On the capacitance of crystalline silicon solar cells in steady state

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Abstract: In this work, an analytical approach is presented for modeling the capacitance of crystalline silicon solar cells. Based on a one-dimensional modeling of the cell, the excess minority carrier density, the photovoltage, and the capacitance are calculated. The motivation of this work are two-fold: to show base doping density and illumination effect on the capacitance of silicon solar cells, and to propose a determination technique for both dark capacitance and base doping density from C-V characteristics.

Key words: Solar cell, doping, capacitance, illumination

1. Introduction
The crystalline silicon solar cell capacitance, also called junction capacitance, is a useful parameter to get indirect information on the intrinsic phenomena inside semiconductor devices and their fundamental properties [1,2].

The capacitive effect also plays an important role when fast transient occurs: for example, with measurements made under flashed irradiance [3,4] and switching circuit in power inverters [5,6].

Given the important role played by solar cell capacitance, it is of great interest to know which parameters could influence it. It is well know that recombination in solar cells is controlled in major part by base doping density; that is, the capacitance should also be controlled by this doping density. Illumination level also plays an important role given that short circuit current is directly proportional to it when the solar cell is not under light concentration.

The aim of this paper is to show how base doping and illumination level can affect the capacitance of the crystalline silicon solar cell. From the continuity equation, we calculate the excess minority carrier density in the base with the associate photovoltage across the junction; the capacitance is then derived and its dependence on base doping and illumination level is discussed. Based on the C-V characteristics, the dark capacitance and the base doping level are determined.

2. Theoretical background
A schematic diagram of the crystalline silicon solar cell showing the different coordinates is given in Figure 1.

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When illuminated, three major phenomena occur inside the solar cell: carrier generation, recombination, and drift/diffusion. In steady state, if we assume a quasineutral base, these phenomena can be described by the following equation:

$$\frac{\partial^2 \delta(x)}{\partial x^2} - \frac{\delta(x)}{L^2} = -\frac{G(x)}{D}.$$  \hspace{0.5cm} (1)

$\delta$ is the excess minority carrier density, $L$ is their diffusion length, and $D$ is their diffusion coefficient.

The diffusion coefficient can be written as [7]:

$$\frac{\partial^2 \delta(x)}{\partial x^2} - \frac{\delta(x)}{L^2} = -\frac{G(x)}{D}.$$ \hspace{0.5cm} (2)

with $N_b$ being the base doping density and $V_t$ the thermal voltage: $V_t = kT/q$. $k$ is the Boltzmann constant, $T$ the temperature, and $q$ the electronic charge.

The associated excess minority carrier lifetime is given by [7]:

$$\frac{\partial^2 \delta(x)}{\partial x^2} - \frac{\delta(x)}{L^2} = -\frac{G(x)}{D} \hspace{0.5cm} \text{(in } \mu\text{s}).$$ \hspace{0.5cm} (3)

The diffusion length can then be deduced as:

$$\frac{\partial^2 \delta(x)}{\partial x^2} - \frac{\delta(x)}{L^2} = -\frac{G(x)}{D}.$$ \hspace{0.5cm} (4)

Carrier generation rate $G(x)$ at depth $x$ in the base is written in the following form [8,9]:

$$G(x) = \sum_{i=1}^{3} a_i \cdot e^{-b_i \cdot x}.$$ \hspace{0.5cm} (5)

$n$ indicates illumination level given to be the ratio between incident power and AM1.5 reference power; coefficients $a_m$ and $b_m$ are tabulated values obtained from solar irradiance and the dependence of the absorption coefficient on the illumination wavelength [10].

The solution of Eq. (1) is:

$$\delta(x) = C_1 \cdot \exp \left( \frac{x}{L} \right) + C_2 \cdot \exp \left( -\frac{x}{L} \right) + \sum_{m=1}^{3} K_m \cdot \exp \left( -b_m \cdot x \right).$$ \hspace{0.5cm} (6)

Coefficients $C_1$ and $C_2$ can be evaluated if we consider the two following boundary conditions:
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- at the junction \((x = 0)\):
  \[
  \frac{\partial \delta(x)}{\partial x} \bigg|_{x=0} = \frac{S_f}{D} \cdot \delta(0), \tag{7}
  \]

- at the backside \((x = H)\):
  \[
  \frac{\partial \delta(x)}{\partial x} \bigg|_{x=0} = \frac{S_f}{D} \cdot \delta(0). \tag{8}
  \]

\(S_b\) is the back surface recombination velocity and \(S_f\) is the junction dynamic velocity; the more carrier flow through the junction, the more \(S_f\) is. That is, \(S_f\) traduces the carrier flow through the junction and is then directly related to the external load conditions \([11]\). Low \(S_f\) values are related to the solar cell operating near the open circuit contrary to high \(S_f\) values, which are related to near short circuit operating conditions.

Since the excess minority carrier density is known, from the Boltzmann relation we can derive the photovoltage across the junction as:

\[
V_{ph} = V_T \cdot \ln \left( \frac{N_B \cdot \delta(x = 0)}{n_i^2} + 1 \right). \tag{9}
\]

\(V_T\) is the thermal voltage and \(n_i\) is the intrinsic carrier concentration.

Now let us define the solar cell capacitance \(C\) as:

\[
C = \frac{dQ}{dV}. \tag{10}
\]

The capacitance \(C\) results from charge variation due mainly to diffusion processes in the solar cell \([12–14]\). When the solar cell is illuminated, there is a photogeneration process associated to recombination and diffusion processes. The diffusion process induces a charge variation into the solar cell accompanied by a charge in the voltage across the cell. It is this charge variation that induces mainly the capacitance of the solar cell under illumination.

The total amount of charge is:

\[
Q = q \cdot \delta(x = 0), \tag{11}
\]

where \(q\) is the electronic charge.

Substituting Eq. (11) into Eq. (10), the capacitance becomes:

\[
C = q \cdot \frac{d\delta(x = 0)}{dV}. \tag{12}
\]

Eq. 12 can be rewritten as:

\[
C = q \cdot \frac{d\delta(x = 0)}{dS_f} \cdot \frac{1}{\frac{dV}{dS_f}}. \tag{13}
\]

Taking into account Eq. (9) and manipulating Eq. (13), we obtain:

\[
C = q \cdot \frac{n_i^2}{V_T} + q \frac{\delta(x = 0)}{V_T} = C_0 + C_d, \tag{14}
\]

with \(C_0 = q \frac{n_i^2}{V_T}\) and \(C_d = q \frac{\delta(x=0)}{V_T}\).

The first term in Eq. (14) is the dark capacitance \(C_0\) of the cell while the second is the diffusion-related capacitance \(C_d\). It is clear that under illumination only diffusion capacitance predominates.
3. Results and discussion

Simulation was performed for various illumination levels, various base doping levels, and various external load conditions.

We present in Figure 2 a plot of the capacitance versus junction dynamic velocity for various base doping density.

This figure shows that the solar cell capacitance decreases with increasing junction dynamic velocity $S_f$. This behavior can be explained by the fact that when $S_f$ increases, carrier flow through the junction increases so that there is less and less free carrier in the base. This lack of permanent excess minority carrier near the junction leads to a decrease in the capacitance.

For low $S_f$ values, the carrier does not cross the junction; it is stored in the base so that we can obtain higher capacitance values, as noted in Figure 2.

That is, when the solar cell is operating near the open circuit, the capacitive effect will be more important so that this could impact negatively on the whole efficiency of the system if the solar cell is connected to a switching circuit or used for measurement purposes.

When the base doping $N_b$ increases, the capacitance decreases and this decrease is more marked for lower $S_f$ values. Effectively, when the base doping increases, impurity concentration increases also, leading to more and more recombination of excess minority carrier and then a decrease of the capacitance.

We now present the capacitance profile versus junction dynamic velocity for various illumination levels in Figure 3.

![Figure 2](image1.png)  
**Figure 2.** Variation of the capacitance as a function of junction dynamic velocity. The values of base doping are indicated in the figure.

![Figure 3](image2.png)  
**Figure 3.** Variation of the capacitance as a function of junction dynamic velocity. The values of illumination level are indicated in the figure.

We note that solar cell capacitance increases as illumination level increases.

There is a balance between illumination level and base doping level given that the increasing illumination level acts as a passivating recombination center contrary to base doping, which increases recombination centers. Care must be taken on increasing the illumination level because there is a limit for the lower injection condition. Beyond a certain illumination level, our assumption of the base quasineutrality will not be true so that we must also take into account the electric field in the base.
If we turn back to Eq. (14), we can write:

$$\frac{C}{\frac{q^2 n_i^2}{V_T}} = 1 + \frac{N_b \cdot \delta(0)}{n_i^2}$$

so that

$$\frac{C}{C_0} = 1 + \frac{N_b \cdot \delta(0)}{n_i^2}.$$  \hspace{1cm} (15)

From Eq. (9) we have:

$$\exp\left(\frac{V_{ph}}{V_T}\right) = 1 + \frac{N_b \cdot \delta(0)}{n_i^2}$$

giving

$$\frac{C}{C_0} = \exp\left(\frac{V_{ph}}{V_T}\right).$$  \hspace{1cm} (16)

This equation can then be written as:

$$\ln(C) - \ln(C_0) = \frac{V_{ph}}{V_T}.$$  \hspace{1cm} (17)

The capacitance (logarithmic scale) as a function of photovoltage is then a straight line as shown in Figure 4.

The capacitance increases as the photovoltage increases effectively because a photovoltage increase is related to more and more stored charge in the base, leading to an increase of the capacitance.

From Eq. (17), we can see that the slope of the straight line is $1/V_T$ and it intercepts the capacitance axis at a value corresponding to the logarithm of the dark capacitance Log$C_0$. For a given cell and from its $C_0$ value, we can deduce the corresponding base doping level by help of the following expression derived from Eq. (14):

$$N_b = \frac{q}{V_T} \cdot \frac{n_i^2}{C_0}.$$  \hspace{1cm} (18)

Figure 5 illustrates the graphical determination of the dark capacitance from the C-V characteristic; for the given example, we have:

$$\log(C_0) \approx -6.75$$

$$C_0 \approx 10^{-6.75} F.$$
4. Conclusion
The capacitance of a crystalline silicon solar cell was investigated; from a one-dimensional model, we pointed out the effects of base doping, illumination level, and junction dynamic velocity (related to operating point) on the capacitance. Based on the C-V characteristics, a graphical method has been proposed for the determination of both dark capacitance $C_0$ and base doping density $N_b$.

References