An Efficient Seismic Analysis Procedure for Torsionally Coupled Multistory Buildings Including Soil-Structure Interaction

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Abstract

In this paper, a simplified methodology of analysis for the seismic response of 3-dimensional irregular high-rise buildings on a rigid footing resting on the surface of a linear elastic half-space is formulated. An efficient method using modal decomposition and carried out in the frequency domain by using the fast Fourier transform to obtain the structural response of torsionally asymmetric buildings, including soil-structure interaction effects, is presented. Applying this algorithm, full advantage is taken of classical normal mode approximation, and the interaction problem is solved easily and effectively within the framework of the Fourier-transformed frequency domain analysis for a fixed-base structure. The matrix formulation of this method produces accurate approximation with less computational effort, in spite of using the frequency dependent impedance functions.

Key words: Torsionally coupled multistory buildings, Soil-structure interaction, Frequency domain analysis, Impedance functions, Modal decomposition.

Introduction

The comprehensive studies (Chandler and Hutchinson, 1986; Cruz and Chopra, 1986b; Hejaj and Chopra, 1989) conducted by a number of researchers in the past few decades and investigations of the effects of past earthquakes have shown that in buildings with non-coincident centers of mass and resistance, significant coupling may occur between the translational and the torsional displacements of the floor diaphragms even when the earthquake induces uniform rigid base translations. However, these studies assumed that the asymmetric structure is supported by a rigid foundation. The elastic response of structures to earthquake ground excitations is influenced by deformability of the foundation medium. Therefore, the rigid foundation assumption represents an approximation to real conditions. It is widely recognized that the dynamic response of a structure supported on soft soil may be different from the response of a similarly excited, identical structure supported on firm ground. The effects of soil-structure interaction (SSI) on the dynamic response of building systems have been the subject of numerous investigations in recent years. Most of the previous SSI studies (Chopra and Guiterrez, 1974; Wu and Smith, 1995; Wu, 1997) on buildings, however, were restricted to 2-dimensional (planar) multistory frames due to the fact that the governing equations of motion for structures with many degrees of freedom, as well as the methods of solving these equations, are relatively complicated and involve a considerable computational costs.

Various analytical and numerical techniques were
also proposed and developed to efficiently simplify SSI analysis, such as transmitting boundaries of different kinds, boundary elements, and infinite elements and their coupling procedures for the modelling of unbounded media (Meek and Wolf, 1992; Aydnoglu, 1993b; Wolf and Song, 1996). However, the complicated formulation and intensive computation necessary to obtain the exact solution to this problem have so far restricted its common application to traditional engineering practice. The foundation on flexible soil may be idealized in structural dynamics as a simple spring-dashpot-mass model and the solution can be performed in the frequency domain. Since most of the complication in the solution of the equations of motion results from the frequency dependence of the dynamic soil stiffness, many studies (Ghaer and Chapel, 1983; Wolf, 1997) have been performed to simulate the SSI phenomenon by representing the soil with frequency-independent impedance functions by constant parameters. However, this proposed approximation may not be valid when the soil surface is shallow relative to the base dimensions of the structure (Tsai et al., 1974). Additional research on SSI proposed by Sivakumaran (1990) and Chandler and Sikaroudi (1992) has been dedicated to the application of modal analysis techniques. Substitution of structural deformations, in combination with the dynamic SSI force-displacement relationships, into the governing equations of the whole system results in integro-differential equations for footing displacements, which are solved numerically by step by step integration in time domain. Such a procedure, given by Sivakumaran et al. (1992), requires a large computational effort, particularly for 3-dimensional asymmetric multistory buildings.

In the present paper, an efficient method using modal decomposition and performed in the frequency domain by using the fast Fourier transform algorithm to obtain the dynamic response of torsionally asymmetric buildings including the SSI effect is presented. The effects of the foundation medium on the structural response are simulated by a series of springs and dashpots representing a theoretical half-space surrounding the base of the structure. The structure investigated herein is presumed to be linear and viscously damped and is supported at the surface of a homogeneous, isotropic, elastic half-space and is excited at the base. However, to accurately represent the elastic half-space, the properties of springs and dashpots are required to be dependent on the frequency of excitation. Thus, the governing equations for the structure foundation system are expressed and solved in the Fourier-transformed frequency domain. The governing equations are developed considering the motions of each floor and the whole system. In this method, initially structural deformations are obtained in terms of foundation displacement using a linear combination of vibrational modes of the building on a rigid foundation, in combination with the dynamic SSI force displacement relationships. In this study, the Fourier transforms are computed very efficiently by the fast Fourier transform algorithm, where they are treated as discrete transforms (Humar and Via, 1993). The numerical approximations of the impedance functions used in the following study are taken from Veletsos and Wei (1971) for lateral and rocking vibration and from Veletsos and Nair (1974) for torsional vibration. To demonstrate the proposed method, a detailed parametric study is made of torsional coupling in a multistory building excited by real earthquake ground motion (Erzincan, 1992, E-W) for different soil stiffnesses by using an original program developed and applied by the authors. For practical purposes the definite part of the record time including the peak value of acceleration of this input motion is considered. The results represented in frequency response curves indicate that this efficient method produces a good approximation to the exact response obtained by the direct method illustrated in previous studies (Çelebi and Gündüz, 2000, 2001). The results presented in this paper show that the seismic responses of asymmetric buildings including soil structure interaction can be significantly different from those without substructure interaction.

**Structure-Foundation Model and Analytical Procedures**

**Assumptions and equation of motion**

The building foundation system considered, as shown in Figure 1, is represented by an N-story 3-dimensional superstructure, consisting of shear frames, resting on a rigid square foundation of mass \( m_f \), with a negligible thickness on the surface of a linear homogeneous elastic half-space. The mass of this building is considered to be concentrated at each floor level, and the floor systems are assumed to be rigid rectangular floor decks supported by relatively massless, axially inextensible columns. The lateral load resisting elements are assumed to be arranged
Figure 1. Idealized 3-dimensional asymmetric building foundation system on an elastic half-space.
so that the system has no axes of symmetry. Thus
the structure is a 2-way torsionally coupled system
with 3 degrees of freedom at each floor, namely hor-
izontal displacements \( u_x \), \( u_y \), and rotation about the
vertical axis \( u_h \), for the \( i \)th floor. In addition, due
to the deformability of the foundation, the system has 5
more displacement degrees of freedom, namely hori-
tzonal translations of the foundation \( u_{ox}, u_{oy}, \) rock-
ing rotations of the foundation \( \gamma_{ox}, \gamma_{oy} \) and the
twist of the foundation \( \theta_o \) (Figure 1). The earth-
quake excitation is defined by \( \ddot{u}_{gy}(t) \) and \( \ddot{u}_{gg}(t) \), the x and y
components of the ground acceleration measured
at the surface of the homogeneous half-space in the
far field, and \( \ddot{u}_{gr}(t), \dot{\gamma}_{gx}(t), \dot{\gamma}_{gy}(t) \) are the rota-
tional acceleration of the base of the building about
the vertical and horizontal axes, respectively. The equa-
tions of motion for a SSI system with 3 \( N + 5 \) degrees
of freedom can be written in the usual matrix form

\[
[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} + [M]\{\ddot{\theta}_o(t)\} + \{\ddot{\gamma}_{ox}(t)\} + \{\ddot{\gamma}_{oy}(t)\} + \{\ddot{\gamma}_{gh}(t)\} = \{0\}
\]

(1a)

where

\[
\{u(t)\} = \begin{cases} u_x(t) \\ u_y(t) \\ u_h(t) \end{cases}, \quad \{\ddot{u}(t)\} = \begin{cases} \ddot{u}_{ox}(t) \\ \ddot{u}_{oy}(t) \\ \ddot{u}_{oh}(t) \end{cases}
\]

\[
\{\ddot{\gamma}_{ox}(t)\} = \begin{cases} \ddot{\gamma}_{ox}(t) \\ \ddot{\gamma}_{oy}(t) \end{cases}, \quad \{\ddot{\gamma}_{gh}(t)\} = \begin{cases} \ddot{\gamma}_{gh}(t) \end{cases}
\]

(1b)

In terms of the linear transformation matrix, Eq. (1)
can be simplified to

\[
[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} + [M]\{\ddot{\theta}_o(t)\} + \{\ddot{\gamma}_{ox}(t)\} + \{\ddot{\gamma}_{oy}(t)\} + \{\ddot{\gamma}_{gh}(t)\} = \{0\}
\]

(2)

where \([T]\) represents the kinematic transfer matrix
for transmitting total motion of the rigid foundation
to the structure and the total displacement vec-
tor, \( \{\ddot{u}_{ot}(t)\} \), and includes horizontal, rocking and
torsional degrees of freedom of the rigid foundation
in addition to the free field motion. The rotation and
rocking acceleration components of the free field
ground motion will be disregarded as indicated in
Eq. (3). That is,

\[
\{\ddot{u}_{ot}(t)\} = \begin{bmatrix} \ddot{u}_{ox}(t) \\ \ddot{u}_{oy}(t) \\ \ddot{\theta}_o(t) \\ \dot{\gamma}_{ox}(t) \\ \dot{\gamma}_{oy}(t) \end{bmatrix},
\]

(3)

\[
[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}_{3N \times 5}
\]

\[
\ddot{u}_{ot}(t) = \ddot{u}_{ox}(t) + \ddot{u}_{gy}(t)
\]

\[
\ddot{u}_{ot}(t) = \ddot{u}_{oy}(t) + \ddot{u}_{gy}(t)
\]

In addition, 5 more equations are needed in order
to completely solve the problem. These equations
are developed by considering the equilibrium of the
whole system. They can be further written in open
form as

\[
(m_o + [1]^T[m_x\{1\}])\ddot{u}_{ox}(t) + [1]^T[m_x]h \ddot{\gamma}_{ox} + [1]^T[m_x]\ddot{u}_x(t) = -(m_o + [1]^T[m_x\{1\}])\ddot{u}_{gx}(t)
\]

(4a)

\[
(m_o + [1]^T[m_y\{1\}])\ddot{u}_{oy}(t) + [1]^T[m_y]h \ddot{\gamma}_{oy} + [1]^T[m_y]\ddot{u}_y(t) = -(m_o + [1]^T[m_y\{1\}])\ddot{u}_{gy}(t)
\]

(4b)
torsional springs and dashpots. Because the foundation stiffness and damping coefficients are dependent on excitation frequency, it is most convenient to consider the response of the building foundation system to harmonic ground motion. Due to these frequencies-based expressions for interaction shear force, moment, and torque, the interaction problem

\[
\begin{align*}
\{1\}^T[m_x]\{h\}u_{ox}(t) + (I_{tx} + \{h\}^T[m_x]\{h\})\gamma_{ox} + \\
h^T[m_x]\{u_x\} + M_{ox}(t) = -\{1\}^T[m_x]\{h\}u_{gy}(t) \\
\{1\}^T[m_y]\{h\}u_{oy}(t) + (I_{ty} + \{h\}^T[m_y]\{h\})\gamma_{oy} + \\
h^T[m_y]\{u_y\} + M_{oy}(t) = -\{1\}^T[m_y]\{h\}u_{gy}(t) \\
(m_o r_o^2 + \{1\}^T[m_y]\{1\}\dot{u}_o(t) + \{1\}^T[m_y]\{\dot{u}_o(t)\} + \\
T_o(t) = 0
\end{align*}
\]

In these equations, \(1\) is the column vector where each element is unity, and \(m_o\) and \(r_o\) are the mass and the radius of gyration of the foundation, respectively. \(I_{tx}, I_{ty}\) are the total of the mass moment of inertia of the floors and the foundation mat with respect to the x and y axes. \(\{h\}\) is the column vector of foundation to story heights, \([m_x], [m_y]\) consist of floor masses of the structure in terms of the x and y axes, and \([m_d]\) is the mass polar moment of inertia about the z axis of the floor mass. \(P_{ox}(t), P_{oy}(t), M_{ox}(t), M_{oy}(t), T_o(t)\) are the horizontal SSI forces, rocking moments and torsional moment, respectively. It is noted that they are the dynamic loads imposed by the structure on the foundation. When the foundation medium is flexible these forces can be related to foundation displacements \(u_{ox}, u_{oy}, \gamma_{ox}, \gamma_{oy}, \theta_o\). Dynamic SSI force displacement equations may be specified in the frequency domain. Equation (4) can be expressed more concisely in the following form:

\[
[M_1]\{\ddot{u}(t)\} + [M_2]\{\ddot{u}_{ox}(t)\} + \{P_o(t)\} = \{0\};
\]

\[
\{P_o(t)\}^T = \{P_{ox}(t) \quad P_{oy}(t) \quad M_{ox}(t) \quad M_{oy}(t) \quad T_o(t)\}
\]

where \([M_1] = [T]^T[M]\) and \([M_2]\) includes components of superstructure, and these refer to the total foundation mass matrix. They are given by

\[
[M_1] = \begin{bmatrix}
\{1\}^T[m_x] & \{0\}^T & \{0\}^T \\
\{0\}^T & \{1\}^T[m_y] & \{0\}^T \\
\{0\}^T & \{1\}^T[m_y] & \{0\}^T \\
\{0\}^T & \{h\}^T[m_x] & \{0\}^T \\
\{0\}^T & \{h\}^T[m_y] & \{0\}^T
\end{bmatrix}_{5 \times 3N}
\]

\[
[M_2] = \begin{bmatrix}
m_o + \{1\}^T[m_x]\{1\} & 0 & \{1\}^T[m_x]\{h\} & 0 & 0 \\
0 & m_o + \{1\}^T[m_y]\{1\} & 0 & \{1\}^T[m_y]\{h\} & 0 \\
0 & 0 & I_{tx} + \{h\}^T[m_x]\{h\} & 0 & 0 \\
0 & \{1\}^T[m_y]\{h\} & 0 & I_{ty} + \{h\}^T[m_y]\{h\} & 0
\end{bmatrix}_{5 \times 5}
\]

**Dynamic soil-structure interaction**

The soil is assumed to be an elastic homogeneous isotropic half-space modeled by a massless rigid plate supported by equivalent translational, rotational and torsional springs and dashpots. Because the foundation stiffness and damping coefficients are dependent on excitation frequency, it is most convenient to consider the response of the building foundation system to harmonic ground motion. Due to these frequencies-based expressions for interaction shear force, moment, and torque, the interaction problem
lends itself readily to formulation in the frequency domain. Thus, the Fourier transform is applied to the equations of motion (2) and (5), and the following, very simple, results are obtained:

\[
(-\omega^2[M] + i\omega[C] + [K])\{\ddot{u}(\omega)\} - \omega^2[M][T]\{\ddot{u}_{ot}(\omega)\} = \{0\};
\]

\[
-\omega^2[M_1]\{\ddot{u}(\omega)\} - \omega^2[M_2]\{\ddot{u}_{ot}(\omega)\} + \left[\ddot{S}(\omega)\right]\{\ddot{u}_{ot}(\omega)\} = \{\ddot{u}_g(\omega)\}
\]

where \{\ddot{u}_g(\omega)\} represents the Fourier transform of free field ground translation. The transformed equations can be rewritten in the matrix notation as follows:

\[
\begin{bmatrix}
K_{hx}(\omega) & 0 & 0 \\
0 & K_{hy}(\omega) & 0 \\
K_{hxz}(\omega) & 0 & K_{hry}(\omega) \\
0 & K_{hyr}(\omega) & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_g(\omega)
\end{bmatrix}
= \begin{bmatrix}
-\omega^2[M] + i\omega[C] + [K] - \omega^2[M][T] \\
-\omega^2[M] \\
\ddot{S}(\omega) \\
-\omega^2[M_2]
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_{ot}(\omega)
\end{bmatrix}
\]

The Fourier transformed dynamic stiffness matrix and the displacement vector consisting of structural and foundation displacement components take the following form:

\[
\begin{bmatrix}
K_{hx}(\omega) & 0 & 0 \\
0 & K_{hy}(\omega) & 0 \\
K_{hxz}(\omega) & 0 & K_{hry}(\omega) \\
0 & K_{hyr}(\omega) & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_g(\omega)
\end{bmatrix}
= \begin{bmatrix}
-\omega^2[M] + i\omega[C] + [K] - \omega^2[M][T] \\
-\omega^2[M] \\
\ddot{S}(\omega) \\
-\omega^2[M_2]
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_{ot}(\omega)
\end{bmatrix}
\]

where \{\ddot{u}(\omega)\} is the Fourier transform of the \{u(t)\}. For harmonic excitation of frequency \omega the dynamic stiffness matrix \[\ddot{S}(\omega)\] of the foundation defined as the ratio of the amplitude of the applied load \{\ddot{P}(\omega)\} to that resulting displacement \{\ddot{u}_{ot}(\omega)\} is written as

\[
\begin{bmatrix}
K_{hx}(\omega) & 0 & 0 \\
0 & K_{hy}(\omega) & 0 \\
K_{hxz}(\omega) & 0 & K_{hry}(\omega) \\
0 & K_{hyr}(\omega) & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_g(\omega)
\end{bmatrix}
= \begin{bmatrix}
-\omega^2[M] + i\omega[C] + [K] - \omega^2[M][T] \\
-\omega^2[M] \\
\ddot{S}(\omega) \\
-\omega^2[M_2]
\end{bmatrix}
\begin{bmatrix}
\ddot{u}(\omega) \\
\ddot{u}_{ot}(\omega)
\end{bmatrix}
\]

The internal damping of soil is also taken into consideration and is characterized by the damping ratio \beta. In this complex variable notation \kappa_j represents the dimensionless spring coefficient and \epsilon_j represents the corresponding damping coefficient depending on \alpha and the Poisson’s ratio \nu. The real parts of the impedance functions signify force components in phase with the displacements and can be termed dynamic stiffness for the foundation. On the other hand, the imaginary parts are force components in phase with the velocities and can be interpreted as energy dissipation by radiation of waves away from the foundation into the soil. Therefore, they may be termed foundation damping coefficients.

Static stiffness coefficients may be defined as \[K_{S\infty j}\] for each degree of freedom – or mode – (for indices: hx and hy, horizontal displacements along the x and y axes, and for indices rx, ry, and t, rotations along the x, y, and z axes, respectively). It is noted that \[K_{hx} = K_{hy} = 8Gr/(2-\nu)\] is the static horizontal force necessary to produce a unit horizontal displace-
ment of the disk with no restriction on the value of the resulting rotation, \( K_{\gamma_1} = K_{\gamma_2} = 8G\gamma^3/3(1-\nu) \) is the static rocking moment necessary to rotate the disk through a unit angle with no restriction on the value of the resulting horizontal displacement, \( K_1 = 16\gamma^2\rho r^3/3 \) is the static torque necessary to twist the disk through a unit angle where \( \rho \) denotes the mass density of soil. The SSI relationships given by Eq. (8) were developed for a massless circular rigid disk. In this study, the research has been applied to a rectangular foundation in an approximate way by use of equivalent values for the radius of the rigid disk \( r_o \) such that the resulting static stiffness coefficients are the same as those corresponding to the rectangular foundation proposed by Thomson and Kobori (1963).

**Modal Analysis**

Based on the assumption that the fixed-base structure possesses classical normal modes, the structural response in the complex frequency domain can be expressed in terms of the mode shapes as

\[
\tilde{\mathbf{u}}(\omega) = \left\{ \tilde{u}_x(\omega), \tilde{u}_y(\omega), \tilde{u}_\theta(\omega) \right\} = [\phi]\{\tilde{q}(\omega)\}
\]  

(12)

where \( \{\tilde{q}(\omega)\} \) is the column vector of the Fourier transform of the modal displacements and \( [\phi] \) is the matrix consisting of normal mode shapes of the fixed-base structure as

\[
\{\tilde{q}(\omega)\} = \begin{bmatrix}
\tilde{q}_1(\omega) \\
\tilde{q}_2(\omega) \\
\vdots \\
\tilde{q}_{3N}(\omega)
\end{bmatrix}
\]

(3N x 1)

(13)

where

\[
[\phi] = \begin{bmatrix}
[\phi_{xx}] & [\phi_{xy}] & [\phi_{x\theta}] \\
[\phi_{yx}] & [\phi_{yy}] & [\phi_{y\theta}] \\
[\phi_{xx}] & [\phi_{xy}] & [\phi_{x\theta}]
\end{bmatrix}
\]

(3N x 3N)

Substituting Eq. (12) into \( \left[ \tilde{K}(\omega) \right]\left\{ \tilde{U}(\omega) \right\} = \left\{ \tilde{P}(\omega) \right\} \) equations of motion of a building system with a rigid foundation in the frequency domain and applying the orthogonality conditions, Eq. (8), when the first row is premultiplied by \( [\phi]^T \), introduces

\[
\left[ \tilde{K}_H(\omega) \right]\{\tilde{U}_H(\omega)\} = \{\tilde{P}_H(\omega)\}
\]

(14a)

and also note that

\[
\left[ \tilde{K}_H(\omega) \right] = \begin{bmatrix}
[H(\omega)] & -\omega^2[I] \\
-\omega^2[I]^T & \tilde{S}(\omega) - \omega^2[M_2]
\end{bmatrix}
\]

(14b)

where \( [I] \) represents the modal participation matrix defined as

\[
[I] = [\phi]^T [M][\phi]
\]

(15)

A diagonal matrix whose elements of modal structural transfer functions and typical element corresponding to the \( k^{th} \) mode are written as

\[
[H(\omega)] = \begin{bmatrix}
H_1(\omega) & 0 & 0 \\
0 & H_2(\omega) & 0 \\
0 & 0 & H_{3N}(\omega)
\end{bmatrix}_{3N x 3N}
\]

(16)

\[
H_k(\omega) = -\omega^2 + 2i\chi_k\omega + \omega_k^2 \quad k = 1, 2, 3 \ldots \ldots \ldots 3N
\]

The modal displacement of the structure can then be written in terms of foundation deformations from the first row of Eq. (14a) as given below; the frequency variation of the configuration vector can be subsequently computed, by summing up modal responses, as

\[
\{\tilde{q}(\omega)\} = \omega^2[H(\omega)]^{-1}[I]\{\tilde{u}_{\text{rot}}(\omega)\}
\]

(17)

Substituting Eq. (17) in the second row of Eq. (14b), the foundation displacement components can be obtained from the following equation:
{\tilde{u}_{sf}(\omega)} = \left[\tilde{D}(\omega)\right]^{-1}\left[\tilde{S}(\omega)\right]\{\tilde{u}_g(\omega)\} = 0 \quad \text{(18)}

in which the total dynamic stiffness matrix, \(\tilde{D}(\omega)\), is expressed as

\[
\tilde{D}(\omega) = \left[\tilde{S}(\omega)\right] - \omega^2([M_2] + \omega^2[\Gamma]^T[H(\omega)][\Gamma]) \quad \text{(19)}
\]

After Eq. (18) is solved for the foundation displacement vector, the modal structural response in the complex frequency domain can be calculated according to Eq. (17). Finally, the structural displacements of each floor in the frequency domain can be calculated by the mode superposition relations as

\[
\{\tilde{u}(\omega)\} = \begin{bmatrix} \tilde{u}_x(\omega) \\ \tilde{u}_y(\omega) \\ \tilde{u}_\theta(\omega) \end{bmatrix} = [\phi]\{q(\omega)\} \quad \text{(20)}
\]

The time-domain solution can be obtained through the inverse Fourier transform of Eq. (21) as follows:

\[
\{u(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{\tilde{u}(\omega)\} e^{i\omega t} d\omega \quad \text{(21)}
\]

With the above computations, a simplified analysis method performed by discrete Fourier transform techniques using the fast Fourier transform algorithm based on modal decomposition with all vibration modes to solve the interaction problem may be summarized in the following analysis procedure:

Defining the structural and soil properties with an earthquake ground acceleration record,

**Step 1.** Modal analysis on the superstructure;

a) Solve the eigenvalue problem, determine the natural frequency and mode shapes.

\[
[M], [K] \Rightarrow \text{diag}[\omega_k^2], [\phi](k = 1, \ldots, 3N).
\]

b) Evaluate the kinematic transfer matrix from Eq. (3) \(\Rightarrow [T]\).

c) Set up total foundation mass matrices from Eq. (6) \(\Rightarrow [M_1], [M_2]\).

d) Determine the modal participation matrix from Eq. (15) \(\Rightarrow [\Gamma]\).

**Step 2.** Frequency domain analysis at each frequency \(\omega\):

a) Fourier transform of ground acceleration, \([\tilde{u}_g(t)] \Rightarrow [\tilde{u}_g(\omega)]\).

b) Determine the modal structural transfer functions from Eq. (16) \(\Rightarrow [\tilde{H}(\omega)]\).

c) Compute the static soil spring constants \(K_{s\infty j}\) (for \(j = h_x, h_y, r_x, r_y\) and \(t\)).

d) Define the foundation impedance functions depending on soil properties \((\rho, v, \beta, \text{and } G)\) from Eq. (11) \(\Rightarrow K_j = K_{s\infty j}(k_j(a_o) + ia_ow_j(a_o))(1 + 2i\beta)\).

e) Set up a total dynamic stiffness matrix from Eq. (19) \(\Rightarrow \tilde{D}(\omega)\).

f) Calculate the translational, rotational and torsional displacements of the foundation from Eq. (18) \(\Rightarrow \{\tilde{u}_{sf}(\omega)\}\).

g) Compute the modal displacements for each floor from Eq. (17) \(\Rightarrow \{\tilde{u}(\omega)\}\).

h) Define the structural displacements from Eq. (20) \(\Rightarrow \{\tilde{u}(\omega)\}\).

**Step 3.** Time domain analysis;

a) Determine the displacement response by using the inverse Fourier transform from Eq. (21) \(\Rightarrow \{u(t)\}\).

b) Calculate the design story shear forces.

c) Design of members.

It is clear that the above design procedure is similar to that for the analysis of a fixed base structure with the addition of steps 2c-2f and the alteration of step 2g to include the deformability of the soil medium.
Numerical Analysis

The application of the proposed method of analysis including all modes of vibration in the calculations to evaluate the dynamic SSI effects on the 3-dimensional buildings with eccentricity in only one direction is considered. The configuration of the soil-structure system is shown in Figure 2. The main parameters of the dynamical structure foundation model and properties of the soil are summarized in this figure. The monosymmetric 2-story building resting on homogeneous elastic soil through a rigid square foundation consists of reinforced concrete frames joined at each floor level by a rigid diaphragm. The floors, including the foundation, were assumed to be identically rectangular, with the length of the building $l = 12$ m and the width of the building $b = 12$ m. In this example, the center of stiffness is assumed to lie at eccentricity $e = 1.0$ m from the center of mass. It has been assumed that the floor slab and the foundation mat have the same eccentricity along the $x$ axis in a direction perpendicular to the input motion, and no eccentricity along the $y$ axis. The mass $m_i = 50$ t and torsional radius of gyration $r_i = 5$ m at each floor level $(i = 1, 2)$ are the same. The rigid foundation mat is idealized as a circular plate of radius $r_o = 5$ m and its mass also taken to be $m_o = 50$ t. It should be noted that the radius of the base mass is taken as the radius of a circle having the same area as the plane of the floor.

It is considered that the structure has the same translational stiffness in the $x$ and $y$ directions, and that the SSI translational and rocking stiffnesses are respectively the same in both horizontal directions. The story stiffnesses of these frames are $K_{BiA_i}$, $K_{B_iG_i}$ and $K_{B_iB_i} = 26500$ kN/m, and the story heights are $h_i = 3$ m, also the same for all floors $(i = 1, 2)$. The damping ratio of the superstructure in each mode of vibration is taken as 5%.

The viscoelastic foundation medium is assumed to have a density of $20$ kN/m$^3$ and a Poisson’s ratio of $\nu = 0.33$, and the material damping ratio is $\beta = 0.05$. To indicate the significance of SSI effects on structural response, the shear wave velocity ($c_s$) of elastic half-space material was selected in a range of $300 \leq c_s \leq 1500$ m/s. The upper limit of $1500$ m/s for the shear wave velocity of the soil may be assumed to refer to a very stiff ground condition (fixed base). The lower limit of $300$ m/s for the shear velocity is chosen for defining soft soil conditions.

For a parametric study, the key parameters controlling the dynamic structural response including the effects of foundation interaction are chosen as the ratio, $m/m_o$, of the superstructure mass to the mass of the foundation, the ratio, $H/r$, of the total height of the structure to the radius of the foundation base taken here as the same as the floor slabs, and the ratio, $e/r$, of the eccentricity to the radius of the foundation base. The practical range of variables considered in this study is taken as $0.5 \leq m/m_o \leq 3$ for the mass ratio, $0.05 \leq e/r \leq 0.25$ for the eccentricity ratio, and $0.5 \leq H/r \leq 2$ for the height ratio.

The responses of this building foundation system to the torsional effects were obtained when it was subjected to the Erzincan, 1992, earthquake (E-W component, $M = 6.8$) as shown in Figure 3 with peak acceleration of 0.5 g as the free field ground motion in a direction perpendicular to the direction of the eccentricity.

In Figure 4, the variation of lateral deflection, rocking and twisting (torsional) components of the foundation base for different shear wave velocities of soil are plotted as a function of time and compared to the corresponding values for a fixed base structure. The maximum values of the foundation displacements for Erzincan, 1992, excitations are $u_o = 0.028$ m, $\gamma_o = 8.85 \times 10^{-3}$rad and $\theta_o = 4.25 \times 10^{-3}$rad for a shear velocity of $c_s = 300$ m/s (soft soil condition) in the case of intermediate high buildings with moderate eccentricity where $m/m_o = 1$, $H/r = 1.2$, and $e/r = 0.2$. However, the corresponding values are $u_o = 0.0025$ m, $\gamma_o = 9 \times 10^{-4}$rad and $\theta_o = 4.2 \times 10^{-4}$rad for relatively rigid based buildings, in which case the structure is considered to be supported by the soil with a shear wave velocity of $c_s = 1500$ m/s, as shown in Figure 4. It may be noted that the deformability of the foundation soil significantly influences the foundation displacements. Therefore, the SSI effects on the structural response are shown to be more important when the soil becomes softer, in soil with a lower value of $c_s$.

Furthermore, in order to demonstrate the application of this modal superposition method to practical problems, a detailed parametric analysis of the same idealized 3-dimensional model has been solved with various controlling parameters $(m/m_o, H/r$, and $e/r)$ to estimate the dynamic behavior of the soil structure system for 2 ground conditions, for stiffer soil conditions, $c_s = 1500$ m/s, and for softer soil conditions, $c_s = 500$ m/s, as shown in Figures 5-7, respectively.
Soil Parameters:

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>0.33</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho$</td>
<td>20 kN/m$^3$</td>
</tr>
<tr>
<td>Material damping</td>
<td>$\beta$</td>
<td>0.05</td>
</tr>
<tr>
<td>Shear velocity</td>
<td>$c_s$</td>
<td>1500, 1000, 700 m/s</td>
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<tr>
<td></td>
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<td></td>
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</table>

Foundation Parameters:

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$m_o$</td>
<td>50 t</td>
</tr>
<tr>
<td>Length &amp; width</td>
<td>$b, d$</td>
<td>12 m, 12 m</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$I_o$</td>
<td>120 t$m^2$</td>
</tr>
<tr>
<td>Ground motion record</td>
<td>$g_u(t)$</td>
<td>Erzincan 1992 (E-W component)</td>
</tr>
</tbody>
</table>

Structural Parameters:

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol (i=1,2)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor mass</td>
<td>$m_i$</td>
<td>50 t</td>
</tr>
<tr>
<td>Frame stiffness</td>
<td>$K_{yi}, K_{xi}, K_{xb}$</td>
<td>26.5 MN/m</td>
</tr>
<tr>
<td>Foundation to story height</td>
<td>$h_i$</td>
<td>3 m × i</td>
</tr>
<tr>
<td>Modal damping ratio</td>
<td>$\xi$</td>
<td>0.05</td>
</tr>
<tr>
<td>Floor moment of inertia</td>
<td>$I_i$</td>
<td>120 t$m^2$</td>
</tr>
<tr>
<td>Elasticity modulus</td>
<td>$E$</td>
<td>28.5 GPa</td>
</tr>
</tbody>
</table>

Figure 2. Configuration of 2-story building and model parameters for numerical examples.
Figure 3. Time history acceleration of free-field input motion at the surface.

Figure 4. Variation of the displacement components of the foundation base depending shear velocity in the case of $m/m_o = 1$, $H/r = 1.2$ and $e/r = 0.2$.

Figure 5. Variation of the lateral displacement components of foundation base depending mass ratio for shear wave velocity of 1500 m/s and 500 m/s in the case of $H/r = 1.2$ and $e/r = 0.2$.

Figure 6. Variation of the rocking displacement components of the foundation base depending on height ratio for a shear wave velocity of 1500 m/s and 500 m/s in the case of $m/m_o = 1$ and $e/r = 0.2$. 
Figure 7. Variation of the torsional displacement components of foundation base depending on the eccentricity ratio for a shear wave velocity of 1500 m/s and 500 m/s in the case of m/m_o = 1 and H/r = 1.2.

Four different values of height to gyration radius ratio (h/r), eccentricity to gyration radius ratio (e/r), and mass of the structure to mass of the foundation ratio (m/m_o), depending on the shear wave velocity of the soil medium, are considered.

It should be noted that an apparent increase occurs in the foundation displacement components as the shear wave velocity of the soil medium decreases. From those time-domain responses, it is observed that in rigid and soft soil conditions, an obvious increase occurs in the horizontal displacement of the foundation as the mass ratio, m/m_o increases, as shown in Figure 5. For instance, the maximum value of the foundation lateral displacement is 2.25 x 10^-3 m in the case of m/m_o = 1 for the rigid foundation medium, whereas the corresponding value is 9.35 x 10^-3 m in the case of m/m_o = 3. In this situation, the foundation mass is assumed to be reasonably small compared to the mass of the superstructure.

Additionally, the lateral displacement of the foundation based on soil with c_s = 500 m/s increases to almost 200 times the lateral displacement of the foundation based on soil with a shear velocity of c_s = 1500 m/s, for the ratio of m/m_o = 3. Furthermore, the rocking rotation of the foundation is affected by any change in the value of the height ratio, H/r, especially when the soil is softer, as shown in Figure 6. It modifies the rocking rotation of the foundation, decreases for short, squat structures and increases for tall, slender structures. The smaller the value of c_s and the greater the value of the height ratio, H/r, the more pronounced the interaction effects become on foundation rotation. For instance, for H/r = 2 and c_s = 1500 m/s, the peak of the rotation angle is approximately 0.022 rad, whereas for c_s = 500 m/s this value is 0.25 rad. For squat structures, these rocking displacements decrease by up to 1.5 x 10^-3 rad and 0.022 rad for rigid and soft soil conditions, respectively.

From Figure 7 it can be seen that the peak values of torsional displacement of the foundation increased significantly in the case of intermediate high buildings with a moderate mass ratio for low soil foundation stiffness when compared to those obtained for rigidly based structures as the eccentricity ratio, e/r, increases. The maximum torsional rotations of the foundation obtained in this analysis are approximately 0.00725 rad and 0.0385 rad for systems having a shear wave velocity of soil with c_s = 1500 m/s and c_s = 500 m/s, respectively, when the eccentricity ratio is taken as e/r = 0.25. Furthermore, it should be noted that when the eccentricity ratio increased from 0.2 to 0.25, the torsional rotations also increased by 2.5 times and 4 times for buildings founded on stiff and soft soils, respectively.

Conclusions

In this study a simplified method of analysis is presented using modal decomposition considering all modes of vibration in the calculation to obtain the structural responses of torsionally asymmetric buildings, including soil-structure interaction effects in the frequency domain, by using the fast Fourier transform. Applying this algorithm, the advantage of classical normal mode approximation is used for almost all vibration modes, and the interaction problem is solved easily and effectively within the framework of the Fourier-transformed frequency domain analysis for a fixed base structure. The matrix formulation of this method produces accurate approximation with less computational effort, despite using frequency dependent impedance functions.

The results presented in this paper indicate that
the earthquake response of a soil-torsionally coupled structure interaction can be significantly different from that calculated with a fixed base model. This work has been based on the assumptions that the foundation is supported at the surface of a viscoelastic half-space and that the superstructure responds within the elastic range. From these time-domain responses, it has been shown that an obvious increase occurs in the horizontal and rocking displacements of the foundation as the predefined mass ratio and height ratio increase, respectively, for soft soil conditions compared to the associated fixed-base system.

This detailed parametric study shows that the effects of deformability of the foundation medium cause the dynamic behavior of the structure foundation system to differ, particularly when the shear wave velocity representing the soil stiffness is lower than 1000 m/s.

For a medium highrise building with moderate or large eccentricity, increased torsional loadings must be taken into account for the combined lateral- or large eccentricity, increased torsional loadings must be taken into account for the combined lateral-torsional response for such structures supported on a moderately and very flexible foundation medium.

Nomenclature

Roman Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_o$</td>
<td>Reissner parameter</td>
</tr>
<tr>
<td>$c_j$</td>
<td>foundation damping coefficients for each degree of freedom - or the mode j</td>
</tr>
<tr>
<td>$c_s$</td>
<td>velocity of shear wave in the soil</td>
</tr>
<tr>
<td>$[C]$</td>
<td>viscous damping matrix of structure</td>
</tr>
<tr>
<td>$[\tilde{D}(\omega)]$</td>
<td>total dynamic stiffness matrix</td>
</tr>
<tr>
<td>$e$</td>
<td>structural (static) eccentricity</td>
</tr>
<tr>
<td>$G$</td>
<td>shear modulus of soil</td>
</tr>
<tr>
<td>$[h]$</td>
<td>column vector of heights</td>
</tr>
<tr>
<td>$[H(\omega)]$</td>
<td>modal structural transfer functions</td>
</tr>
<tr>
<td>$I_{tx}, I_{ty}$</td>
<td>the total of the mass moment of inertia of the floors and the foundation mat with respect to the x and y axes</td>
</tr>
<tr>
<td>$k_j$</td>
<td>dimensionless spring coefficients for mode j</td>
</tr>
<tr>
<td>$K_{hx}, K_{hy}$</td>
<td>static translational stiffness of foundation in the x and y directions, respectively</td>
</tr>
<tr>
<td>$K_{\gamma rz}, K_{\gamma ry}$</td>
<td>static rocking moment of foundation about the x and y axes, respectively</td>
</tr>
<tr>
<td>$K_t$</td>
<td>static torque of foundation</td>
</tr>
<tr>
<td>$K_j$</td>
<td>foundation impedance functions for the mode j</td>
</tr>
<tr>
<td>$K_{S\infty j}$</td>
<td>static stiffness for the half-space for the mode j</td>
</tr>
<tr>
<td>$[K]$</td>
<td>stiffness matrix of structure</td>
</tr>
<tr>
<td>$m_i, m_o$</td>
<td>floor and foundation mass</td>
</tr>
<tr>
<td>$M_{oz}(t), M_{oy}(t)$</td>
<td>overturning moments at foundation-soil interface in the x and y directions</td>
</tr>
<tr>
<td>$[m_x], [m_y]$</td>
<td>mass matrices consisting of floor masses with respect to the x and y axes</td>
</tr>
<tr>
<td>$[m_\theta]$</td>
<td>mass polar moment of inertia, about the z axis of the floor mass</td>
</tr>
<tr>
<td>$[M]$</td>
<td>mass matrix of structure</td>
</tr>
<tr>
<td>$P_{oz}(t), P_{oy}(t)$</td>
<td>horizontal soil-structure interaction forces in the x and y directions, respectively</td>
</tr>
<tr>
<td>${\hat{P}(\omega)}$</td>
<td>frequency dependent external force vector</td>
</tr>
<tr>
<td>${\hat{q}(\omega)}$</td>
<td>column vector of Fourier transform of the modal displacements</td>
</tr>
<tr>
<td>$r, r_o$</td>
<td>radius of floor and foundation (base) disk, respectively</td>
</tr>
<tr>
<td>$[\tilde{S}(\omega)]$</td>
<td>dynamic-stiffness matrix of the foundation defined as the ratio of the amplitude of the applied load ${\hat{P}(\omega)}$</td>
</tr>
<tr>
<td>$T_o(t)$</td>
<td>torsional soil-structure interaction moment</td>
</tr>
<tr>
<td>$[T]$</td>
<td>kinematic transfer matrix</td>
</tr>
<tr>
<td>$u_{xi}, u_{yi}$</td>
<td>horizontal displacements of ith floor in the x and y directions, respectively</td>
</tr>
<tr>
<td>$u_{\theta i}$</td>
<td>rotation about vertical axis</td>
</tr>
<tr>
<td>$u_{oz}, u_{oy}$</td>
<td>horizontal translations of the foundation with respect to the x and y axes</td>
</tr>
<tr>
<td>$u_{gx}(t), u_{gy}(t)$</td>
<td>free field ground acceleration measured at the surface</td>
</tr>
<tr>
<td>${u(t)}$</td>
<td>column vectors of the structural displacements relative to the rigid foundation</td>
</tr>
<tr>
<td>${\tilde{u}(\omega)}$</td>
<td>Fourier transform of the ${u(t)}$</td>
</tr>
<tr>
<td>${\tilde{u}_{\theta i}(t)}$</td>
<td>the rotational acceleration of the base of the building about the vertical axis</td>
</tr>
<tr>
<td>${\tilde{u}_{\theta i}(t)}$</td>
<td>the acceleration vector of the rigid foundation motion</td>
</tr>
<tr>
<td>${\tilde{u}_{\theta i}(\omega)}$</td>
<td>Fourier transform of the total displacement vector of the rigid foundation in addition to the free field motion</td>
</tr>
</tbody>
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Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>internal damping ratio of soil</td>
</tr>
<tr>
<td>$[\phi]$</td>
<td>normal mode matrix</td>
</tr>
</tbody>
</table>
γ_{ox}, γ_{oy} rocking rotations of the foundation in the directions of the x and y axes, respectively

\ddot{\gamma}_{gx}(t), \ddot{\gamma}_{gy}(t) the rotational acceleration of the base of the building along horizontal axes

[Γ] modal participation matrix

References


