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Investigation of Forces and Stresses Acting on a Shoulder-Hand System Considering Strains in Muscles

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Abstract

A model is proposed to investigate the forces and stresses acting on bones in a shoulder-hand system due to straining of muscles during daily physical activities. This model is proposed as a truss system consisting of beams, or rods, corresponding to the considered bones and muscles, connected by frictionless joints. One of the hard loading positions, which occurs when one does pull-ups while holding onto a bar (a common fitness activity), is considered in the proposed model as an example. The stress is found to increase from the elbow end of the humerus bone toward the shoulder end. The stress in the example study reached a maximum value of approximately 8000 N/cm^2 near the shoulder end during the pull-up motion, which is about 20% less than the maximum stress calculated when the straining of muscles was neglected. The difference between these 2 maximum values of stress indicates the effect of strained muscles, which was first taken into consideration in this study. This stress value is considerably high when the strength of bones is taken into account. It is also necessary to take into account the stress fluctuations in the human body during various daily activities; fluctuating stresses acting on an implant may create fatigue cracks at the shoulder end of the humerus in addition to a high stress concentration factor due to shape and size changes in the humerus bone.

Key words: Biomechanical modeling, Humerus bone, Stresses on shoulder-hand system.

Introduction

Nowadays, total joint replacements for the hip, knee, shoulder, and other joints are routine in orthopedic surgery. Implants that are inserted during orthopedic treatment in bones that have suffered trauma are affected by various factors in the human body such as mechanical and thermal loading and corrosion. The character and intensity of mechanical loads in bones must be analyzed in detail for optimal implant design. These analyses require some surgical simulations (Fung, 1990; Winter, 1990; Gordon and Robertson, 1997; Ozkaya and Nrodin, 1999; Alexander, 2002; Schneck and Bronzino, 2003). Surgical simulations are particularly appropriate for the large volume and expense of joint replacement procedures in orthopedics (O'Toole et al., 1995; Anglin et al., 2000; Szivek et al., 2000).

The human shoulder is highly flexible, and therefore provides a large range of motion to the arm and hand, despite the expense of precarious stability of the articulations. The shoulder is flexible enough to provide the arm with an enormous range of motion, yet it can simultaneously provide a stable platform for the arm even when very strenuous forces are exerted against the environment (Kirsch and Acosta, 2002). Presently used shoulder endoprostheses show unsatisfactory results in terms of functionality and long-term fixation. Complications are mainly due to joint pathology and insufficient bone material for glenoid component fixation (patient factors), application of less efficient materials as compared to anatomical tissues and the introduction of interfaces (design factors), and difficulties with the insertion and alignment of the components (surgical factors) (Oosterom et al., 2002; Wakabayashi et al., 2003; Debski et al., 2005).

The maximum acceptable workload for each element of a joint can be calculated individually as a function of the external load and the geometry of the articulating surfaces, muscles, and ligaments. The proposed model provides sets of humerus positions that are acceptable in terms of muscle and ligament strength and stresses at the bone-on-bone contact points. The force applied at the hand is a critical element in assessing individual force limitations during different activities (Gielo-Perczak and Leamon, 2002).

In the present study, a model is proposed to investigate the forces and stresses acting on bones in a shoulder-hand system due to muscle straining occurring during daily activities. This model is proposed as a truss system consisting of rods corresponding to the considered bones and muscles connected by frictionless joints. One of the hard loading positions, which occurs when one does pull-ups while holding onto a bar (a common fitness activity), is considered in the proposed model as an example. In earlier studies (Linde et al., 1992; Anglin et al., 1999; Au et al., 2005), stresses acting on bones due to external loads were evaluated without considering the stress that developed due to strained muscles. In this work, the stresses that developed on bones due to both external loads and strained muscles were studied.

Modeling of Forces Formed in the Shoulder-Hand System

The model developed for 1 of the 2 shoulder-hand systems of the body is shown in Figure 1. The radius-ulna and humerus bones and muscle groups are modeled via the rods AD and BD and BC, AC, and CD, respectively. The rods are connected by frictionless joints A, B, C, and D.

The hand area affected by the reaction force (Q) and the shoulder area affected by G (which is half of the body weight) are represented by points A and B in Figure 1, respectively. Angles φ_1 and φ_2 given in Figure 1 are related by:

$$\varphi_2 = \arctan(\mathrm{m} \tan \varphi_1), \qquad (1)$$

where the parameter $\mathrm{m} = \frac{\mathrm{OD}}{\mathrm{OC}}$.

As the geometry of this static system and the applied forces are symmetrical, we can write $F_1 = F_2$ and $F_3 = F_4$. The internal forces occurring in the rods were found using static equilibrium conditions (Timoshenko and Young, 1983):

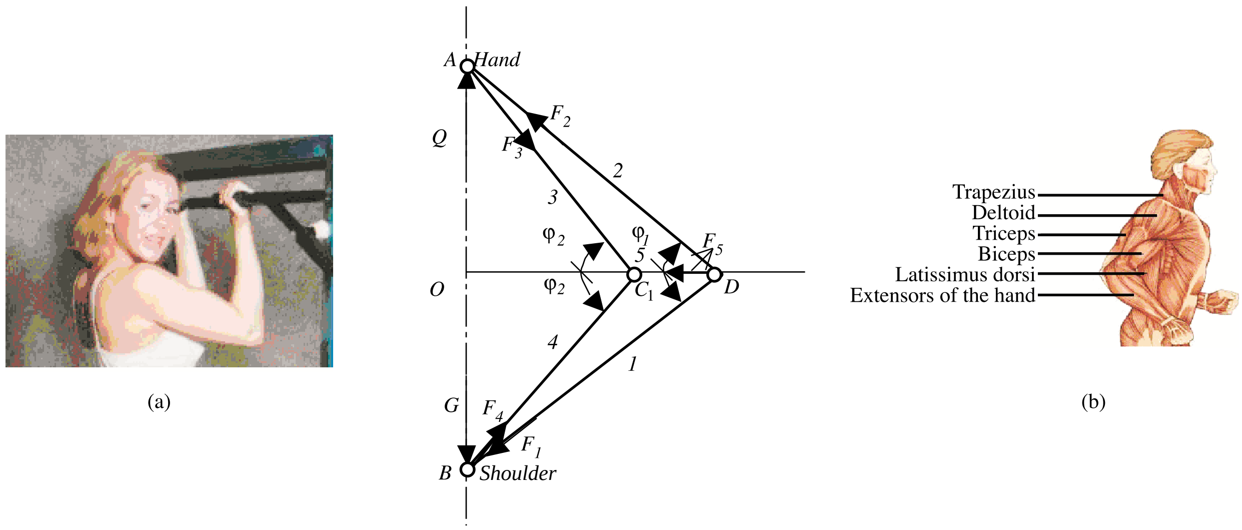

Figure 1. (a) The pull-up motion of a person (b) Model developed for shoulder-hand system during pull-up motion (1-Humerus, 2-Radius-Ulna, 3-Extensors of the hand-brachioradialus, palmarislongus, extensordigitorum muscles, 4-Biceps, triceps, deltoid, brachialus muscles, 5-Extensor carpi radials longus and pronator teres muscles).

$$F_1 = G\frac{\cos\varphi_2}{\sin(\varphi_2-\varphi_1)} \quad , \quad F_3 = G\frac{\cos\varphi_1}{\sin(\varphi_2-\varphi_1)} \quad , \quad F_5 = 2G\frac{\cos\varphi_1.\cos\varphi_2}{\sin(\varphi_2-\varphi_1)}$$

Using Eq. (1), the above relations can be written:

$$F_1 = G\frac{\cos\left[\arctan(\mathrm{m}\tan\varphi_1)\right]}{\sin\left[\arctan(\mathrm{m}\tan\varphi_1) - \varphi_1\right]} \quad ; \quad F_3 = G\frac{\cos\varphi_1}{\sin\left[\arctan(\mathrm{m}\tan\varphi_1) - \varphi_1\right]} \quad ;$$

$$F_5 = 2G\frac{\cos\varphi_1\cos\left[\arctan(\mathrm{m}\tan\varphi_1)\right]}{\sin\left[\arctan(\mathrm{m}\tan\varphi_1) - \varphi_1\right]}. \tag{2}$$

These equations show that the parameter m strongly affects the results of force calculations. Therefore, the value for m should be chosen cautiously.

After the forces acting on the bones and muscles are determined, stresses that formed on these elements can be investigated by analytical and experimental methods (Belyaev, 1953; Timoshenko, 1976).

Stresses in the Humerus Bone

The effects on the bones due to forces resulting from physical activities are analyzed by assuming small deformations and linearly elastic behavior. Related works show that the application of such hypotheses is typically sufficient for orthopedic treatments and the design of implants and prostheses (Liebowitz, 1972; Martens et al., 1983; Cowin, 1989; Linde et al., 1992; Anglin et al., 1999; Au et al., 2005).

When length, vertical section dimensions, and the working conditions of the humerus bone are taken into account, we can assume a design scheme for investigation of stresses consisting of a simple beam supported at the ends and loaded by eccentric forces $F = F_1$ (Figure 2). The natural curvature of the humerus, the initial maximum deflection in the middle section (y_0), and the forces acting at the shoulder end of the humerus at point B with eccentricity (e) are taken into consideration.

The initial deflection of the beam (y_0) is small with respect to length ℓ; therefore, the equation of the axis of the beam (y_1) can be assumed as sinusoidal during the unloading condition:

$$y_1 = y_0\sin\frac{\pi x}{\ell}. \tag{3}$$

Deflections formed by forces acting on the humerus can be found according to $y - y_1$:

$$y - y_1 = a\sin\frac{\pi x}{\ell}, \tag{4}$$

where y is the total deflection of the beam axis and a is the deflection of the beam at $x = \frac{\ell}{2}$.

Using Eq. (3), we obtain

$$y = (a + y_0)\sin\frac{\pi x}{\ell} \quad . \tag{5}$$

The energy U due to bending is

$$U = \int_0^\ell \frac{M_b^2 dx}{2EI} = \frac{1}{2}\int_0^\ell EI\left[(y - y_1)''\right]^2 dx,$$

where E is the elastic modulus of the humerus, I is the axial moment of inertia of the section, M_b is the bending moment, and the symbol $(.)''$ denotes the second derivative with respect to x.

Then we obtain

$$U = \frac{\pi^4 a^2}{2\ell^4}k \quad , \tag{6}$$

where

$$k = \int_0^\ell EI\sin^2\frac{\pi x}{\ell}dx. \tag{7}$$

By differentiating Eq. (6) with respect to a, we obtain

$$dU = \frac{\pi^4 a}{\ell^4}k da. \tag{8}$$

The displacement Δ (Figure 2) can be determined as the difference between length ℓ and the projection of the deflection curve on the straight line joining points A and C. Clearly, it can then be stated that

$$\Delta = \int_0^\ell (dx - ds\cos\vartheta).$$

However, for small deflections,

$$|\Delta| = \frac{1}{2}\int_0^\ell (y')^2 dx, \tag{9}$$

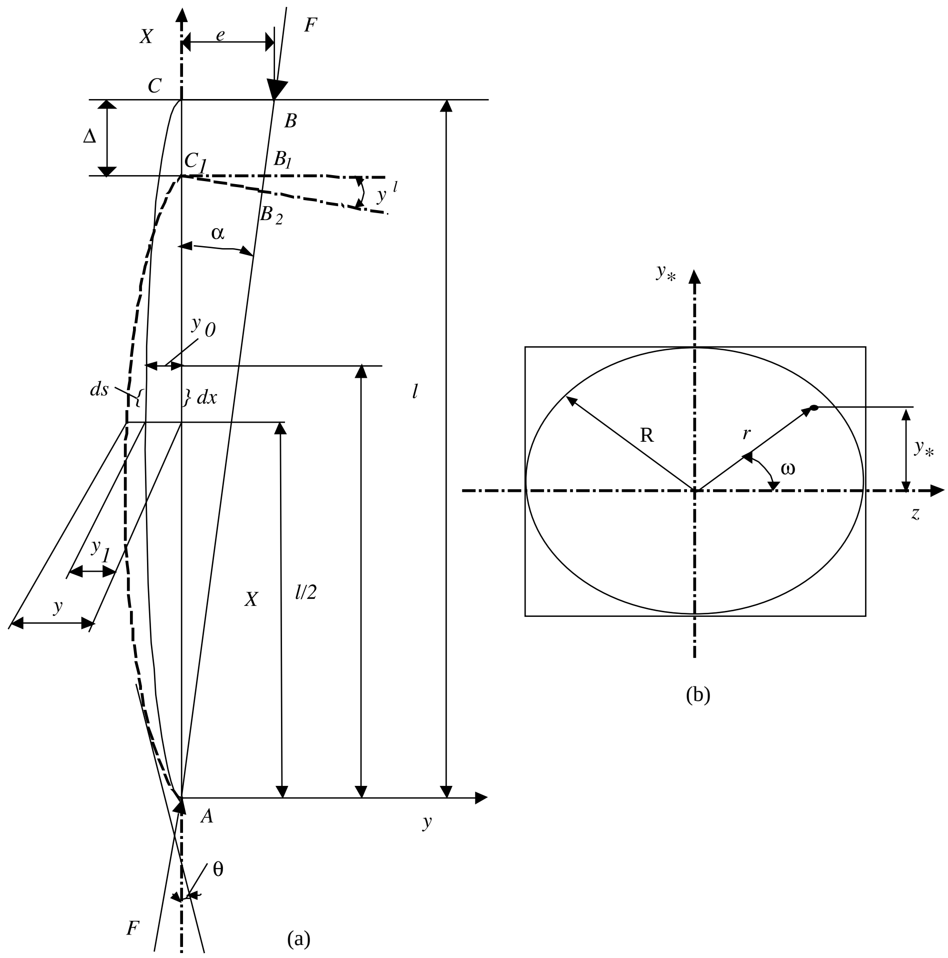

Figure 2. Investigation of stresses on the humerus bone: (a) Calculation scheme of stresses, (b) Cross-section of humerus bone.

where the symbol $(.)'$ denotes the first derivative with respect to x.

If we consider Eq. (5), Eq. (9) becomes

$$|\Delta| = \frac{\pi^2 (a+y_0)^2}{4\ell}. \qquad (10)$$

The displacement of the beam in the vertical direction (Figure 2) is given as

$$\Delta_F = BB_1 + B_1B_2 = |\Delta| + |y'|\, e, \qquad (11)$$

where e is as defined above.

Taking into account that $|y'|_{x=\ell} = (a+y_0)\frac{\pi}{\ell}$ in Eq. (5), we obtain

$$\Delta_F = \frac{\pi^2 (a+y_0)^2}{4\ell} + \frac{\pi e (a+y_0)}{\ell}. \qquad (12)$$

By differentiating this displacement with respect to a,

$$d\Delta_F = \left[\frac{\pi^2 (a+y_0)}{2\ell} + \frac{\pi e}{\ell} \right] da. \qquad (13)$$

Work done by the $F_x$component of the force F due to the above displacement is

$$dU_F = F_x d\Delta_F.$$

Introducing Eq. (13) into the above relation yields

$$dU_F = F \left[\frac{\pi^2 (a+y_0)}{2\ell} + \frac{\pi e}{\ell} \right] \cos\alpha da. \qquad (14)$$

We can write Eq. (14) in the following form:

$$dU = dU_F \quad .$$

Taking into account Eqs. (8) and (14), we obtain

$$\frac{\pi^4 a}{\ell^4} k da = \left[\frac{\pi^2 (a+y_0)}{2\ell} + \frac{\pi e}{\ell} \right] F\text{cos}\alpha da.$$

Next, by simple rearrangement, we find

$$a = \frac{F\ell^3(\pi y_0 + 2e)\cos\alpha}{\pi(2\pi^2 k - \ell^3 F\cos\alpha)}. \quad (15)$$

Introducing Eq. (15) into Eq. (5), we obtain

$$y = \frac{2(e\ell^3 F\cos\alpha + \pi^3 k y_0)}{\pi(2\pi^2 k - \ell^3 F\cos\alpha)}\sin\frac{\pi x}{\ell}. \quad (16)$$

The bending moment M_b acting on the humerus is given as

$$M_b = F\left[(\ell - x)\sin\alpha - (e + y)\text{cos}\alpha\right].$$

Then by introducing Eq. (16) into the above relation,

$$\begin{aligned} M_b = \; & F\Big\{(\ell - x)\text{sin}\alpha \\ & - \left[e + \frac{2(e\ell^3 F\text{cos}\alpha + \pi^3 k y_0)}{\pi(2\pi^2 k - \ell^3 F\text{cos}\alpha)}\sin\frac{\pi x}{\ell}\right]\text{cos}\alpha\Big\}. \end{aligned} \quad (17)$$

If we assume that bending occurs only along the *xoy* plane, we obtain normal bending stresses (σ_{xb}) affecting the vertical section of the beam by using

$$\iint_A \sigma_{xb} y_* dA = M_b \quad ,$$

where y_* is the ordinate of the section points (Figure 2) and A is the area of the cross section.

When stress distribution in the vertical section of the beam is taken into account, the normal bending stress (σ_{xb}) can be calculated from the following equation:

$$\sigma_{xb} = \frac{E y_*}{\iint_A E y_*^2 dA} M_b,$$

or by using Eq. (17) we obtain

$$\sigma_{xb} = \frac{EF y_*}{\int\int_A E y_*^2 dA}\left\{(\ell - x)\text{sin}\alpha\text{-}\left[e + \frac{2(F\ell^3 e\text{cos}\alpha + \pi^3 k y_0)}{\pi(2\pi^2 k - F\ell^3\text{cos}\alpha)}\sin\frac{\pi x}{\ell}\right]\cos\alpha\right\}. \quad (18)$$

The total normal stresses (σ_{xT}) in the beam, which consist of axial compressive stresses ($Fcos\alpha/A$) and bending stress (σ_{xb}), can be calculated from the following equation:

$$\sigma_{xT} = \frac{F\text{cos}\alpha}{A} \pm \frac{EF y_*}{\int\int_A E y_*^2 dA}\left\{(\ell - x)\text{sin}\alpha\text{-}\left[e + \frac{2(F\ell^3 e\text{cos}\alpha + \pi^3 k y_0)}{\pi(2\pi^2 k - F\ell^3\text{cos}\alpha)}\sin\frac{\pi x}{\ell}\right]\cos\alpha\right\}. \quad (19)$$

Calculation of Stresses in the Humerus Bone as a Case Study

A person with a weight of 800 N is considered as an example scenario for the case study. During the pull-up motion of this person, one hand will carry half of the weight, which is 400 N. We assume that the cross section of the humerus bone is circular with a radius (R) and its modulus of elasticity, E= 89 × 10^4N/cm^2, is constant along its length for simplification of calculations (Liebowitz, 1972). However, the modulus of elasticity changes at the cross section of the bone according to the polar radius (r), as given by the function

$$E = N\left[1 + n\left(\frac{r}{R}\right)^2\right],$$

where $r = \sqrt{y_*^2 + z^2}$. Taking into account the modulus of elasticity of the humerus, N, its value is assumed to be approximately equal to 45 × 10^4 N/cm^2 (Liebowitz, 1972). Additionally, it has been assumed that the parameter $n = 1.00$ to simplify calculations.

The dimensions of the humerus bone, which is considered as a beam, are assumed to have the following values (Figure 2):

ℓ = 22.50 cm, R = 1.00 cm, y_0= 0.25 cm, e =1.00 cm, $\alpha = \arctan\frac{e}{\ell} \approx 2.5^0$, $I = \frac{\pi R^4}{4} \cong 0.79 cm^4$, and G = 400 N. Assuming $\varphi_1 = 25^0$and m = 1.25, we can find the force acting on the humerus using Eq. (2):

$$F = F_1 = 400\frac{\cos\left[\arctan(1.25\tan 25^0)\right]}{\sin\left[\arctan(1.25\tan 25^0) - 25^0\right]} \approx 3784 \text{ N}$$

Taking into account that the elemental area is $dA =$

$rdrd\omega$, $y_* = r\sin\omega$, and parameter k from Eq. (7) is

$$k = EI\int_0^\ell \sin^2\frac{\pi x}{\ell}dx = \frac{IE\ell}{2},$$

from Eq. (18), we find

$$\sigma_{xb} = \frac{12(R^2+nr^2)}{\pi R^6(3+2n)}y_*F\left\{(\ell-x)\sin\alpha\text{-}\left[e+\frac{2(e\ell^2F\cos\alpha+0.5\pi^3EIy_0)}{\pi(\pi^2EI-\ell^2F\cos\alpha)}\sin\frac{\pi x}{\ell}\right]\cos\alpha\right\}.$$

For points on only the xoy plane (z= 0) and at the humerus boundaries of $y_* = r = \pm 1$, stresses can be given as

$$\sigma_{xb} = \pm\frac{24}{5\pi}3784\left\{(22.5-x)\sin\alpha\text{-}\left[1+\frac{2(22.5^2\times 3784\times\cos\alpha+0.5\pi^3\times 90.10^4\times 0.79\times 0.25)}{\pi(\pi^2 90.10^4\times 0.79-22.5^2\times 3784\times\cos\alpha)}\sin\frac{\pi x}{22.5}\right]\cos\alpha\right\}$$

or

$$\sigma_{xb} = \pm 5782\left\{(22.5-x)\sin\alpha\text{-}\left[1+0.58\sin(0.14x)\right]\cos\alpha\right\}.$$

The total stress will be the sum of the bending stress (σ_{xb}) calculated from the above equation and the axial compressive stress ($F\cos\alpha/A$=1200 N/cm^2), as given in Figure 3.

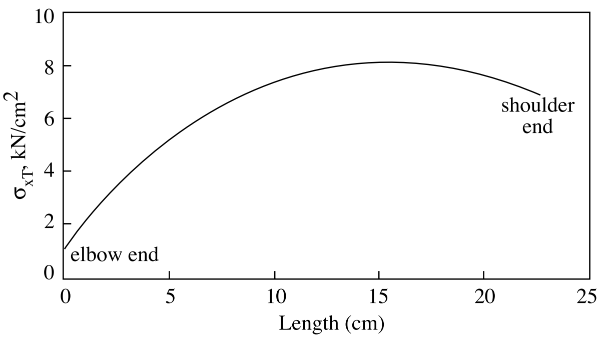

Figure 3. Total normal stress distribution along the length of the humerus bone for m = 1.25 in Eq. (1).

In general, the stress increases from the end of the humerus bone near the elbow toward the end near the shoulder. The stress reaches a maximum value of 8000 N/cm^2 near the shoulder end of the humerus bone for the studied condition. As shown in Figure 3, this stress value is considerably high. However, this result strongly depends on the value of m in Eq. (1), as shown in Figure 4.

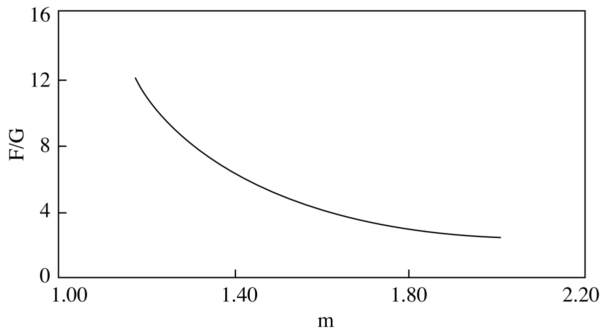

Figure 4. F/G variation as a function of the value of m.

To evaluate the results obtained from the method presented in this study, a comparison is made relative to previous related studies (Fung, 1990; Linde et al., 1992; Anglin et al., 1999; Ozkaya and Nrodin, 1999; Szivek et al., 2000; Currey, 2002; Au et al., 2005), which neglect the effect of muscle straining. This comparison is made below by applying the method of the previous studies using the case study example data from the present study.

The elbow region of the humerus (point D, Figure 1) is assumed to be fixed, although in actuality it is a hinged joint. Therefore, the investigated case of the humerus can be considered as a cantilever beam, loaded with the force G at the free end, point B. Its length, ℓ, is 22.50 cm. The origin of the coordinate axes is selected as point B. The x-axis is along the humerus axis oriented toward the end, point D. The force, G, has an angle of $\varphi_1 = 25^0$ with respect to the normal of the x-axis. The x-component of the force, G$\sin\varphi_1$, occurs on the beam, resulting in a tension of σ_x = (G/A) $\sin\varphi_1$, where A represents the cross-sectional area of the humerus, which is equal to πR^2. The normal component of force, G, with respect to the axis of the beam is G$\cos\varphi_1$. This component causes a bending stress, which can be expressed as $\sigma_x = \pm$ M/W. The bending moment, M, is xGcosφ_1. Variation in the bending moment is linear along the beam and reaches its maximum value at point D, where x is equal to ℓ and the section modulus, W, is equal to $\pi R^3/4$. The total stress is obtained by superimposing the tension and bending stresses and can be calculated with the following expression:

$$\begin{aligned}\sigma\text{max} &= \frac{G}{\pi R^2}\left(\frac{4\ell}{R}\cos\varphi_1 + \sin\varphi_1\right)\\ &= \frac{400}{\pi}\left(4\times 22.50\times\cos 25^o + \sin 25^o\right)\\ &= 10,439.33\text{N/cm}^2\end{aligned}$$

This resulting stress state has a different character and maximum value relative to the results of the method demonstrated in this study above. The above resulting maximum stress value (≅10,439 N/cm^2), where strained muscles are neglected, is significantly different from the result of the method presented in this study (≅8000 N/cm^2). The difference between these 2 maximum stress values exceeds 20% and is clearly due to the neglect of the effect of strained muscles by the previous studies' approach. The case presented in this study considering the effect of strained muscles better reflects actual conditions.

Conclusion

The stresses exerted on the bones in a shoulder-hand system during the pull-up motion onto a bar, a common fitness activity, due to both external loads and strained muscles were evaluated in this study. The maximum stress in the considered example was calculated to be approximately 8000 N/cm^2, which is about 20% less than the stress calculated when muscle straining was neglected. The difference between these 2 maximum stress values represents the effect of strained muscles, which was first taken into consideration in this study.

During the design of an implant at the shoulder end of the humerus bone, the stress distribution calculated in this study must be taken into consideration, particularly with respect to the strength of the bones. It is also necessary to take into account the stress fluctuations in the human body occurring during daily activities; fluctuating stresses acting on an implant may create fatigue cracks at the shoulder end of the humerus and, additionally, may impose a high stress concentration factor due to shape and cross-sectional size changes in the humerus bone. As a result, the stress distribution in the humerus bone calculated in this study may help in the appropriate design of an implant for a humerus bone.

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