Comparison of Methods for Predicting Discharge in Straight Compound Channels Using the Apparent Shear Stress Concept

Tülay ÖZBEK, Müsteyde B. KOÇYİĞİT, Önder KOÇYİĞİT
Gazi University, Faculty of Engineering and Architecture,
Civil Engineering Department, Maltepe, Ankara-TURKEY
e-mail: tozbek@gazi.edu.tr

Kagan CEBE
Barsu Ltd. Şti., Çetineçmez Blv., 8. Cad. 10/5, Ankara-TURKEY

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Abstract
Some experimental results from the SERC Flood Channel Facility at Hydraulic Research, Wallingford, UK, were used for computing apparent shear stress and discharge in symmetrical compound channels with varying floodplain widths. Three assumed interface planes (vertical, horizontal and diagonal) between the main channel and the floodplain sub-sections were considered. The apparent shear stresses across those interfaces were computed and the ratios of these stresses to the average main channel shear stresses were determined. Then discharge values in the sub-sections and in the whole cross-section were evaluated. The results showed that the performance of these computation methods depends on their ability to accurately predict apparent shear stress. Diagonal and horizontal division methods provided better results than the vertical division method, with the diagonal method giving the most satisfactory results.

Key words: Compound channels, Division channel method, SERC-FCF, Apparent shear stress.

Introduction
During recent decades, there has been considerable interest in flood channel research parallel to the considerable damage caused by floods and the increased awareness of environmental issues. It has thus become crucial for water authority engineers engaged in river engineering to find economical and hydraulically efficient solutions to engineering problems and to ensure that these solutions are environmentally sensitive and sustainable and ensure flood protection.

A major area of uncertainty in river channel analysis is that of accurately predicting the capability of river channels with floodplains, which are termed compound channels. Cross-sections of these compound channels are generally characterised by a deep main channel, bounded on one or both sides by a relatively shallow floodplain, which is rougher, often vegetated and has slower velocities than the main channel.

At low depths, when the flow is only in the main channel, conventional methods are used to assess discharge capacity. However, when overbank flow occurs, for instance for a river in flood, the classical formulae for discharge capacity estimation do not yield reliable solutions and may lead either to overestimation of discharge capacity, which is dangerous, or to underestimation of capacity, which may cause waste of resources. This problem has led to a thorough investigation of the flow mechanism in compound channels, leading to studies involving either improvement of the classical discharge estimation methods, or development of new computational methods for an accurate prediction of discharge capacity.

After a brief review of the methods proposed for flow computation in compound channels by various authors, this paper concentrates on the divided chan-
nel method (DCM), in which the compound cross-
section is divided into relatively large homogeneous sub-areas. Vertical (V), horizontal (H) and diagonal (D) imaginary interface planes were considered between the main channel and the floodplain subsections. Some experimental results of symmetrical compound channels with varying floodplain widths from the SERC Flood Channel Facility at Hydraulic Research, Wallingford, UK, were used for computing discharge and apparent shear stresses on these assumed interfaces. The ratios of these apparent shear stresses to the average main channel and boundary shear stresses were computed and, based on these ratios and discharge computations, the performance of the methods was investigated.

**Review**

Over the years, considerable research has been undertaken to investigate flow in compound channels, aimed at understanding the structure of flow and at the development of accurate methods of discharge estimation. When overbank flow occurs, the flow in the main channel is considerably faster than that in the floodplain(s). These large velocity gradients occurring at the main channel/floodplain interface create turbulence and result in momentum transfer from the deeper faster flowing main channel to shallower slower flowing floodplain discharge. Thus, there is a momentum transfer mechanism between the main channel and floodplain, retarding velocity and discharge in the main channel and increasing the corresponding parameters on the floodplain. Furthermore, the roughness of floodplains often exceeds that in the main channel and thus significantly enhances the effect of the mechanism. This mechanism, which takes the form of a bank of vortices having vertical axes along the channel floodplain interface, was first recognised by Sellin (1964) and Zheleznyakov (1971), who demonstrated the presence of these vortices by experimental means. Since the recognition of the momentum transfer mechanism, a large number of studies, including theoretical work and small/large scaled laboratory experiments, have been carried out to investigate and quantify this mechanism, which also explains the reason for the failure of traditional methods of discharge estimation in compound channels. Many investigators, such as Myers and Elsawy (1975), Myers (1978), Knight and Demetriou (1983) and Wormleaton et al. (1982), Knight and Hamed (1984), performed laboratory experiments and measured boundary shear stress distributions to quantify the momentum transfer mechanism in terms of apparent shear force acting on the channel/floodplain interface. They showed that the apparent shear stress is many times greater than the average shear stress around the solid boundaries. Hence, various empirical relationships have been developed expressing the apparent shear stress as a function of wide ranges of geometry and flow variables (Myers et al., 1999).

A very important advance in the investigation of compound channel research is the SERC - Flood Channel Facility (SERC-FCF), a major experimental facility described by Knight and Sellin (1987). The SERC-FCF is a flume of length 56 m and width 10 m and involves a wide ranging experimental investigation to build up a bridge between small laboratory scale and full-sized rivers. The experimental programme of the FCF involves work on straight, skewed and meandering channels and investigates sediment transport problems. Details of the SERC-FCF and its associated instrumentation are presented by various authors such as Myers and Brennen (1990) and Wormleaton and Merret (1990).

Some field studies were also undertaken by various researchers (Knight et al., 1989; Martin and Myers, 1991) but fieldwork may prove difficult partly due to problems encountered during the acquisition of data and partly due to work conditions, which might be dangerous during flooding.

Investigators such as Wormleaton et al. (1982), Knight and Demetriou (1983), Prinos and Townsend (1984) Wormleaton and Hadjiyapoulos (1985) and Myers et al. (1987) stated that the application of conventional methods of discharge estimation results in large errors due to complex turbulent transfer at the main channel/floodplain interface. Martin and Myers (1991) stated that conventional methods of discharge estimation for compound river channels were shown to incur errors of up to ± 25%, confirming previous laboratory findings. A number of studies have also been undertaken, aimed at investigating flow resistance in compound channels, including those by Pasche and Rouve (1985), Prinos and Townsend (1985) and Myers and Brennen (1990). In addition to the extensive research of flow in rigid or fixed bed channels, a number of recent studies (Cassells et al., 2001; Knight and Brown, 2001; Lyness et al., 2001) have focused on discharge prediction in straight mobile bed compound channels, examining the impact of sediment movement in the main channel on the discharge capacity of the main channel and floodplain.
The simplest model of computing uniform flow in a compound channel is the single channel method (SCM), in which the channel is treated as a single unit with some appropriate averaging for the friction coefficient. In this SCM, the composite character of the channel is discarded and the velocity is assumed to be uniform in the whole cross-section. It has been shown by Myers and Brennen (1990) that with the application of this model the discharge capacity is significantly underestimated at low overbank flow depths due to the uniform velocity assumption.

The most commonly used method for calculating discharge in compound channels is the DCM, in which the compound cross-section is divided into hydraulically homogeneous sub-areas, as shown in Figure 1 (Wormleaton and Hadjipanos, 1985), in such a way that the velocity in each subsection can be assumed to be uniform. The division lines between the sub-sections can either be vertical, horizontal or diagonal, with the most common and practical choice being the vertical ones (Bousmar and Zech, 1999). These imaginary division interfaces were originally assumed to be shear-free and therefore were not included in the wetted perimeters of the adjacent subdivisions when discharge was computed. Using boundary shear stress measurements, Myers (1978) showed that these division planes were not shear-free, due to a turbulent interaction between the main channel and floodplain, and an apparent shear force must be present to produce a balance between the gravitational and boundary resistance forces.

Ackers (1991, 1992) proposed an approach using the traditional DCM with a vertical division plane and a large amount of previously published experimental data, with which he developed empirical correction coefficients he termed “discharge adjustment factors”. He applied these coefficients to discharge given by the DCM to correct for the momentum interaction effects.

Bretschneider and Ozbek (1997) proposed an empirical equation for the Darcy-Weissbach friction factor on the vertical interface plane using dimensional and regression analysis. They showed that the friction factor on the assumed vertical interface was affected by the ratio of the floodplain width to the main channel width as well as by $H / (H-h)$.

Lambert and Sellin (1996) carried out a similar study using DCM in which they used the mixing length approximation to calculate a correction factor for the momentum interaction effects. In their approach, they divided the compound channel into small vertical elements and used a mixing length hypothesis to describe the variation of the apparent lateral shear stress across the channel.

In another study, Lambert and Myers (1998) developed a new approach, termed the weighted divided channel method (WDCM), in which the location of the main channel and floodplain interface is variable and dependent upon a weighting coefficient ($\xi$), where it is used to give improved estimates of the mean flow velocity in both the main channel and the floodplain. The weighting factor varies between zero and unity and represents a range of channel subdivisions between the traditional vertical division ($\xi = 1$) and the horizontal division ($\xi = 0$).

Using experimental observations and data from a natural compound river channel, Myers et al. (2001) showed that the SCM significantly underestimates the compound discharge for low flow depths, but becomes more accurate at greater depths for the smooth boundary laboratory data and the river data.

Ozbek (1996, 2000) assumed a vertical interface at the main channel/floodplain interface and computed the apparent shear stress according to the DVWK approach presented in Merkblatter 220, which is used in Germany for estimating the discharge capacity of compound channels. Ozbek used experimental data from the SERC-FCF for symmetrical compound channels with floodplains of varying widths (Series 1, 2 and 3 of Phase A), for main channels with varying slopes (Series 2, 8 and 10) and for asymmetrical compound channels with only one floodplain. She found that the discharge estimation procedure given by DVWK-Merkblatter 220 gave satisfactory results when the ratio of floodplain width to the main channel width was equal to unity. She also found that for asymmetrical compound channels, the friction factor assumed to be present across the vertical division plane decreased dramatically when $H / (H-h) > 5$.

Figure 1. Compound channel cross-section with horizontal (H), diagonal (D) or vertical (V) planes shown as 1-1, 1-2 and 1-3, respectively (Wormleaton and Merrett, 1990).
They also found that the DCM exhibits reasonable accuracy when applied to laboratory data with a smooth floodplain, but shows significant errors of up to 35% for rough floodplain data, and up to 27% for river data.

**Theoretical background**

In this study, a compound channel is divided into a main channel and floodplains along some imaginary interface (V, H and D) as suggested by Wormleaton et al. (1982). Six methods using the vertical, horizontal and diagonal division planes were used for the estimation of discharge capacity. These methods either exclude the interface from the wetted perimeter of the adjacent sub-sections, assuming zero mean shear stress on the interface (referred to as the Ve, He and De methods), or include the interface in the wetted perimeter of the main channel sub-section accounting for the drag effect of the slower floodplains but exclude it from the floodplain, expressing an acceleration effect (referred to as Vi, Hi and Di methods) (Wormleaton et al., 1982).

The Universal equation was used to compute average velocity rather than Chezy or Manning-Strickler uniform flow equations due to the dimensionless friction factor $f$ of the equation. Once the individual discharges in the main channel and floodplain sub-sections for any assumed interface are computed, they are summed to obtain the total discharge of the compound channel.

**Computation of apparent shear stress**

Experimental data of Series 1, 2 and 3 of Phase A from the SERC-FCF include measurements of straight symmetric compound channels. Details of the compound geometry used in this study are given in the Table, where $m_c$ and $m_f$ are the slopes and $b_c$ and $b_f$ are the widths of the main channel and floodplain, respectively.

Using Vi, Hi and Di methods, in which the interface plane is included in the wetted perimeter of the main channel sub-section, the apparent shear stress on the assumed division interface plane was computed using the idea of a balance of forces within individual sub-sections together with the measured shear force values.

Using the Darcy-Weissbach uniform flow equation for velocity distribution, a theoretical average shear stress $\tau_{c,ave}$ around the total boundary of the main channel sub-division including the division plane interfaces can be written as

$$\tau_{c,ave} = f_{c,ave} \frac{\rho}{8} v_c^2$$  \hspace{1cm} (1)

where $f_{c,ave}$ is a theoretical average Darcy-Weissbach friction factor assumed to be present on the wetted perimeter of the main channel sub-section and $v_c$ is the mean velocity of the main channel. Similarly, the friction factor on an assumed division plane $i$, ($f_{app,i}$) (where $i = V$, H and D) and on the main channel solid boundary ($f_{c,bnd}$) can be expressed in terms of apparent shear stress on that division plane $\tau_{app,i}$ and boundary shear stress on the main channel $\tau_{c,bnd}$, which are given, respectively, as

$$\tau_{app,i} = f_{app,i} \frac{\rho}{8} v_C^2$$  \hspace{1cm} (2)

$$\tau_{c,bnd} = f_{c,bnd} \frac{\rho}{8} v_C^2$$  \hspace{1cm} (3)

Thus, the Darcy-Weissbach friction factor on any assumed division plane and on the wetted perimeter of the main channel could be determined using Eqs. (2) and (3), respectively, and the measured shear stress and shear force values from the SERC-FCF experiments.

The application of vertical, horizontal and diagonal division plane methods involved the assumption that the apparent shear stress on the division plane

<table>
<thead>
<tr>
<th>Series No.</th>
<th>$b_f$ (m)</th>
<th>$b_c$ (m)</th>
<th>$m_f$</th>
<th>$m_c$</th>
<th>$b_f/b_c$</th>
<th>$H$ (mm)</th>
<th>$h$ (mm)</th>
<th>$H/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>4.10</td>
<td>0.75</td>
<td>0</td>
<td>1</td>
<td>5.47</td>
<td>157.9</td>
<td>150</td>
<td>1.05</td>
</tr>
<tr>
<td>02</td>
<td>2.25</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>3.00</td>
<td>166.7</td>
<td>150</td>
<td>1.11</td>
</tr>
<tr>
<td>03</td>
<td>0.75</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>1.00</td>
<td>176.5</td>
<td>150</td>
<td>1.18</td>
</tr>
</tbody>
</table>

*Table.* Geometrical properties of the straight compound channel for Series 1, 2 and 3.
is either equal to zero or equivalent to the boundary shear stress on the main channel solid boundary \( \tau_{c,bnd} \). According to Eqs. (1)-(3), \( \tau_{app,i}/\tau_{c,ave} = f_{app,i}/f_{c,ave} \) and \( \tau_{app,i}/\tau_{c,bnd} = f_{app,i}/f_{c,bnd} \). Therefore, when \( \tau_{app,i}/\tau_{c,bnd} = f_{app,i}/f_{c,bnd} \), the apparent shear stress is zero, implying that the interface is ignored as a wetted perimeter. This case corresponds to the Ve, He and De methods. When the apparent shear stress is considered equal to the shear stress on the main channel solid boundary as \( \tau_{app,i}/\tau_{c,bnd} = f_{app,i}/f_{c,bnd} = 1 \), the friction factor on the assumed division plane is equal to that on the main channel solid boundary corresponding to the Vi, Hi and Di methods.

It should also be noted here that there are other approaches, in which the friction factor on the assumed division plane is not considered equal to that on the main channel solid boundary. One of such approaches is the DVWK (1991) approach, which is given as

\[
\text{For } \frac{H}{(H-h)} < 3 \quad f_{app,i} = f_{c,bnd} \tag{4a}
\]

\[
\text{For } \frac{H}{(H-h)} \geq 3 \quad f_{app,i} = 3f_{c,bnd} \tag{4b}
\]

where \( H \) = depth of flow in the main channel and \( h = \) depth of the main channel bed below the floodplain. Hence, the empirical friction factor assumed to be on the division interface plane is determined according to the depth ratio and the friction factor on the solid boundary of the main channel (Baduna, 1996; Ozbek, 1999).

Interpretation of results of apparent shear stress ratios

In this study, data from symmetrical compound sections with varying floodplain widths were used. The notations in Figure 1 were used and all of the geometries had the same main channel dimensions of \( b_c = 0.75 \) m and \( h = 0.15 \) m. The floodplain width ratios, \( b_f/b_c \), for geometries 1 to 3 are 5.47, 3 and 1, respectively. For all 3 series, the depth ratio, \( H/(H-h_f) \), varies in the range \( 1 < H/(H-h_f) < 1.7 \). Geometries 1 to 3 have smooth floodplains with a cement mortar finish similar to the main channel.

Figure 2 shows the variation of the friction factor on the vertical interface to the friction factor on the main channel boundary, \( f_{app,V}/f_{c,ave} \), which is also equal to the ratio of apparent shear stress across the vertical interface plane to the average main channel shear stress, \( \tau_{app,V}/\tau_{c,ave} \). With depth ratio \( H/(H-h_f) \) for Series 1 to 3. It can clearly be seen from the figure that the friction factor \( f_{app,V} \) on the vertical interface plane is much larger than that on the main channel wetted perimeter at low water depths. In other words, the apparent shear stress on the vertical interface \( \tau_{app,V} \) is much larger than the average main channel shear stress expressing the momentum transfer at the main channel/floodplain interface. As shown in Figure 2, in all 3 cases the ratio \( f_{app,V}/f_{c,ave} \) increases with decreases in the depth the ratio and becomes much larger than unity. This thus proves that the assumptions of apparent shear stress being equal to zero or equal to the average main channel shear stress are not valid. This also shows that the momentum transfer effect is much larger than that considered in conventional methods used for discharge estimation. The apparent shear stress on the vertical interface greatly depends on the velocity difference between the main channel and the floodplains \( \Delta v \), depth ratio \( H/(H-h_f) \) and floodplain width ratio \( b_f/b_c \) (Ozbek, 1996).

According to these results, 2 basic approaches can be suggested for discharge estimation. The first is to use the vertical interface and to enable the inclusion of a large apparent shear stress for the calculation of discharge, and the second is to use a different division plane, such as horizontal or diagonal planes, on which either the apparent shear stress is either zero or equal to the average main channel shear stress (Wormleaton et al., 1982).
Similar graphics are plotted for diagonal and horizontal division plane methods (Figures 3 and 4, respectively). It can be seen that the apparent shear stress on the diagonal and horizontal interfaces is much smaller than that across the vertical interface. Furthermore, as the depth over the floodplain increases, the apparent shear stress is reduced and eventually becomes negative. This negative value indicates that there is a momentum transfer from the part of the main channel section above the level of the interface to the part below it. A similar result was also observed by Wormleaton et al. (1982).

It should also be noted here that although similar results and conclusions were obtained by Wormleaton et al. (1982), they were not illustrated on the same graphics in this study. This is because of differences in the experimental channels used in both studies, in which the SERC-FCF was a very large-scale experimental facility, while the laboratory scale of the experimental channel used by Wormleaton et al. (1982) was small. Furthermore, channel properties, such as the bed slope and the roughness coefficient, and the ranges of the relative bed width ratio and the relative water depth ratio for both channels were different.

Computation of discharge

Experimental discharge values for the main channel and floodplain sub-sections were easily determined from the measured discharge data for the vertical division method. However, in the case of the horizontal and diagonal division methods, in order to obtain discharges in the whole cross-section or within the individual sub-sections the point velocity readings, which were recorded at 10 mm vertical, and 100 mm values are summed to determine the capacity of the whole cross-section. Instead of using the widely applied uniform flow equation like Chezy and Manning-Strickler, the Universal equation was applied to compute the average velocity of the sub-sections in order to avoid errors induced by the dimensional coefficients of other uniform flow equations. The Universal equation is derived from the Prandtl-Colebrook approach, which can be defined as an application of the Darcy-Weissbach equation to open channel hydraulics. The difference between the Universal equation and the Prandtl-Colebrook approach is the shape coefficients introduced in the equation to correct velocity distribution in compact cross-sections. Bolrich and Preisler (1992) suggested that for a trapezoidal cross-section the shape coefficient takes the value of 2.90 and 3.16 for smooth and rough surfaces, respectively. This approach might also introduce some error but it is thought that this error is much smaller than that introduced through other dimensional coefficients. Hence, the average velocity of a sub-section is computed by

\[ v_C = -4 \log \left( \frac{SF_{\text{smooth}} \cdot \nu}{8R \sqrt{2gRI}} + \frac{k/R}{4SF_{\text{rough}}} \right) \sqrt{2gRI} \]  

where \( R \) is the hydraulic radius of that cross-section, \( k \) is the roughness length and \( SF \) is the shape factor.

Once the average velocity was determined, the discharge value of that sub-section was computed by multiplying the area of the sub-section by the computed average velocity within that section. Then comparisons of the computed and the measured discharge values were made by using the error ratio given as
where $Q^{\text{comp}}$ is the computed discharge and $Q^{\text{meas}}$ is the measured discharge of a sub-section or whole cross-section.

In Figures 5, 6 and 7 error percentages for the overall discharge computed by the Universal equation are given for Series 1, 2 and 3, respectively. It can be seen that in the case of the Vi method, total discharge in the main channel section is estimated more accurately than that in the floodplain section and is better when compared with the Ve method. The Vi method gave the best predictions for Series 3 where the floodplain width ratio $b_f/b_c = 1$.

For all 3 series considered, Hi and He methods underestimated the discharge capacity of the compound channel and proved that it should not be used for the discharge estimation of smooth compound channels. Further details of the computation method and findings can be found in Cebe (2002).

Conclusions

A series of experimental data from the SERC-FCF was used for computing the apparent shear stress in straight symmetrical compound channels with varying floodplain widths. Three assumed interface planes (V, H and D) were considered and the performances of 6 traditional discharge estimation methods, Ve, He, De, Vi, Hi and Di, were compared.

An empirical Darcy-Weissbach friction factor, equal to the friction factor on the main channel solid boundary, was used to quantify the momentum transfer at the interaction of the main channel/floodplain interface. The apparent shear stress on the assumed interfaces was thus estimated. Once the apparent shear stress across the vertical, horizontal and diagonal interface planes was computed, the ratios of these stresses to the average main channel shear stress were determined. It was found that the apparent shear stress ratios for horizontal and diagonal interface planes were much smaller than that for the vertical plane. It can therefore be concluded that horizontal and diagonal interface planes are preferable to the vertical planes for discharge calculations.

The De method might be regarded as the most realistic approach according to the apparent shear stress ratios. Although the Vi method gave much smaller apparent shear stress than expected, the er-
The apparent shear stress ratios showed that the Hi and Di methods gave unrealistic results and should not be used for discharge computation. In the case of the vertical interface plane, the apparent shear stress is much larger than the average main channel shear stress. It was thus concluded that the assumption of an average friction factor as used in the Vi method is not a correct approach. However, the reason for the small error percentages for overall discharge, despite the apparent shear stress computations being wrong for the Vi method, can be explained by the small perimeter of the interface planes (Wormleaton et al., 1982).

The reason for the unexpected error percentages in overall discharge values for the De method was the longer interface plane and the larger effect on the discharge calculation of even a very small error in the computation of apparent shear stress. Overall, it can be concluded that the De method is more practical and gives better results when apparent shear stress is neglected for all 3 $b_f/b_c$ ratios.

### Nomenclature

- $H$: depth of flow in main channel
- $h$: depth of main channel bed below floodplain
- $hf$: depth of flow in the floodplain
- $b_c, b_f$: width of main channel bed and floodplain, respectively
- $\tau_{c,ave}$: average main channel shear stress
- $\tau_{c,bnd}$: main channel solid boundary shear stress
- $\tau_{app,i}$: apparent shear stress on unspecified interface $i$
- $\tau_{app,V}$: apparent shear stress on vertical interface
- $v_c$: main channel velocity
- $\nu_*$: shear stress velocity
- $f_{app,i}$: friction factor for unspecified $i$ interface
- $f_{c,bnd}$: friction factor on the main channel solid boundary
- $f_{c,ave}$: friction factor on the main channel including interface length
- $f_{app,V}$: friction factor on the vertical interface plane
- $\rho$: density of water
- $\Delta v$: velocity difference between main channel and floodplain

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