \rho = 0_X)\).

(iii) Fully stratified if \(\alpha \in \tau\) for all \(\alpha \in L\).

(iv) Weakly induced if each nonempty element of \(\tau\) is a lower semi-continuous mapping
    from \((X, [\tau])\) to \(L\).

(v) Induced if it is both fully stratified and weakly induced.

The family of all crisp open (resp. closed) sets in \(\tau\) is denoted by \([\tau]\) (resp. \([\tau']\)).
Obviously, \((X, [\tau])\) is a crisp topological space.

Theorem 2.5 [7]: A topological space \((X, T)\) is a \(T_2\)-space iff an L-fts \((L^X, \omega_L(T))\) is a \(LFT_2\)-space.

Theorem 2.6 [6]: For fully stratified L-fts \((L^X, \tau)\) and \(\mu \in L^X\), if for each \(\alpha \in M(L), \mu_{\omega \alpha} \in [\tau']\), then \(\mu \in \tau'\), where \(\mu_{\omega \alpha} = \{x \in X : \mu(x) \geq \alpha \text{ and } \alpha \in M(L)\}\).

Theorem 2.7 [9]: Each strong \(Q\)-compact L-fuzzy set in a fully stratified \(LFT_2\)-space is L-fuzzy closed.
Theorem 2.8 [9]: Every L-fuzzy closed set of a $Q_\alpha$-compact (resp., strong $Q$-compact) L-fts is $Q_\alpha$-compact (resp., strong $Q$-compact).

Theorem 2.9 [9]: Let $(X, T)$ be a topological space. Then L-fuzzy set $\mu \in L^X$ is $Q_\alpha$-compact in $(L^X, \omega_L(T))$ iff $\mu_{\text{wc}}$ is compact in $(X, T)$, for all $\alpha \in M(L)$.

Theorem 2.10 [9]: Let $(L^Y, \Delta)$ be an $LFT_2$-space and $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ be an L-fuzzy continuous mapping [12] and $\mu \in L^X$ be a strong $Q$-compact in $(L^X, \tau)$, then $f(\mu)$ is a strong $Q$-compact L-fuzzy set in $(L^Y, \Delta)$.

Theorem 2.11 [9]: Let $(L^X, \tau)$ be an $LFR_2$-space. Then every strong nearly $Q$-compact set is strong $Q$-compact.

Theorem 2.12 [9]: Let $(L^X, \tau)$ be an induced L-fts. Then the concepts of $N$-compactness and strong $Q$-compactness are equivalent.

Definition 2.13 [12,15]: Let $(L^X, \tau)$ be an L-fts. An L-fuzzy net in $(L^X, \tau)$ is a mapping $S : D \rightarrow M(L^X)$ denoted by $S = \{S(n), n \in D\}$, where $D$ is a directed set. $S$ is said to be in $\mu \in L^X$ if $\forall n \in D, S(n) \leq \mu$.

Definition 2.14 [12,13]: The non empty family $L \subset L^X$ is called an L-fuzzy ideal if, for each $\mu_1, \mu_2 \in L^X$ the following satisfies:

(i) If $\mu_1 \leq \mu_2$ and $\mu_2 \in L$, then $\mu_1 \in L$.

(ii) If $\mu_1, \mu_2 \in L$, then $\mu_1 \lor \mu_2 \in L$.

(iii) $1_X \notin L$.

Theorem 2.15 [12,13]: Let $(L^X, \tau)$ be an L-fts, $\mu \in L^X$ and $x_\alpha \in M(L^X)$. Then $x_\alpha \leq \text{cl}(\mu)$ iff there exists an L-fuzzy net in $\mu$ (resp., an L-fuzzy ideal $L$ not containing $\mu$) which converges to $x_\alpha$ (see Definitions 3.9 and 3.11).

Other unexplained notations and definitions in this paper can be found in [1,2,9,12,13].

3. L-fuzzy C-closure and C-interior operators.

In this section, we introduce and study the concepts of C-closure operator and C-interior operator by having the aid of the notion of $Q_\alpha$-compactness and discuss their properties. Then we present the concepts of C-limit and C-cluster points of L-fuzzy nets and L-fuzzy ideals.
**Definition 3.1:** Let \((L^X, \tau)\) be an L-fits and \(\mu \in L^X\). An L-fuzzy point \(x_\alpha \in M(L^X)\) is called an C-adherent (resp. \(N^*\)-adherent) point of \(\mu\), written as \(x_\alpha \leq C.cl(\mu)\) (resp. \(x_\alpha \leq N^*.cl(\mu)\)) iff \(\mu \not\subseteq \lambda\) for each \(\lambda \in CR_{x_\alpha}\) (resp. \(\lambda \in N^*R_{x_\alpha}\)), where \(CR_{x_\alpha}\) (resp. \(N^*R_{x_\alpha}\)) is the family of all strong \(Q\)-compact (resp. strong nearly \(Q\)-compact) R-nbds of \(x_\alpha\). \(C.cl(\mu)\) (resp. \(N^*.cl(\mu)\)) is said to be C-closure (resp. \(N^*\)-closure) of \(\mu\). If \(C.cl(\mu) \subseteq \mu\) (resp. \(N^*.cl(\mu) \subseteq \mu\)), then \(\mu\) is called L-fuzzy C-closed (resp. \(N^*\)-closed). The complement of an L-fuzzy C-closed (resp. \(N^*\)-closed) set is called L-fuzzy C-open (resp. \(N^*\)-open) set.

In [1], Chen and Wang have introduced the concept of L-fuzzy \(N\)-closed sets by using \(N\)-compactness due to Zhao [14]. It is easy to see that every L-fuzzy \(N^*\)-closed set is \(N\)-closed. So the properties and characterizations of \(N^*\)-closed set and its related notions are similar to those of \(N\)-closed set.

**Theorem 3.2:** Let \((L^X, \tau)\) be an L-fits and \(\mu, \eta \in L^X\). Then the following statements hold:

(i) \(\mu \leq cl(\mu) \leq N^*.cl(\mu) \leq C.cl(\mu)\).

(ii) If \(\mu \leq \eta\) then \(C.cl(\mu) \leq C.cl(\eta)\).

(iii) \(C.cl(C.cl(\mu)) = C.cl(\mu)\).

(iv) \(C.cl(\mu) = \wedge\{\rho \in L^X : \rho\) is a C-closed set containing \(\mu\}\).

**Proof:** It is similar to that of Theorem 3.1 in [2].

**Theorem 3.3:** Let \((L^X, \tau)\) be an L-fits. The following statements hold:

(i) \(1_X\) and \(0_X\) are both C-closed.

(ii) Every strong \(Q\)-compact closed set is C-closed.

(iii) The union of finite C-closed sets is C-closed.

(iv) The intersection of arbitrary C-closed sets is C-closed.

(v) \(\mu \in L^X\) is C-closed iff there exists \(\eta \in CR_{x_\alpha}\) such that \(\mu \leq \eta\) for each \(x_\alpha \in M(L^X)\) with \(x_\alpha \not\subseteq \mu\).
Proof: It is similar to that of Theorem 3.2 in [2].

**Theorem 3.4:** Let $(L^X, \tau)$ be an L-fts and $\mu \in L^X$. Then the families

$$\tau_C = \{\mu \in L^X : \mu' = C.cl(\mu')\} \text{ and } \tau_{\mathcal{N}^*} = \{\mu \in L^X : \mu' = \mathcal{N}^*.cl(\mu')\}$$

of all L-fuzzy C-open and $\mathcal{N}^*$-open sets in $X$ are L-fuzzy topologies on $X$ associated with $\tau$. We call $(L^X, \tau_C)$ and $(L^X, \tau_{\mathcal{N}^*})$ L-fuzzy C-space and L-fuzzy $\mathcal{N}^*$-space, respectively, induced by $(L^X, \tau)$.

Proof: It is an immediate consequence of Definition 3.1 and Theorems 3.2 and 3.3.

**Theorem 3.5:** Let $(L^X, \tau)$ be an L-fts. Then

(i) $\tau_C \subseteq \tau_{\mathcal{N}^*} \subseteq \tau$.

(ii) If $(L^X, \tau)$ is strong $Q$-compact (resp. strong nearly $Q$-compact), then $\tau = \tau_C$ (resp. $\tau = \tau_{\mathcal{N}^*}$).

(iii) If $(L^X, \tau)$ is LFR$_2$-space, then $\tau_C = \tau_{\mathcal{N}^*}$.

(iv) If $(L^X, \tau)$ is induced L-fts, then $\tau_{\mathcal{N}^*} = \tau_N[1]$.

Proof: Follows from Theorems 2.11, 2.12 and 3.4.

**Definition 3.6:** Let $(L^X, \tau)$ be an L-fts, $\mu \in L^X$ and $C.int(\mu) = \forall \{\rho \in L^X : \rho \text{ is an L-fuzzy C-open set contained in } \mu\}$. We say that $C.int(\mu)$ is the C-interior of $\mu$.

The following theorem shows the relationships between C-closure operator and C-interior operator.

**Theorem 3.7:** Let $(L^X, \tau)$ be an L-fts and $\mu \in L^X$. Then the following are true.

(i) $\mu$ is C-open iff $\mu = C.int(\mu)$.

(ii) $C.int(\mu) \leq int(\mu) \leq \mu$.

(iii) $C.int(\mu) = (C.cl(\mu'))'$.

(iii) If $\eta \in L^X$ and $\mu \leq \eta$, then $C.int(\mu) \leq C.int(\eta)$.

(iv) $C.int(C.int(\mu)) = C.int(\mu)$.  

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Dually, we have the following results.

**Theorem 3.8:** Let \((L^X, \tau)\) be an \(L\)-fts. The following statements hold:

(i) \(1_X\) and \(0_X\) are both \(C\)-open.

(ii) The intersection of finite \(C\)-open sets is \(C\)-open.

(iii) The union of arbitrary \(C\)-open sets is \(C\)-open.

**Definition 3.9:** Let \(S\) be an \(L\)-fuzzy net in an \(L\)-fts \((L^X, \tau)\) and \(x_\alpha \in M(L^X)\). Then \(x_\alpha\) is said to be a:

(i) limit point of \(S\) [12] or \(S\) converges to \(x_\alpha\), in symbol \(S \rightarrow x_\alpha\), if

\[
(\forall \lambda \in R_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda).
\]

(ii) \(C\)-limit point of \(S\) or \(S\) \(C\)-converges to \(x_\alpha\), in symbol \(S^{C} \rightarrow x_\alpha\), if

\[
(\forall \lambda \in CR_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda).
\]

The union of all limit (resp. \(C\)-limit) points of \(S\) is denoted by \(\lim(S)\) (resp. \(C:lim(S)\)).

**Proposition 3.10:** Suppose that \(S\) is an \(L\)-fuzzy net in \((L^X, \tau)\), \(\mu \in L^X\) and \(x_\alpha \in M(L^X)\). Then the following results are true:

(i) \(x_\alpha \leq C:lim(S)\) iff \(S^{C} \rightarrow x_\alpha\).

(ii) \(\lim(S) \leq C:lim(S)\).

(iii) \(x_\alpha \leq C:cl(\mu)\) iff there is an \(L\)-fuzzy net in \(\mu\) which \(C\)-converges to \(x_\alpha\).

(iv) \(C:lim(S)\) is an \(L\)-fuzzy \(C\)-closed set in \(L^X\).

**Proof:** (i) Let \(S^{C} \rightarrow x_\alpha\), so by definition \(x_\alpha \leq C:lim(S)\). Conversely, let \(x_\alpha \leq C:lim(S)\) and \(\lambda \in CR_{x_\alpha}\). Since \(x_\alpha \not\leq \lambda\), we have \(C:lim(S) \geq \alpha > \lambda(x)\). Thus \(C:lim(S) \not\leq \lambda\).

Therefore there exists \(y_\beta \in M(L^X)\) such that \(S^{C} \rightarrow y_\beta\), but \(y_\beta \not\leq \lambda\) and so \(\lambda \in CR_{y_\beta}\). Hence

\[
(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda). \text{ Thus } S^{C} \rightarrow x_\alpha.
\]

(ii) Let \(x_\alpha \leq \lim(S)\) and \(\eta \in CR_{x_\alpha}\). Since \(CR_{x_\alpha} \leq R_{x_\alpha}\), then \(\eta \in R_{x_\alpha}\). And since \(x_\alpha \leq \lim(S)\), then

\[
(\forall \lambda \in R_{x_\alpha})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda) \text{ and so } S(m) \not\leq \eta. \text{ Hence } x_\alpha \leq C:lim(S). \text{ So } \lim(S) \leq C:lim(S).
\]

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(iii) Let \( x_\alpha \leq C.c.l(\mu) \). Then \( (\forall \lambda \in CR_{x_\alpha})(\mu \not\leq \lambda) \) and so there exists \( \alpha(\mu, \lambda) \in L \setminus \{0\} \) such that \( x_{\alpha(\mu, \lambda)} \leq \mu \) and \( x_{\alpha(\mu, \lambda)} \not\leq \lambda \). Since the pair \( (CR_{x_\alpha}, \geq) \) is a directed set, we can define an \( L \)-fuzzy set \( S : CR_{x_{\alpha}} \to M(L^X) \) given by \( S(\lambda) = x_{\alpha(\mu, \lambda)}, \forall \lambda \in CR_{x_{\alpha}} \). Then \( S \) is an \( L \)-fuzzy net in \( \mu \). Now let \( \rho \in CR_{x_{\alpha}} \) such that \( \rho \geq \lambda \), so we have the situation in which there exists \( S(\rho) = x_{\alpha(\mu, \rho)} > \rho \geq \lambda \). Then \( x_{\alpha(\mu, \rho)} \not\leq \lambda \). So \( S \supseteq x_{\alpha} \). Conversely, let \( S \) be an \( L \)-fuzzy set in \( \mu \) with \( S \supseteq x_{\alpha} \). Then \( (\forall \lambda \in CR_{x_{\alpha}})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \lambda) \). Since \( S \) is an \( L \)-fuzzy net in \( \mu \), then \( \mu \geq S(m) > \lambda \). Hence \( (\mu \not\leq \lambda)(\forall \lambda \in CR_{x_{\alpha}}) \). So \( x_{\alpha} \leq C.c.l(\mu) \).

(iv) Let \( x_\alpha \leq C.c.l(C.lim(S)) \) and \( \lambda \in CR_{x_{\alpha}} \). Then \( C.lim(S) \not\leq \lambda \). So there exists \( y_{\beta} \in M(L^X) \) such that \( y_{\beta} \leq C.lim(S) \) and \( y_{\beta} \not\leq \lambda \). Then \( (\forall \rho \in CR_{y_{\beta}})(\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq \rho) \) and so \( S(m) \not\leq \lambda \). Hence \( x_\alpha \leq C.lim(S) \). Thus \( C.c.l(C.lim(S)) \leq C.lim(S) \) and so \( C.lim(S) \) is a \( L \)-fuzzy \( C \)-closed set.

**Definition 3.11:** Let \( L \) be an \( L \)-fuzzy ideal in an \( L \)-ftes \((L^X, \tau)\) and \( x_\alpha \in M(L^X) \). Then \( x_\alpha \) is said to be:

(i) a limit point of \( L \) [13] or \( L \) converges to \( x_\alpha \), in symbol \( L \to x_\alpha \), if \( R_{x_\alpha} \subseteq L \).

(ii) \( C \)-limit point of \( L \) or \( L \) \( C \)-converges to \( x_\alpha \), in symbol \( L \supseteq x_\alpha \), if \( CR_{x_\alpha} \subseteq L \).

The union of all limit points (resp., \( C \)-limit points) of \( L \) is denoted by \( \text{lim}(L) \) (resp. \( C.lim(L) \)).

**Proposition 3.12:** Suppose that \( L \) is an \( L \)-fuzzy ideal in \((L^X, \tau)\), \( \mu \in L^X \) and \( x_\alpha \in M(L^X) \). Then the following results are true:

(i) \( x_\alpha \leq C.lim(L) \) iff \( L \supseteq x_\alpha \).

(ii) \( \text{lim}(L) \leq C.lim(L) \).

(iii) \( x_\alpha \leq C.c.l(\mu) \) iff there is an \( L \)-fuzzy ideal \( L \) which \( C \)-converges to \( x_\alpha \) and \( \mu \not\leq L \).

(iv) \( C.lim(L) \) is an \( L \)-fuzzy \( C \)-closed set in \( L^X \).

**Proof:** The proof of the statements (i), (ii) and (iv) are similar to the correspondence statements of Proposition 3.10.
(iii) Let $x_\alpha \leq C.c.l(\mu)$. Let $\mathcal{L}(CR_{x_\alpha}) = \{ \rho \in L^X : \exists \lambda \in CR_{x_\alpha} : \rho \leq \lambda \}$. It easy to show that $\mathcal{L}(CR_{x_\alpha})$ is an $L$-fuzzy ideal. Now we show that $\mu \notin \mathcal{L}(CR_{x_\alpha})$. Since $x_\alpha \leq C.c.l(\mu)$, then for each $\lambda \in CR_{x_\alpha}$, $\mu \not\leq \lambda$. So by definition of $\mathcal{L}(CR_{x_\alpha})$ we have $\mu \not\in \mathcal{L}(CR_{x_\alpha})$. Finally, we show that $\mathcal{L}^C_{x_\alpha}$. Let $\lambda \in CR_{x_\alpha}$ and since $\lambda \leq \lambda$, then $\lambda \in \mathcal{L}(CR_{x_\alpha})$. So $CR_{x_\alpha} \subseteq \mathcal{L}(CR_{x_\alpha})$. Thus $\mathcal{L}^C_{x_\alpha}$. Conversely, let $\mathcal{L}$ be an $L$-fuzzy ideal, $\mu \notin \mathcal{L}$ and $\mathcal{L}^C_{x_\alpha}$. Then for each $\lambda \in CR_{x_\alpha}$, $\lambda \in \mathcal{L}$. Since $\lambda \in \mathcal{L}$, $\mu \notin \mathcal{L}$, then $\mu \not\leq \lambda$ and so $x_\alpha \leq C.c.l(\mu)$.


**Definition 4.1:** An $L$–fuzzy mapping $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ is said to be:

(i) An $L$–fuzzy $C$–continuous if $f^{-1}(\eta) \in \tau'$ for each strong $Q$–compact $L$–fuzzy closed set $\eta$ in $L^Y$.

(ii) An $L$–fuzzy $C$–continuous at $L$–fuzzy point $x_\alpha \in M(L^X)$ if $f^{-1}(\lambda) \in R_{x_\alpha}$ for each $\lambda \in CR_f(x_\alpha)$.

**Theorem 4.2:** A mapping $f : (X, T_1) \rightarrow (Y, T_2)$ is $C$–continuous iff an $L$–fuzzy mapping $f : (L^X, \omega_L(T_1)) \rightarrow (L^Y, \omega_L(T_2))$ is $L$–fuzzy $C$–continuous.

**Proof:** Let $f : (L^X, T_1) \rightarrow (L^Y, T_2)$ be $C$–continuous and let $\mu \in L^Y$ be strong $Q$–compact $L$–fuzzy closed. Then by Theorem 3.2 in [6] and Theorem 2.9, we have $\mu_{w_\alpha} \subseteq Y$ is compact and closed in $(Y, T_2)$, $\forall \alpha \in M(L)$. Since $f^{-1}(\mu_{w_\alpha}) = (f^{-1}(\mu))_{w_\alpha}$, then $f^{-1}(\mu_{w_\alpha}) \in T_1$ for each $\alpha \in M(L)$ and so $f^{-1}(\mu) \in \omega_L(T_1) = (\omega_L(T_1))'$. Thus $f : (L^X, \omega_L(T_1)) \rightarrow (L^Y, \omega_L(T_2))$ is $L$–fuzzy $C$–continuous. Conversely, let $f : (L^X, \omega_L(T_1)) \rightarrow (L^Y, \omega_L(T_2))$ be $L$–fuzzy $C$–continuous and let $A \subseteq Y$ be compact and closed. Then, by Theorem 2.9, $1_\lambda \in L^Y$ is $Q_\alpha$–compact and $L$–fuzzy closed in $(L^Y, \omega_L(T_2))$. Since $1_{f^{-1}(A)} = f^{-1}(1_\lambda) \in \omega_L(T_1)$ so $f^{-1}(A) \in T_1$. Hence $f : (X, T_1) \rightarrow (Y, T_2)$ is $C$–continuous.

**Theorem 4.3:** Let $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ be an $L$–fuzzy mapping. Then the following are equivalent:

(i) $f$ is $L$–fuzzy $C$–continuous;

(ii) $f$ is $L$–fuzzy $C$–continuous at $x_\alpha$ for each $L$–fuzzy point $x_\alpha \in M(L^X)$;
(iii) For each \( \eta \in \Delta \) with \( \eta' \) is strong \( Q \)-compact, then \( f^{-1}(\eta) \in \tau \).

These statements are implied by

(iv) If \( \eta \in L^Y \) is strong \( Q \)-compact, then \( f^{-1}(\eta) \in \tau' \).

Moreover, if \( (L^Y, \Delta) \) is fully stratified \( LFT_2 \)-space, all the statements are equivalent.

\textbf{Proof} : (i) \( \implies \) (ii) Suppose that \( f \) is \( L \)-fuzzy \( C \)-continuous, \( x_\alpha \in M(L^X) \) and \( \lambda \in CR_f(x_\alpha) \), then \( f^{-1}(\lambda) \in \tau' \). Since \( f(x_\alpha) \not\leq \lambda \) is equivalent to \( x_\alpha \not\leq f^{-1}(\lambda) \), so \( f^{-1}(\lambda) \in R_{x_\alpha} \), and hence \( f \) is \( L \)-fuzzy \( C \)-continuous at \( x_\alpha \).

(ii) \( \implies \) (i) Let \( f \) be an \( L \)-fuzzy \( C \)-continuous at \( x_\alpha \) for each \( x_\alpha \in M(L^X) \). If \( f \) is not \( L \)-fuzzy \( C \)-continuous, then there is \( \text{C-closed L-fuzzy set } \eta \in L^Y \) with \( c\ell(f^{-1}(\eta)) \not\leq f^{-1}(\eta) \). Then there exists \( x_\alpha \in M(L^X) \) such that \( x_\alpha \not\leq c\ell(f^{-1}(\eta)) \) and \( x_\alpha \not\leq f^{-1}(\eta) \). Since \( x_\alpha \not\leq f^{-1}(\eta) \) implies that \( f(x_\alpha) \not\leq \eta \), so \( \eta \in CR_f(x_\alpha) \). But \( f^{-1}(\eta) \not\in R_{x_\alpha} \), a contradiction. Therefore, \( f \) must be \( L \)-fuzzy \( C \)-continuous.

(i) \( \iff \) (iii) Let \( \eta \in \Delta \) with \( \eta' \) is strong \( Q \)-compact. By (iv), we have \( f^{-1}(\eta') \in \tau' \). Thus, \( f^{-1}(\eta) = (f^{-1}(\eta'))' \in \tau \).

Now suppose that \( (L^Y, \Delta) \) is fully stratified \( LFT_2 \)-space.

(iii) \( \implies \) (iv) Let \( \eta \in L^Y \) be strong \( Q \)-compact set. Since \( (L^Y, \Delta) \) is fully stratified \( LFT_2 \)-space, then \( \eta \in \Delta' \) and so \( \eta' \in \Delta \). By (iii), \( f^{-1}(\eta') \in \tau \). Thus \( f^{-1}(\eta) = (f^{-1}(\eta'))' \in \tau' \).

By view of Theorems 4.2 and 4.3 the following example shows that \( LFT_2 \) is necessary when showing (i) implies (iii) in the above Theorem.

\textbf{Example 4.4}: Let \( X = \{1, 2, 3\} \), \( Y = R \), \( \tau = \omega_L(S) \), where \( S = \{X, \emptyset, \{3\}, \{2, 3\}\} \) and \( \Delta = \omega_L(T) \), where \( T \) be a topology on \( Y \) generated by \( \{(-\infty, -r) \cup (r, \infty) : r \in Y\} \). Then the mapping \( f : (X, S) \to (Y, T) \) defined by \( f(x) = x \) for each \( x \in X \) is \( C \)-continuous (See, Example 1 in [5]). Hence by Theorem 4.2, the mapping \( f : (L^X, \omega_L(S)) \to (L^Y, \omega_L(T)) \) is \( L \)-fuzzy \( C \)-continuous but does not satisfy statement (iii) in Theorem 4.3.

\textbf{Theorem 4.5}: Let \( f : (L^X, \tau) \to (L^Y, \Delta) \) be an surjective \( L \)-fuzzy mapping. Then the following conditions are equivalent:

(i) \( f \) is \( L \)-fuzzy \( C \)-continuous;
(ii) For each $\mu \in L^X$, $f(\text{cl}(\mu)) \leq C.\text{cl}(f(\mu))$.

(iii) For each $\eta \in L^Y$, $\text{cl}(f^{-1}(\eta)) \leq f^{-1}(C.\text{cl}(\eta))$.

(iv) For each $\eta \in L^Y$, $f^{-1}(C.\text{int}(\eta)) \leq \text{int}(f^{-1}(\eta))$.

(v) $f^{-1}(\rho)$ is $L$-fuzzy open in $L^X$, for each $L$-fuzzy C-open set $\rho$ in $L^Y$.

(vi) $f^{-1}(\lambda)$ is $L$-fuzzy closed in $L^X$, for each $L$-fuzzy C-closed set $\lambda$ in $L^Y$.

**Proof:**

(i) $\implies$ (ii) Let $\mu \in L^X$ and $x_\alpha \in M(L^X)$ with $x_\alpha \leq \text{cl}(\mu)$. Then $f(x_\alpha) \leq f(\text{cl}(\mu))$. Let $\lambda \in CR_f(x_\alpha)$. So by (i), $f^{-1}(\lambda) \in R_{x_\alpha}$. Since $x_\alpha \leq \text{cl}(\mu)$ and $f^{-1}(\lambda) \in R_{x_\alpha}$, then $\mu \not\leq f^{-1}(\lambda)$. Thus $f(\mu) \not\leq \lambda$ and $\lambda \in CR_f(x_\alpha)$ and so $f(x_\alpha) \leq C.\text{cl}(f(\mu))$. Hence $f(\text{cl}(\mu)) \leq C.\text{cl}(f(\mu))$.

(ii) $\implies$ (iii) Let $\eta \in L^Y$. Then $f^{-1}(\eta) \in L^X$. By (ii), we have $f(\text{cl}(f^{-1}(\eta))) \leq C.\text{cl}(f^{-1}(\eta)) \leq C.\text{cl}(\eta)$ and so $f(\text{cl}(f^{-1}(\eta))) \leq C.\text{cl}(\eta)$. Thus $f^{-1}(f(\text{cl}(f^{-1}(\eta)))) \leq f^{-1}(C.\text{cl}(\eta))$. Since $\text{cl}(f^{-1}(\eta)) \leq f^{-1}(f(\text{cl}(f^{-1}(\eta))))$, then $\text{cl}(f^{-1}(\eta)) \leq f^{-1}(C.\text{cl}(\eta))$.

(iii) $\implies$ (iv) Let $\eta \in L^Y$. By (iii), $f(\text{cl}(f^{-1}(\eta))) \leq f^{-1}(C.\text{cl}(\eta))$. Since $\text{cl}(f^{-1}(\eta)) = \text{cl}(\text{int}(f^{-1}(\eta)))'$ and $f^{-1}(C.\text{cl}(\eta)) = (f^{-1}(C.\text{int}(\eta)))'$, so $(\text{int}(f^{-1}(\eta)))' \leq (f^{-1}(C.\text{int}(\eta)))'$ and by the complement, $\text{int}(f^{-1}(\eta)) \geq f^{-1}(C.\text{int}(\eta))$.

(iv) $\implies$ (v) Let $\rho$ be C-open in $L^Y$. Then $f^{-1}(\rho) = f^{-1}(C.\text{int}(\rho))$ and by (iv), $f^{-1}(C.\text{int}(\rho)) \leq \text{int}(f^{-1}(\rho))$, so $f^{-1}(\rho) \leq \text{int}(f^{-1}(\rho))$. Thus $f^{-1}(\rho) \in \tau$.

(v) $\implies$ (vi) Let $\lambda$ be C-closed in $L^Y$. By (v), $f^{-1}(\lambda) \in \tau$. Then $(f^{-1}(\lambda))' = f^{-1}(\lambda') \in \tau$. So $f^{-1}(\lambda) \in \tau'$.

(vi) $\implies$ (i) Let $\eta$ be strongly Q-compact and closed set in $L^Y$. Then by Theorem 3.3 (ii), we have $\eta$ is C-closed set in $L^Y$. Hence by (vi), $f^{-1}(\eta) \in \tau'$. Hence $f$ is $L$-fuzzy C-continuous.

**Theorem 4.6:** Every $L$-fuzzy continuous mapping in the sense of Wang [12] is $L$-fuzzy C-continuous.

**Proof:** Straightforward.

By view of Theorems 4.2 and 4.6 the following example shows that not every $L$-fuzzy C-continuous mapping is $L$-fuzzy continuous.

**Example 4.7:** Let $R$ be the set of reals with the usual topology $T_U$ and define $f : L^X \to L^Y$.
Then $f$ is $C$-continuous but not continuous (See, Example 2 in [5]). Hence by Theorem 4.2, $f : (L^R, \omega_L(T_U)) \rightarrow (L^R, \omega_L(T_U))$ is $L$-fuzzy $C$-continuous but not $L$-fuzzy continuous.

In the following two Theorems we discuss the conditions which the $L$-fuzzy $C$-continuity is equivalent to the $L$-fuzzy continuity.

**Theorem 4.8:** A mapping $f : (L^X, \tau) \rightarrow (L^Y, \Delta_C)$ is $L$-fuzzy continuous iff $f$ is $L$-fuzzy $C$-continuous.

**Proof:** Since $\Delta'_C \subseteq \Delta'_C$, then necessity is evident. Now, suppose that $f$ is $L$-fuzzy $C$-continuous and $\eta \in \Delta'_C$. Then by Theorem 4.5 (iii) we have $f^{-1}(\eta) = f^{-1}(C, c\ell(\eta)) \supseteq c\ell(f^{-1}(\eta))$ and so $f^{-1}(\eta) \subseteq \tau'$.

**Theorem 4.9:** Let $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ be an $L$-fuzzy mapping and $(L^Y, \Delta)$ be strong $Q$-compact space. Then $f$ is $L$-fuzzy continuous iff $f$ is $L$-fuzzy $C$-continuous.

**Proof:** By Theorem 4.6 we need only to investigate the sufficiency. Let $\eta \in \Delta'$. Since $(L^Y, \Delta)$ is strong $Q$-compact then, by Theorem 2.8, $\eta$ is strong $Q$-compact and so $\eta$ is $L$-fuzzy $C$-closed set. By $L$-fuzzy $C$-continuity of $f$, we have $f^{-1}(\eta) \subseteq \tau'$. Hence $f$ is $L$-fuzzy continuous.

In [1] Chen and Wang have introduced and studied the concept of $L$-fuzzy $N$-continuous mapping by using nearly $N$-compactness due to Chen and Wang [1]. Here we redefine this concept by using strong nearly $Q$-compactness due to Nouh [9]. However, its detailed treatment is beyond the scope of this paper and will be dealt elsewhere.

**Definition 4.10:** An $L$-fuzzy mapping $f : (L^X, \tau) \rightarrow (L^Y, \Delta)$ is said to be:

(i) An $L$-fuzzy $N^*$-continuous if $f^{-1}(\eta) \subseteq \tau'$ for each strong nearly $Q$-compact $L$-fuzzy closed set $\eta$ in $L^Y$.

(ii) An $L$-fuzzy $N^*$-continuous at $L$-fuzzy point $x_\alpha \in M(L^X)$ if $f^{-1}(\lambda) \subseteq R_{x_\alpha}$ for each $\lambda \in N^* R_{f(x_\alpha)}$.

**Theorem 4.11:** Every $L$-fuzzy $N^*$-continuous mapping is $L$-fuzzy $N$-continuous in the sense of Chen and Wang [1].
Proof: Follows from the fact that every N-compact L-fuzzy set is strong $Q$-compact.

The converse of Theorem 4.11 is not true in general as can be seen from Example 1 in [15]. However, we have the following result.

**Theorem 4.12:** Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L-fuzzy mapping and $(L^Y, \Delta)$ be induced L-fts. Then $f$ is L-fuzzy $N^*$-continuous iff $f$ is L-fuzzy N-continuous.

**Proof:** Follows from Theorems 2.12 and 4.11.

**Theorem 4.13:** Every L-fuzzy $N^*$-continuous mapping is L-fuzzy C-continuous.

**Proof:** Follows from the fact that every strong $Q$-compact set is strong nearly $Q$-compact.

**Theorem 4.14:** Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L-fuzzy mapping and $(L^Y, \Delta)$ be $LFR_2$-space. Then $f$ is L-fuzzy $N^*$-continuous iff $f$ is L-fuzzy C-continuous.

**Proof:** Follows from Theorems 2.11 and 4.13.

**Remark 4.15:** For an L-fuzzy mapping $f : (L^X, \tau) \to (L^Y, \Delta)$, we obtain the following implications:

- L-fuzzy continuity $\Rightarrow$ L-fuzzy $N^*$-continuity $\Rightarrow$ L-fuzzy C-continuity.

The following counterexample shows that none of these implications are reversible.

**Counterexample 4.16:** Let $(L^X, \tau)$ and $(L^X, \Delta)$ be two L-fts’s, where $(L^X, \tau)$ is fully stratified $LFT_2$ and $(L^X, \Delta)$ is not $LFR_2$. Let $f : (L^X, \tau) \to (L^X, \Delta)$ be the identity mapping. Then:

(i) If $\Delta$ is strictly finer than $\tau$ and $(L^X, \tau)$ is $LFR_2$, then $f$ is L-fuzzy $N^*$-continuous but not L-fuzzy continuous.

(ii) If $\tau \neq \Delta$ and $(L^X, \tau)$ is not $LFR_2$, then $f$ is L-fuzzy C-continuous but not L-fuzzy $N^*$-continuous.

However, if $(L^Y, \Delta)$ is strong $Q$-compact (resp. $LFR_2$) space, then Theorem 4.9 (resp. Theorem 4.14) implies that the concepts of L-fuzzy continuity (resp. $N^*$-continuity) and L-fuzzy C-continuity are equivalent.

**Definition 4.17** [1]: Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L-fuzzy mapping and $A \subseteq X$. Define an L-fuzzy mapping $f|_A : L^A \to L^Y$ as follows:

$$(f|_A)(\mu) = f(\mu) \land 1_A = f(\mu^*), \text{ for each } \mu \in L^A$$

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And call $f|_A$ the restriction of $f$ on $A$. Where $\mu^*$ denote the extension of $\mu$ in $L^X$, that is for each $x \in X$,

$$
\mu^*(x) = \begin{cases} 
\mu(x) & : x \in A \\
0 & : x \notin A
\end{cases}
$$

**Theorem 4.18:** If $f : (L^X, \tau) \to (L^Y, \Delta)$ is an L-fuzzy C-continuous and $A \subseteq X$, then $f|_A : (L^A, \tau_A) \to (L^Y, \Delta)$ is an L-fuzzy C-continuous mapping.

**Proof:** Let $\mu \in L^Y$ be C-closed. Since $f$ is L-fuzzy C-continuous, so $f^{-1}(\mu) \in \tau'$ and $(f|_A)^{-1}(\mu) = f^{-1}(\mu) \land 1_A \in \tau_A^*$. Hence $f|_A : (L^A, \tau_A) \to (L^Y, \Delta)$ is L-fuzzy C-continuous.

The composition of two L-fuzzy C-continuous mappings need not be L-fuzzy C-continuous (See, Example 3.14 in [3]). However, we have the following result.

**Theorem 4.19:** If $f : (L^X, \tau_1) \to (L^Y, \tau_2)$ is L-fuzzy continuous mapping and $g : (L^Y, \tau_2) \to (L^Z, \tau_3)$ is L-fuzzy C-continuous mapping, then $g \circ f : (L^X, \tau_1) \to (L^Z, \tau_3)$ is L-fuzzy C-continuous.

**Proof:** Obvious.

**Theorem 4.20:** If $(L^X, \tau), (L^Y, \Delta)$ are L-fits’s and $1_X = 1_A \lor 1_B$, where $1_A$ and $1_B$ are closed of $L^X$ and $f : (L^X, \tau) \to (L^Y, \Delta)$ is L-fuzzy mapping such that $f|_A$ and $f|_B$ are L-fuzzy C-continuous, then $f$ is L-fuzzy C-continuous.

**Proof:** Let $1_A, 1_B \in \tau'$. Let $\mu \in L^Y$ be C-closed. Then $(f|_A)^{-1}(\mu) \lor (f|_B)^{-1}(\mu) = (f^{-1}(\mu) \land 1_A) \lor (f^{-1}(\mu) \lor 1_B) = f^{-1}(\mu) \lor (1_A \lor 1_B) = f^{-1}(\mu) \lor 1_X = f^{-1}(\mu)$. Hence $f^{-1}(\mu) = (f|_A)^{-1}(\mu) \lor (f|_B)^{-1}(\mu) \in \tau'$. So $f : (L^X, \tau) \to (L^Y, \Delta)$ is L-fuzzy C-continuous.

5. **More Characterizations of L-fuzzy C-continuous mappings**

**Theorem 5.1:** Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be L-fuzzy C-continuous and $(L^Y, \Delta)$ be a fully stratified $LFT_2$-space. If $f(1_X)$ is contained in some strong $Q$-compact set of $L^Y$, then $f$ is L-fuzzy continuous.

**Proof:** Let $\mu \in L^Y$ be a strong $Q$-compact containing $f(1_X)$ and let $\rho \in \Delta'$. Since $\mu$ is strong $Q$-compact in $(L^Y, \Delta)$ which is fully stratified $LFT_2$-space, so $\mu \in \Delta'$. Thus $\mu \land \rho \in \Delta'$. Hence by Theorem 2.8, $\mu \land \rho \in L^Y$ is strong $Q$-compact. Thus
\[ \mu \land \rho \in L^Y \text{ is } C\text{-closed. Since } f \text{ is } L\text{-fuzzy } C\text{-continuous, then } f^{-1}(\mu \land \rho) \in \tau'. \] But,
\[ f^{-1}(\mu \land \rho) = f^{-1}(\mu) \land f^{-1}(\rho) = f^{-1}(\rho) \land 1_X = f^{-1}(\rho). \]
So \( f^{-1}(\rho) \in \tau' \). Hence \( f \) is \( L\text{-fuzzy continuous.} \)

**Theorem 5.2:** Let \((L^X, \tau)\) be an \( L\)-fts and \((L^Y, \Delta)\) be fully stratified \( LFT_2\)-space. If \( f: (L^X, \tau) \to (L^Y, \Delta)\) is a bijective and \( L\)-fuzzy continuous, then \( f^{-1}: (L^Y, \Delta) \to (L^X, \tau)\) is \( L\)-fuzzy \( C\)-continuous.

**Proof:** Let \( \eta \in L^X \) be strong \( Q\)-compact. Since \( f \) is \( L\)-fuzzy continuous, then by Theorem 2.10, \( f(\eta) \) is strong \( Q\)-compact. Since \((L^Y, \Delta)\) is fully stratified \( LFT_2\)-space, then \( f(\eta) \in \Delta' \). Hence by Theorem 4.3, \( f^{-1} \) is \( L\)-fuzzy \( C\)-continuous.

**Corollary 5.3:** Let \((L^X, \tau)\) be a strong \( Q\)-compact space and \((L^Y, \Delta)\) be fully stratified \( LFT_2\)-space. If \( f: (L^X, \tau) \to (L^Y, \Delta)\) is a bijective and \( L\)-fuzzy continuous, then \( f \) is an \( L\)-fuzzy homeomorphism.

**Proof:** Follows from Theorems 5.1 and 5.2.

**Theorem 5.4:** Let \( f: (L^X, \tau) \to (L^Y, \Delta)\) be an surjective \( L\)-fuzzy mapping. Then the following conditions are equivalent:

(i) \( f \) is \( L\)-fuzzy \( C\)-continuous;

(ii) For each \( x_\alpha \in M(L^X) \) and each \( L\)-fuzzy net \( S \) in \( L^X \), \( f(S) \subseteq f(x_\alpha) \) if \( S \to x_\alpha \).

(iii) \( f(\lim(S)) \leq C.\lim(f(S)) \), for each \( L\)-fuzzy net \( S \) in \( L^X \).

**Proof:** (i) \( \implies \) (ii) Let \( x_\alpha \in M(L^X) \) and \( S = \{x_\alpha^n : n \in D\} \) be an \( L\)-fuzzy net in \( L^X \) which converges to \( x_\alpha \). Let \( \eta \in CR(f(x_\alpha)) \), by (i) \( f^{-1}(\eta) \in R_{x_\alpha} \). Since \( S \to x_\alpha \), then \((\exists n \in D)(\forall m \in D, m \geq n)(S(m) \not\leq f^{-1}(\eta)) \). Then \( f(S(m)) \not\leq ff^{-1}(\eta) = \eta \). Thus \( f(S(m)) \not\leq \eta \). Hence \( f(S) \subseteq f(x_\alpha) \).

(ii) \( \implies \) (iii) Let \( x_\alpha \leq \lim(S) \), then \( f(x_\alpha) \leq f(f(S)) \), by (ii) \( f(x_\alpha) \leq C.\lim(f(S)) \). Thus \( f(\lim(S)) \leq C.\lim(f(S)) \).

(iii) \( \implies \) (i) Let \( \eta \in L^Y \) be \( L\)-fuzzy \( C\)-closed and \( x_\alpha \in M(L^X) \) with \( x_\alpha \leq cf^{-1}(\eta) \), by Theorem 2.15, there exists an \( L\)-fuzzy net \( S \) in \( f^{-1}(\eta) \) which converges to \( x_\alpha \). Thus \( x_\alpha \leq \lim(S) \) and so \( f(x_\alpha) \leq f(\lim(S)) \). By (iii), \( f(x_\alpha) \leq f(\lim(S)) \leq C.\lim f(S) \) and so, \( f(S) \subseteq f(x_\alpha) \). Since \( S \) is \( L\)-fuzzy net in \( f^{-1}(\eta) \), then for each \( n \in D \), \( S(n) \leq f^{-1}(\eta) \).
and so $f(S(n)) \leq ff^{-1}(\eta) \leq \eta$. Hence $f(S(n)) \leq \eta$ for each $n \in D$. Thus $f(S)$ is L-fuzzy net in $\eta$. So we have $f(S) \subseteq f(x_\alpha)$ and $f(S)$ is L-fuzzy net in $\eta$ so by Proposition 3.10 (iii), $f(x_\alpha) \leq C.\ell(\eta)$. But since $\eta$ is C-closed, so $\eta = C.\ell(\eta)$. Thus $f(x_\alpha) \leq \eta$. Hence $x_\alpha \leq f^{-1}(\eta)$. So $C.\ell(f^{-1}(\eta)) \leq f^{-1}(\eta)$. Thus $f^{-1}(\eta) \in \tau'$. Then $f$ is L-fuzzy C-continuous.

**Theorem 5.5:** Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L-fuzzy mapping. Then the following conditions are equivalent:

(i) $f$ is L-fuzzy C-continuous;

(ii) For each $x_\alpha \in M(L^X)$ and each L-fuzzy ideal $\mathcal{L}$ in $L^X$ which converges to $x_\alpha$ in $L^X$, $f^*(\mathcal{L})$ C-converges to $f(x_\alpha)$, where $f^*(\mathcal{L}) = \{ \eta \in L^Y : \exists \mu \in \mathcal{L} \text{ such that for any } x_\alpha \in M(L^X), f(x_\alpha) \not\leq \eta \text{ if } x_\alpha \not\leq \mu \}$ is an L-fuzzy ideal in $L^Y$.

(iii) $f(lim(\mathcal{L})) \leq C.lim(f^*(\mathcal{L}))$, for each L-fuzzy ideal $\mathcal{L}$ in $L^X$.

**Proof:** The proof is similar to that of Theorem 5.4.

Similarly, we have the following result.

**Theorem 5.6:** Let $f : (L^X, \tau) \to (L^Y, \Delta)$ be an L-fuzzy mapping. Then the following conditions are equivalent:

(i) $f$ is L-fuzzy C-continuous;

(ii) For each $x_\alpha \in M(L^X)$ and each L-fuzzy ideal $\mathcal{L}$ in $L^X$ which converges to $x_\alpha$ in $L^X$, then $(f(\mathcal{L}))'$ C-converges to $f(x_\alpha)$.

(iii) $f(lim(\mathcal{L})) \leq C.lim((f(\mathcal{L}))')$, for each L-fuzzy ideal $\mathcal{L}$ in $L^X$.

**References**


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