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Self-Tuning and Conventional Control of an Industrial Scale Packed Distillation Column

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Abstract: The performance of self-tuning and a variety of conventional control strategies are examined when applied to the overhead product composition control of a packed distillation column. These control strategies are tested under various sets of conditions on a packed distillation column situated B.P. Chemicals, South Wales, U.K. Conventional PID control action is employed throughout-being considered the most likely type of control action for this application. The controller parameters were estimated using three different closed loop response tuning criteria for discrete controllers, viz. those due to Yuwana and Seborg (YS), Jutan and Rodriguez (YS-JR) and Wardle and Heathcock (YS-WH). An additional criterion (YS-JR-decrease gain) is proposed in the present work and is found to give better control than YS-JR method in a number of cases. The best conventional PID action is compared with self-tuning PID control. The success of the various control actions are estimated using an integral square of the error (ISE) criterion and it is shown that self-tuning PID control provides better control than conventional PID action for the cases studied.

Key Words: Self-Tuning Control, Conventional Control, Packed Distillation Column

Endüstriyel Ölçekli Bir Dolgulu Distilasyon Kolununun Kendinden Ayarlamalı ve Geleneksel Kontrolu

Özet: Kendinden ayarlamalı ve geleneksel kontrol çeşitlerinin performansı, bir dolgulu distilasyon kolonunun üst ürün derişimi kontrolü için yapılan uygulamalar ile araştırıldı. Bu kontrol stratejileri, B.P. Chemicals, Güney Wales, İngiltere'de halen çalışmakta bulunan dolgulu distilasyon kolonu üzerinde, farklı çalışma koşulları için denendi. Bu uygulamada, en çok kullanılan tip olan, geleneksel PID kontrol (sabit parametreli) kullanıldı. Geleneksel kontrol parametreleri, Yuwana-Seborg (YS), Jutan-Rodriguez (YS-JR) ve Wardle-Heathcock (YS-WH) yöntemlerine göre hesaplandı. Bu çalışmada, bir değiştirilmiş metot (YS-JR-azalan kazanç) daha önerildi ve bu yöntemin bazı durumlarda YS-JR-metodundan daha iyi kontrol ettiği bulundu. En iyi geleneksel PID sonucu, Kendinden- ayarlamalı PID kontrol sonucu ile karşılaştırıldı. Farklı kontrol yöntemlerinin başarısı, hata karelerinin integrali (ISE) yöntemi ile hesaplandı ve bu makalede ele alınan problemler için kendinden ayarlamalı PID kontrolun geleneksel PID kontrolden daha iyi olduğu gösterildi.

Anahtar Sözcükler : Kendinden Ayarlamalı Kontrol, Geleneksel Kontrol, Dolgulu Distilasyon Kolonu

Self-Tuning PID Controller

Conventional controller design methods produce constant coefficient algorithms based upon an assumed linear time-invariant system. The basis of a self-tuning system is an algorithm that will automatically change its parameters to meet a particular requirement or situation. This is achieved by the addition of a mechanism which monitors the system and adjusts the cofficients of the corresponding controller to maintain a required performance.

Bearing in mind the considerable use of PI and PID controllers in the process industries, it is not surprising that three-term control is employed in association with pole placement techniques.

Due to the low order of the PID control law, assumptions have to be made concerning the order of the controlled process to be sure that the self-tuning algorithm corresponds to the applied PID framework (Wellstead and Zarrop (1991)).

Although the application of fixed parameter PID controllers to packed distillation columns has received some attention (Lee(1976)); Molander and Breitholtz (1987), there do not appear to have been any studies concerning the application of self-tuning controllers to this type of epuipment. Wittenmark and Aström (1980) observed that a self-tuning PID (STPID) controller acts simply as a well-tuned PID controller. This means that it should be possible to apply the self-tuner to the same processes as can the conventional PID controller, and that a pole-placement based STPID algorithm is likely to produce a robust, low order controller.

In order to convert the velocity form of the PID algorithm into a self-tuning equivalent, consider the discrete time PID control algorithm (Wellstead and Zarrop (1991)):

$$u(t) = \frac{S}{R}[r(t)-y(t)] \qquad (1)$$

where r(t) represents the set point, and

$$S = s_0+s_2z^{-1}+s_2z^{-2} \qquad (2)$$

and

$$R = \left(1-z^{-1}\right) \qquad (3)$$

Hence

$$u(t) = \frac{\left[r(t)-y(t)\right]\left[s_0+s_1z^{-1}+s_2z^{-2}\right]}{\left[1-z^{-1}\right]} \qquad (4)$$

It is assumed that the process can be adequately repsented by the CARMA(Controlled Auto Regressive Moving Average) model (Clarke and Gawthrop (1979)), viz:

$$y(t) = \frac{B}{A}u(t-1) + \frac{C}{A}e(t) \qquad (5)$$

A,B and C are polynomials in the backward shift operator z^{-1} and k is the system time delay associated with the control input. A and C are monic and A and B represent the poles and zeros respectively of the discrete time system. C contains the zeros of the process noise and e(t) is an uncorrelated random sequence.

Substituting the control equation into this CARMA process model yields the following closed-loop equation:

$$y(t) = \frac{z^{-1}BS}{AR+z^{-1}BS}r(t) + \frac{RC}{AR+z^{-1}BS}e(t) \qquad (6)$$

The properties of this closed-loop can be vaired by placing the poles of the characteristic eqution (i. e. the denominator of equation (6)) utilizing a 'tailoring polynomial' T, where the poles of T are chosen by the system designer. Thus the characteristic equation is:

$$AR+z^{-1}BS = T \qquad (7)$$

The coefficients of the A and B polynomials are estimated from the Bierman UDU^T algorithm (Bierman (1977)) and the coefficients of the T polynomial are defined by user. Then the parameters of the S polynomial (s_0, s_1, s_2) can be determined by solving the set of simultaneous equations obtained from equation (7) that constitute the characteristic equation. The degrees of the polynomials in the characteristic equation are:

$$n_a+n_r = n_b+n_s+1 = n_t \qquad (8)$$

where the degrees of the S and R polynomials must be two (n_s=2) and one (n_r=1) respectively because of the polynomial representation of velocity form of the PID algorithm (Hapoğlu (1993)). Then n_a must be equal to n_b+2 and n_t must be equal to $n_b+3(=n_a+1)$. If all the coefficients of the T polynomial are user-defined, then we must select a second order A polynomial (n_a=2 and thus n_b=0 and n_t=3) to make sure that a unique set of PID controller coefficients can be obtained from the design. If the estimated model is reduced to first order, then the new form of the algorithm is called self-tuning PI(D). When the order of the estimated model is more than two (n_a>2), the user is not able to define the coefficients of T polynomial uniquely, i.e. to obtain a single set of PID controller coefficients. For instance, if the order of the A polynomial is three, i.e. n_a=3, then n_b=1 and n_t=4, and two different equations will be obtained from the STPID procedure for the determination of s_2, viz:

$$s_2 = \frac{t_4-a_3}{b_1} \qquad (9)$$

and

$$s_2 = \frac{t_3b_0^2-a_3b_0^2-a_2b_0^2-t_2b_1b_0+b_0a_2b_1}{b_0^3} + \frac{a_1b_0b_1+t_1b_1^2-a_1b_1^2+b_1^2}{b_0^3} \qquad (10)$$

As a result the coefficients of the T polynomial must be selected according to:

$$t_4 = \frac{a_3b_0^3+b_1t_3b_0^2-a_3b_1b_0^2-a_2b_1b_0^2-b_0t_2b_1^2}{b_0^3} + \frac{a_2b_0b_1^2+a_1b_0b_1^2+t_1b_1^3-a_1b_1^3+b_1^3}{b_0^3} \qquad (11)$$

where coefficients t_1, t_2, t_3 can be defined by the user. A similar argument applies with higher order polynomials. Thus, if the poles of the characteristic equation are to be easily placed, all the coefficients of the T polynomial should be user defined and hence we must choose a third order T polynomial. Therefore, we must use a system transfer function of the form:

$$y(t) = \frac{b_0 z^{-1}}{1+a_1 z^{-1}+a_2 z^{-2}} u(t) \qquad (12)$$

The closed loop setpoint following relationship is obtained by combining the system model equations (12) and the controller equation (4), i.e:

$$y(t) = \frac{b_0 z^{-1}(s_0+s_1+s_2)}{(1-z^{-1})(1+a_1 z^{-2}+a_2 z^{-1})+b_0 z^{-1}(s_0 s_1 z^{-1}+s_2 z^{-2})} r(t) \qquad (13)$$

The equivalent chosen closed loop T polynomial is of the form:

$$T = 1+t_1 z^{-1}+t_2 z^{-2}+t_3 z^{-3} \qquad (14)$$

The controller coefficients can be found by equating the real denominator of the closed loop equation (13) with equation (14) thus:

$$(1-z^{-1})(1+a_1 z^{-1}+a_2 z^{-2})+b_0 z^{-1}(s_0+s_1 z^{-1}+s_2 z^{-2}) = 1+t_1 z^{-1} t_2 z^{-2}+t_3 z^{-3} \qquad (15)$$

By comparing coefficients of powers of z^{-1}, we obtain:

$$s_0 = \frac{(t_1-a_1+1)}{b_0} \qquad (16)$$

$$s_1 = \frac{(t_2-a_2+a_1)}{b_0} \qquad (17)$$

and

$$s_2 = \frac{(t_3+a_2)}{b_0} \qquad (18)$$

The necessary increment in the control signal can now be obtained from (Jacquot (1981)):

$$\Delta u = s_0 \in(t)+s_1 \in(t-1)+s_2 \in(t-2) \qquad (19)$$

In the present work, the steps in the operation of the self-tuning PID algorithm may be given as:

(a) Apply a pseudo random binary sequence (prbs) to the system as a forcing function and attain the plant output

(b) Estimate A and B from the CARMA model using the Bierman U-D uptade algorithm

(c) Calculate s_1, s_2 and s_3 from equations (16), (17) and (18).

(d) Use equation (19) to obtain the incremental control signal.

(e) Output the updated control signal to the pro cess.

(f) Return to (a).

Conventional Controller

Conventional three term controllers employ proportinal, integral and derivative control actions. A new method for tuning controllers was proposed by Yuwana and Seborg (denoted by YS) in 1982 and later modified by Jutan and Rodriguez (1984) (denoted by YS-JR) and Wardle Heathock (1992) (denoted by YS-WH). An additional criterion (the YS-JR-decrease gain method) has been proposed in the present work and is found to give better control than YS-JR method in terms of the integral square of the error in a number of cases. Yuwana and Seborg (1982) considered the closed-loop response of a typical system under proportional control with a proportional gain of K (Figure 1). The process transfer function is assumed to be unknown. However, its functional form is represented by a first-order plus dead time model, i.e:

$$G_m(S) = \frac{K_m e^{-d_m S}}{\tau_m S+1} \qquad (20)$$

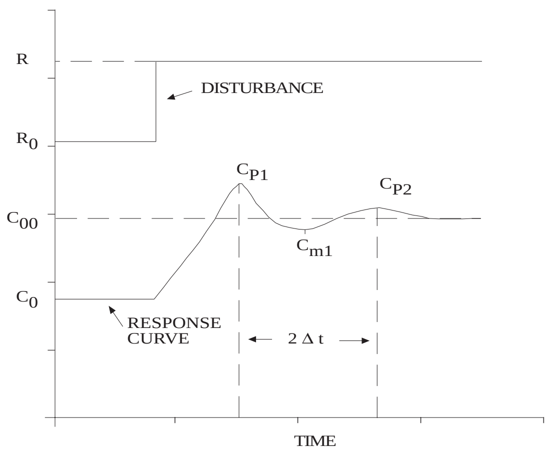

Figure 1. Response of a typical system to a step change in set point of magnitude A - Yuwana and Seborg method.

where K_m is the model gain, τ_m is the model time constant and d_m is the model dead time. These parameters are calculated using the symbols given in Figure 1 according to:

$$K_m = \frac{|C_\infty - C_0|}{K\left[|R-R_0|\ |C_\infty - C_0|\right]} \tag{21}$$

$$C_\infty = \frac{C_{P_2}C_{P_1} - C_{m_1}^2}{C_{P_1} + C_{P_2} - 2C_{m_1}} \tag{22}$$

$$\alpha_1 = \frac{C_\infty - 2C_{m_1}}{C_{P_1} - C_\infty} \tag{23}$$

$$\zeta = -\frac{\ln\alpha_1}{\left[\pi^2 + (\ln\alpha_1)^2\right]^{0.5}} \tag{24}$$

$$K_b = KK_m \tag{25}$$

$$\beta_1 = \zeta(K_b+1)^{0.5} + \left[\zeta^2(K_b+1)+K_b\right]^{0.5} \tag{26}$$

$$\beta_2 = \left[(1-\zeta^2)(K_b+1)\right]^{0.5} \tag{27}$$

$$\tau_m = \frac{\Delta t\beta_1\beta_2}{\pi} \tag{28}$$

$$d_m = \frac{2\Delta t\beta_2}{\pi\beta_1} \tag{29}$$

In addition to the Yuwana and Seborg parameters (K_m, τ_m, d_m), the Integral Time Absolute Error (ITAE) tuning relationship (Miller et al (1967)) was applied to the process model equation (20) to provide optimal tuning parameters for a PID controller of the form:

$$G_c = K_c\left(1 + \frac{1}{\tau_1 s} + \tau_D s\right) \tag{30}$$

where K_c is calculated from the ITAE tuning formulae as:

$$K_c = \frac{1.36}{K_m}\left(\frac{d_m}{\tau_m}\right)^{-0.947} \tag{31}$$

Substituting equation (21) in equation (31):

$$K_c = K\,\frac{1.36\left(|R-R_0| - |C_\infty - C_0|\right)}{|C_\infty - C_0|}\left(\frac{d_m}{\tau_m}\right)^{-0.947} = KK_a \tag{32}$$

where

$$K_a = \frac{1.36\left(|R-R_0| - |C_\infty - C_0|\right)}{|C_\infty - C_0|}\left(\frac{d_m}{\tau_m}\right)^{-0.947} \tag{33}$$

STPID and Conventional Control of a Packed Distillation Column

In the present work, single variable STPID and Conventional control algorithms are applied to a column containing a 15.6 m high packed section using 30 mm and 45 mm Intalox saddles (currently operating at the BP Chemicals site at Baglan Bay in South Wales, U.K.) (see Table 1 and 2). This column is distiling benzene-toluene mixture.

Table 1. Packing and column data for the 15.6 m column at BP Chemicals, Baglan Bay, South Wales, U.K.

		Rectifying Section	Stripping Section
Packing Type		Intalox Saddles	Intalox Saddles
Packing size, mm		30	45
Contoct Surface S_B, m^2/m^3		175	148
Free space %		96.5	97.1
Packing Factor F_p		41	24
Wetting rate correlation estimated from		Coulson et al (1978)	Coulson et al (1978)
Flooding rate correlation estimated from		British Petroleum and Brown and Von Rosenberg (1963)	British Petroleum and Brown and Von Rosenberg (1963)
Over-all mass transfer coefficient estimated from		British Petroleum and Yoshida et al (1954)	British Petroleum and Yoshida et al (1954)
Reboiler holdup, kmol	2.8		
Time delay in condenser, h	0.0167		
Height of the packing, m		7.78	7.78
Diameter of the column m		0.711	0.711
Holdup correlation for packing estimated from		British Petroleum	British Petroleum

Table 2. Steady-state operating conditions for 15.6 m column

Feed quality	Liquid at its boiling point
Feed composition based on fictitious molar masses	0.48 mole fraction of benzene
Feed composition based on normal molar masses	0.50 mole fraction of benzene
Feed flow rate based on fictitious molar masses	35 kmol/h
Feed flow rate based on normal molar masses	36.3 kmol/h
Distillate flow rate based on fictitious molar masses	10.0 kmol/h
Distillate flow rate based on normal molar masses	10.8 kmol/h
Bottom product flow rate based on fictitious molar masses	25.0 kmol/h
Bottom product flow rate based on normal molar masses	25.5 kmol/h
Reflux flowrate based on fictitious molar masses	20.0 kmol/h
Vapour flow rate based on fictitious molar masses	30.0 kmol/h
Internal reflux ratio	0.67

The steady-state profile of the column is shown in Figure 2. The flow model used for simulation and the resulting equations solved by orthogonal collocation on finite elements employing Hermite polynomials (Wardle and Hapoğlu (1994)).

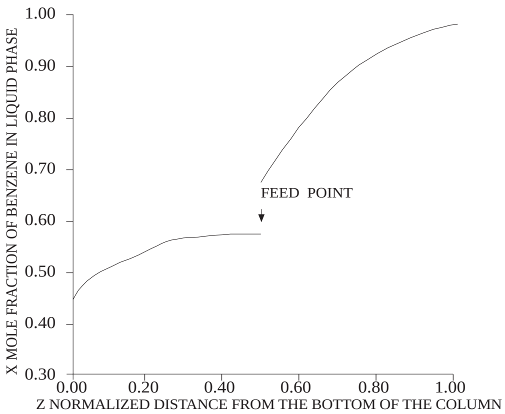

Figure 2. Steady-state profile for the 15.6 m simulated column.

Table 3. Comparison of closed-loop ISE values for the STPID control for the 15.6 m column distilling a mixture of benzene and toluene.

Sampling interval Δt, min	t_1	20 % decrease in feed composition ISEx10^5	40 % decrease in feed composition ISEx10^5
1	-0.1	67.2	153
1	-0.2	30.6	134
1	-0.3	9.10	106
1	-0.4	3.56	69.1
1	-0.5	2.21	27.3
1	-0.6	1.81	14.4
1	-0.7	1.77	11.3
1	-0.8	2.05	11.5
1	-0.9	3.27	16.7

Note that the dead time produced by the condenser (1 min) is excluded from all the response curves presented in this work.

PID control action was employed throughout being considered the most likely type of control action for this application. The controller parameters were estimated using three different closed loop response tuning criteria for discrete controllers, viz. those due to Yuwana and Seborg (1982), Jutan and Rodriguez (1984), Wardle and Heathcock (1992) and an additional criterion which has been proposed in this work.

The parameters τ_I, τ_D and K_a are largely independent of the gain K which is chosen by the user in order to obtain the setpoint response curve presented in Figure 1. Thus changes in K only affect K_c and the higher the selected gain K the higher is the calculated PID controller gain K_c. However, if the gain K of the proportional controller employed in the setpoint test is too high, then the system becomes unstable and a satisfactory setpoint response curve cannot be obtained. An extra criterion is required to choose the test gain K in order to obtain the best value of K_c for PID control using the YS procedure. As a means of comparing the performance of the PID controller, the integral square of the error (ISE) is computed for the closed-loop process output, where:

$$ISE = \sum_{t=0}^{t_1} \left(y(t)-r(t)\right)^2 \tag{34}$$

This extra criterion is that the test proportional controller gain K should be such that it gives a response curve (Figure 1) with a decay ratio of about 25 percent (Hapoğlu (1993)). To obtain a set point response curve with 25 percent decay ratio could be time con-

Table 4. Comparison of closed-loop ISE values for PID and STPID control of the overhead product composition in the 15.6 m column distilling a mixture of benzene and toluene.

Sampling interval Δ, min	Method				20 % decrease in feed composition ISEx10^5	40 % decrease in feed composition ISEx10^5
		t_1	t_2	t_3		
1	STPID	-.07	0	-0.157	1.77	11.3
		K_c	τ_I, min	τ_D, min		
1	YS-JR	1.06	0.15	0.053	268	199
1	YS-JR-decrease gain	0.250	0.15	0.053	4.61	49.1
1	YS-JR-decrease gain	0.200	0.15	0.053	4.02	31.3
1	YS-JR-decrease gain	0.175	0.15	0.053	4.05	29.1
1	YS-JR-decrease gain	0.150	0.15	0.053	4.26	29.3
1	YS	0.121	0.26	0.17	6.83	45.6
1	YS-WH	0.300	0.26	0.17	3.26	21.0
1	YS-WH	0.350	0.26	0.17	3.04	20.1
1	YS-WH	0.400	0.26	0.17	2.96	20.8
1	YS-WH	0.450	0.26	0.17	3.01	23.7

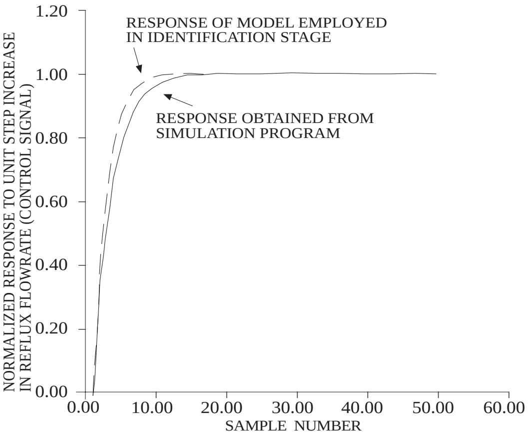

Figure 3. Open loop step responses obtained using simulation program and identified model.

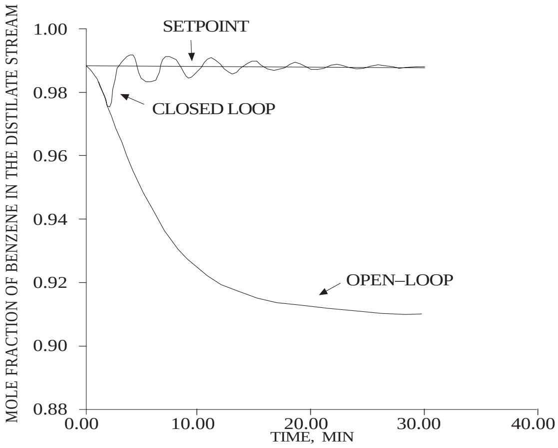

Figure 6. Self-tuning PID control of overhead product composition. τ_I=-0.7

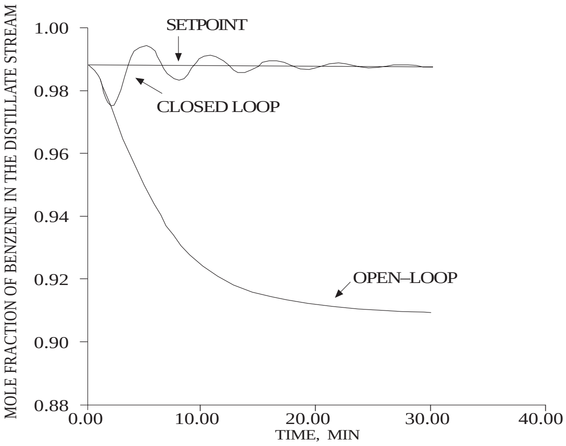

Figure 4. PID control of overhead product composition. Method: YS-WH, K_c=0.35, τ_I=0.26 min, τ_D=0.17 min

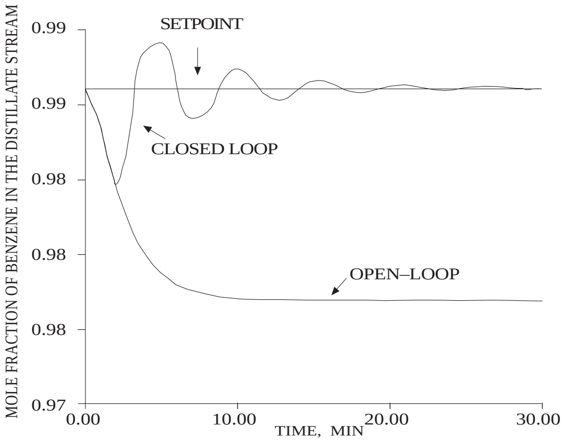

Figure 7. PID control of overhead product composition. Method: YS-WH, K_c=0.4, τ_I=0.26 min, τ_D=0.17 min

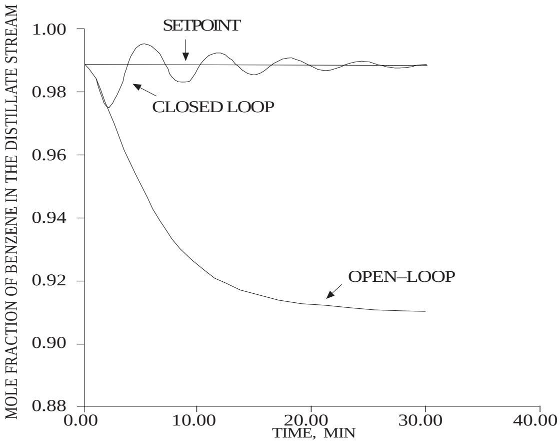

Figure 5. PID control of overhead product composition. Method: YS-JR-decrease gain, K_c=0.175, τ_I=0.15 min, τ_D=0.053 min

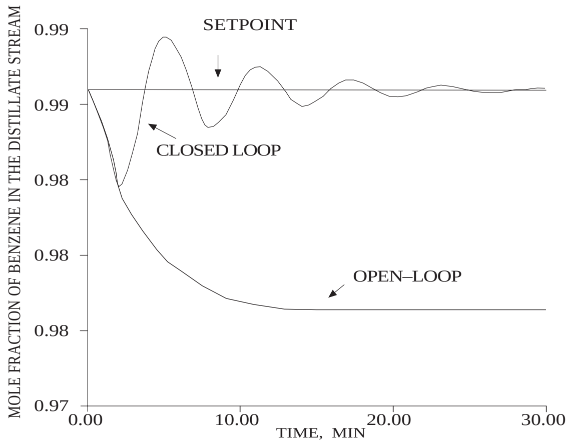

Figure 8. PID control of overhead product composition. Method: YS-JR-decrease gain, K_c=0.2, τ_I=0.15 min, τ_D=0.053 min

suming. The user may prefer to employ a small value of K to calculate the PID parameters using the Yuwana and Seborg (1982) procedure and simply vary K_c to obtain the smallest value of ISE for the closed-loop controlled variable. Wardle and Heathcock (1992) previously reported this procedure but they only tried increasing K_c (this is termed the YS-WH). Additionally, in this work, the effect of decreasing K_c is examined (called the YS-JR-decreased gain approach)(see Table 4).

In this work, the STPID algorithm used a system transfer function of the form:

$$y(t) = \frac{A}{B} u(t-1) \quad (35)$$

The identified model for the 15.6 column was:

$$y(t) = \frac{0.348}{1-0.473z^{-1}-0.157z^{-2}} u(t-1) \quad (36)$$

The response of the overhead composition (mole fraction of the more volatile component) obtained from the computer simulation programma and of the identified models to a unit step increase in manipulated variable (reflux flowrate) is shown in Figure 3. Agreement is sufficiently close for the identified model to be used for controller design in the cases studied. The properties of the controller can be varied by changing the pole locations of the polynomial T in the closed-loop characteristic equation (14). The coefficients of the T polynomial (t_1, t_2 and t_3) are defined by the user. In order to simplify the tuning procedure, standard practice is to hold t_2 and t_3 equal to its minimum possible value and to vary t_1 only in order to tune the controller. Table 3 shows the tuning of STPID. Figures 6 and 9 show the best STPID control of overhead composition in the face of a 40 and a 20 percent step increase in feed composition respectively. Table 4 shows the conventional controller parameters obtained by using the YS, YS-JR, YS-WH and 'YS-JR-decrease gain' methods and self-tuning PID control. A 1 min sampling interval was employed in each case and the corresponding self-tuning PID controller gives a smaller ISE value than any of the fixed parameter procedures (Figures 4 to 9).

Conclusions

Conventional three-term PID action remains the most widely utilized form of control in the process industries though many such controllers are often inefficiently tuned with a consequent degradation of control performance.

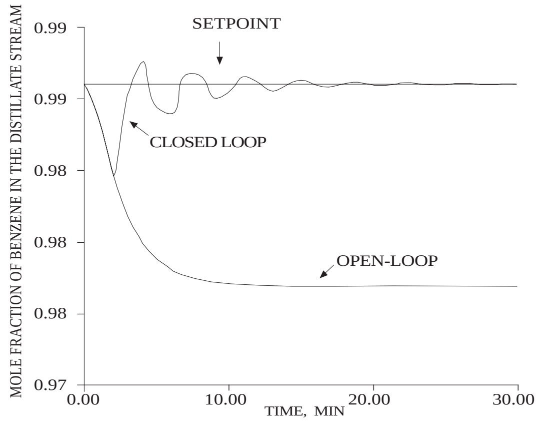

Figure 9. Self-tuning PID control of overhead product composition. τ_1=-0.7

A method for tuning controllers was proposed by Yuwana and Seborg in 1982 and later modified by Jutan and Rodriguez (1984) and Wardle and Heathock (1992). This relatively new tuning procedure avoids significant disadvantages associated with the two most popular methods, viz. the time-consuming, trial and error tests associated with the continuous cycling approach and the open-loop perturbation requirement for the process reaction curve method. The Yuwana and Seborg procedure should be attractive for practical applications since only a single, closed-loop test is required and since the recommended controller settings can be calculated analytically. An additional criterion (the YS-JR-decrease gain method) has been proposed in the present work and is found to give better control than YS-JR method in terms of the ISE in a number of cases. The Wardle and Heathock and 'YS-JR-decrease gain' methods were found to be very reliable and gave controller settings which were robust. The procedures are all simple to employ and require only minimal computer usage.

Conventional controller design is generally based upon simple process models, thus on-line 'fine-tuning' is often required after the controller has been initially set up. On the other hand, self-tuning controllers can provide good control even if an accurate process model is not avaliable or if the process dynamics vary with time.

The application of adaptive self-tuning PID, to a packed distillation column has been examined. It can be seen that STPID action is the most successful control strategy in every case (in terms of ISE).

Nomenclature

A monic polynomial in the z-domian representing the poles of the discrete-time system

a_i parameters of A polynomial

B polynomial in the z-domain representing the zeros of the discrete time system

b_i parameters of B polynomial

C monic polynomial in z-domain representing the zeros of the process noise

d_m process model dead-time

e white noise

G_m process transfer function

K_m process model gain

n_a degree of polynomial A

n_b degree of polynomial B

r(t) set-point at time t

S a polynomial

T a polynomial

u(t) input variable at time t

y(t) output variable at time t

$\in(t)$ difference between the measured variable and set point at time t

τ_m process model time constant

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