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Elastic-Plastic Deformation of a Tube with Free Ends Subjected to Internal Energy Generation

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Abstract

The elastic-plastic deformation of a tube with free ends subjected to uniform internal energy generation is evaluated using Tresca's yield condition and the associated flow rule. Yielding starts at the outer boundary and later another plastic region develops at the center, both of them corresponding to an 'edge regime' of Tresca's prism. The expansion of the plastic regions with increasing thermal load and the distribution of stresses and plastic strains are displayed graphically and discussed.

Key Words: Elastic-plastic, thermal stresses, internal energy, tube.

İç Enerji Üretimli Uçları Serbest Tüpte Elastik-Plastik Deformasyon

Özet

Bu çalışmada, iç enerji üretimi altında, uçları serbest tüpteki elastik-plastik deformasyon Tresca akma kriteri ve ilgili akma kuralı kullanarak incelenmiştir. Plastik akma dış yüzeyde başladıktan sonra iç yüzeyde ikinci bir plastik bölge meydana gelmektedir. Her iki plastik bölge Tresca prizmasının 'kenar rejim'inde yer almaktadır. Artan ısı yükü ile plastik bölgelerin yayılması, gerilme dağılımı ve plastik şekil değiştirmeler grafikler halinde verilerek irdelenmiştir.

Anahtar Sözcükler: Elastik-plastik, ısıl gerilmeler, iç enerji, tüp.

Introduction

The determination of stress distribution in elastic-plastic bodies with cylindrical or spherical symmetry has received considerable attention due to its importance in engineering design and applications (Gamer, 1991; Orçan and Gamer, 1991; Orç an, 1994). Most of the existing solutions to elastic-plastic problems are based on Tresca's yield condition and its associated flow rule leading to analytical or quasi-analytical solutions. More recently, a number of papers have appeared in the literature in which the effect of the temperature dependence of the yield stress on the expansion of plastic regions is investigated (Bengeri and Mack, 1994; Gülgeç and Orçan, 2000; Orçan and Gülgeç, 2000).

The exact solution of an elastic-plastic tube subjected to mechanical as well as thermal loads was obtained by Bland (1956) for a linearly work hardening material. The treatment was based on the assumption that the axial stress in the plastic region is the intermediate principal stress and that no edge regime of Tresca's prism is encountered. Hence, the solution presented in Bland (1956) is not valid in the absence of internal pressure and/or for large temperature gradients.

Kammash (1960) investigated the elastic-plastic stresses in tubes subjected to a uniform internal heat source in the state of plane strain for a linearly strain hardening material. It was found that for tubes with insulated inner surfaces, yielding commences at the inner boundary and propagates outward upon further thermal loading. The theoretical results were compared with the experimentally available failure loads of graphite tubes. However, the different phases of plastic deformation that occur before the fully plastic state is reached were not considered and hence the theoretical analysis presented there is not complete.

The aim of the present paper is to derive the complete solution of the elastic-plastic behaviour of a tube with free ends subjected to a uniform energy source q''' per unit volume per unit time, which increases monotonously with time. Although the temperature distribution under consideration is the same as that considered by Kammash (1960), the elastic-plastic behaviours in the two cases are entirely different. In the present case, yielding starts at the outer surface of the tube and later at the inner surface, where the stress states lie in two different edge regimes of Tresca's prism.

In Section 2, the thermoelastic behaviour and the specific temperature distribution are given. The first stage of elastic-plastic deformation is analyzed in Section 3, and the second stage in Section 4. Finally, the numerical results are presented and discussed in Section 5.

Elastic Behaviour

The stresses and displacement in a tube under the state of generalized plane strain and subjected to a temperature distribution T are given by

$$\sigma_r = -\frac{E\alpha}{1-\nu}\theta\ (a,r) + \frac{C_1}{2} + \frac{C_2}{r^2}, \tag{1}$$

$$\sigma_\theta = \frac{E\alpha}{1-\nu}[\theta\ (a,r) - T] + \frac{C_1}{2} - \frac{C_2}{r^2}, \tag{2}$$

$$\sigma_z = -\frac{E\alpha}{1-\nu}T + \nu C_1 + E\varepsilon_z, \tag{3}$$

$$\begin{aligned} Eu = &-(1+\nu)\tfrac{C_2}{r} \\ &+\Big[\tfrac{1+\nu}{1-\nu}E\alpha\theta\,(a,r) + \tfrac{1}{2}(1+\nu)(1-2\nu)\,C_1 \\ &-\nu E\varepsilon_z\Big]r, \end{aligned} \tag{4}$$

where $\theta := \theta\,(a,r) = \frac{1}{r^2}\int\limits_a^r T\,r\,dr$.

The integration constants C_1 and C_2 are evaluated using the condition that σ_r vanishes at the inner and outer surfaces, $r = a$ and $r = b$, as

$$C_1 = \frac{2b^2}{(b^2-a^2)(1-\nu)}E\alpha\theta\,(a,b)\,, \quad C_2 = -\frac{a^2}{2}C_1. \tag{5}$$

Temperature distribution

For an insulated inner surface ($\frac{dT}{dr} = 0$ at $r = a$) and zero reference temperature at $r = b$, the temperature distribution in a tube subjected to quasi-static uniform internal energy generation q''' is given by

$$T\,(r) = \frac{q'''}{4\lambda}\left[\left(b^2 - r^2\right) + 2a^2\,\ln r/b\right] \tag{6}$$

where λ is the thermal conductivity.

The distribution of stress and displacement in the elastic phase for the thermal loading under consideration is obtained by inserting the temperature from (6) into (1)-(4). This solution is valid until the difference of any two principal stresses reaches the uniaxial yield limit σ_0 of the material. It can be shown that for a tube with free ends this occurs at the outer surface where the stress image points lie on an edge of Tresca's prism with $\sigma_\theta = \sigma_z > \sigma_r$. The corresponding load parameter is determined by the condition that $\sigma_\theta - \sigma_r = \sigma_z - \sigma_r = \sigma_0$ at $r = b$, as

$$\bar{q}_1''' = \frac{E\alpha\,q_1''',b^2}{\lambda\sigma_0} = \frac{2\,(1-\nu)\left(1-Q^2\right)}{(0.25 - Q^2 + 0.75Q^4 - Q^4\,\ln Q)} \tag{7}$$

where $Q := a/b$.

First Stage of Plastic Deformation

For $\bar{q}''' = \bar{q}_1'''$, as in the related cases, two plastic regions (plastic regions I and II) governed by different forms of the yield condition emerge at the outer boundary and spread inward with increasing load parameter. The derivation of the basic equations for the stresses, displacement and plastic strains in plastic regions I and II is given as follows:

Plastic Region I: $r_4 < r \leq b$

In plastic region I, the stress image points lie on an edge of Tresca's prism and the yield condition takes the form

$$\sigma_\theta - \sigma_r = \sigma_0, \quad \sigma_z - \sigma_r = \sigma_0. \tag{8}$$

The distributions of stresses, displacement and plastic strains are given by

$$\sigma_r = \sigma_0 \ln r + C_3, \tag{9}$$

$$\sigma_\theta = \sigma_z = \sigma_0 (1 + \ln r) + C_3, \tag{10}$$

$$Eu = \frac{1-2\nu}{2} r \left[\sigma_0 \left(\frac{1}{2} + 3 \ln r\right) + 3C_3\right] + 3\alpha E r\theta - \frac{1}{2} E\varepsilon_z r + \frac{C_4}{r}, \tag{11}$$

$$E\varepsilon_\theta^p = \frac{1-2\nu}{2} (\sigma_0 \ln r + C_3) - \frac{3-2\nu}{4}\sigma_0 + E\alpha (3\theta - T) - \frac{1}{2} E\varepsilon_z + \frac{C_4}{r^2}, \tag{12}$$

$$E\varepsilon_r^p = \frac{1-2\nu}{2} (\sigma_0 \ln r + C_3) + \frac{7-6\nu}{4}\sigma_0 - E\alpha (3\theta - 2T) - \frac{1}{2} E\varepsilon_z - \frac{C_4}{r^2}, \tag{13}$$

$$E\varepsilon_z^p = E\varepsilon_z - [(1-\nu)\sigma_0 + (1-2\nu)(\sigma_0 \ln r + C_3)] - E\alpha T. \tag{14}$$

Plastic Region II : quad $r_3 < r < r_4$

In this region due to the inequality $\sigma_r < \sigma_z < \sigma_\theta$ the yield condition is given by

$$\sigma_\theta - \sigma_r = \sigma_0 \tag{15}$$

The stresses, displacement and plastic strains can be derived as follows:

$$\sigma_r = \sigma_0 \ln r + C_5, \tag{16}$$

$$\sigma_\theta = \sigma_0 (\ln r + 1) + C_5, \tag{17}$$

$$\sigma_z = \nu [\sigma_0 (2 \ln r + 1) + 2C_5] + E\varepsilon_z - E\alpha T, \tag{18}$$

$$Eu = (1+\nu)(1-2\nu) r (\sigma_0 \ln r + C_5) + 2(1+\nu) E\theta r - \nu E\varepsilon_z r + \frac{C_6}{r}, \tag{19}$$

$$E\varepsilon_\theta^p = -E\varepsilon_r^p = -\left(1-\nu^2\right)\sigma_0 + (1+\nu) E\alpha (2\theta - T) + \frac{C_6}{r^2}, \tag{20}$$

$$\varepsilon_z^p = 0. \tag{21}$$

Conditions and solution procedure

The general expressions for the distribution of stresses, displacement and plastic strains in the first stage of plastic deformation contain nine unknowns, which are functions of the load parameter: Integration constants C_1, C_2, ... ,C_6, the uniform axial strain ε_z, the elastic-plastic interface radius r_3 and the interface radius r_4 separating plastic regions I and II. Two of the boundary conditions follow from the fact that the inner and outer surfaces of the tube are free of traction. Three conditions at each interface radius should also be chosen appropriately from the available five nonredundant conditions of continuity of stresses, displacement and plastic strains. Here it is convenient to choose the following conditions: $\sigma_r^{el} = \sigma_r^{II}$, $\sigma_\theta^{el} - \sigma_r^{el} = \sigma_0$, $(\varepsilon_\theta^p)^{II} = 0$ at $r = r_3$ and $\sigma_r^I = \sigma_r^{II}$, $\sigma_\theta^{II} = \sigma_z^{II}$, $(\varepsilon_\theta^p)^I = (\varepsilon_\theta^p)^{II}$ at $r = r_4$. When these conditions are enforced the unknown constants are found to be

$$C_1 = -2\frac{C_2}{a^2} = -\frac{2r_3^2}{a^2 - r_3^2}\left[\sigma_0 \ln r_3/b + \frac{E\alpha}{1-\nu}\theta(a, r_3)\right], \tag{22}$$

$$C_3 = C_5 = -\sigma_0 \ln b, \tag{23}$$

$$C_4 = C_6 + r_4^2\left\{-\frac{(1 + 2\nu - 4\nu^2)}{4}\sigma_0 + 2(1+\nu)E\alpha\theta(r_3, r_4) - t - \frac{(1-2\nu)}{2}\sigma_0 \ln r_4/b - \nu E\alpha T(r_4) + \frac{1}{2}E\varepsilon_z\right\}, \tag{24}$$

$$C_6 = r_3^2\left\{(1-\nu^2)\sigma_0 + (1+\nu)E\alpha T(r_3)\right\}, \tag{25}$$

$$E\varepsilon_z = \sigma_0[1 - \nu + (1-2\nu)\ln r_4/b] + E\alpha T(r_4). \tag{26}$$

The interface radius r_3 is related to the load parameter by the nonlinear equation

$$\frac{E\alpha}{1-\nu}\left[T(r_3) + \frac{2r_3^2}{a^2 - r_3^2}\theta(a, r_3)\right] + \sigma_0\left[1 + \frac{2a^2}{a^2 - r_3^2}\ln r_3/b\right] = 0 \tag{27}$$

and the free end condition can be written as

$$I^{el}(a, r_3) + I^{II}(r_3, r_4) + I^I(r_4, b) = 0 \tag{28}$$

The integrals $I(s,t) = \int_s^t r\sigma_z\, dr$, where s and t denote two arbitrary interface radii, are given in the Appendix for each region.

Second Stage of Plastic Deformation

At the critical load parameter q_2''' further plastic flow sets in at the inner boundary where $\sigma_\theta = \sigma_z < \sigma_r$. Two plastic regions, one corresponding to an edge regime (plastic region III) and the other to a side regime (plastic region IV), emerge simultaneously and expand outward with increasing load parameter. The derivation of stresses, displacement and plastic strains in plastic regions III and IV is given as follows:

Plastic Region III: $a \leq r < r_1$

In this plastic region the stress image points lie on another edge of Tresca's prism, and the yield condition takes the form

$$\sigma_r - \sigma_z = \sigma_0, \quad \sigma_r - \sigma_\theta = \sigma_0. \tag{29}$$

Integration of the equilibrium equation with the yield condition (29) leads to

$$\sigma_r = -\sigma_0 \, \ln r + C_7, \tag{30}$$

$$\sigma_\theta = \sigma_z = -\sigma_0 \, (1 + \ln r) + C_7. \tag{31}$$

Since the dilatation is purely elastic the following differential equation can be derived:

$$\frac{du}{dr} + \frac{u}{r} = \frac{1-2\nu}{E}[-3 \, (\sigma_0 \, \ln r - C_7) - 2\sigma_0] + 3\alpha T - \varepsilon_z \tag{32}$$

which can be integrated to give

$$Eu = \frac{1-2\nu}{2} r \left[-\sigma_0 \left(\frac{1}{2} + 3 \, \ln r \right) + 3C_7 \right] + 3 \, r\alpha E\theta - \frac{1}{2} E\varepsilon_z \, r + \frac{C_8}{r}, \tag{33}$$

Therefrom, the plastic parts of the strain components are derived as follows:

$$E\varepsilon_\theta^p = \frac{1-2\nu}{2} \, (-\sigma_0 \, \ln r + C_7) - \frac{3-2\nu}{4} \sigma_0 + E\alpha \, (3\theta - T) - \frac{1}{2} E\varepsilon_z + \frac{C_8}{r^2}, \tag{34}$$

$$E\varepsilon_r^p = \frac{1-2\nu}{2} \, (-\sigma_0 \, \ln r + C_7) - \frac{7-6\nu}{4} \sigma_0 - E\alpha \, (3\theta - 2T) - \frac{1}{2} E\varepsilon_z - \frac{C_8}{r^2}, \tag{35}$$

$$E\varepsilon_z^p = E\varepsilon_z + (1-\nu) \, \sigma_0 - (1-2\nu) \, (-\sigma_0 \, \ln r + C_7) - E\alpha T. \tag{36}$$

Plastic Region IV: $r_1 < r < r_2$

Here the yield condition has the form

$$\sigma_r - \sigma_z = \sigma_0. \tag{37}$$

According to the associated flow rule

$$d\varepsilon_r^p = -d\varepsilon_z^p, \quad d\varepsilon_\theta^p = 0 \tag{38}$$

Hence, the total strains can be written as:

$$\varepsilon_r = \varepsilon_r^{el} + \varepsilon_z^{el} + 2\alpha T - \varepsilon_z, \tag{39}$$

$$\varepsilon_\theta = \varepsilon_\theta^{el} + \alpha T. \tag{40}$$

Inserting the total strains into the compatibility equation, expressing their elastic parts in terms of stresses and making use of the equilibrium equation together with the yield condition (37), there follows the differential equation

$$r^2\frac{d^2\sigma_\theta}{dr^2}+3r\frac{d\sigma_\theta}{dr}-(1-2\nu)\,\sigma_\theta=-E\varepsilon_z-\sigma_0+E\alpha\left(T-r\frac{dT}{dr}-r^2\frac{d^2T}{dr^2}\right) \tag{41}$$

The general solution of (41) is

$$\sigma_\theta=C_9r^{-1+M}+C_{10}r^{-1-M}+\frac{1}{1-2\nu}\,(\sigma_0+E\varepsilon_z)+\frac{E\alpha}{2}[(2-M)\,\theta_1+(2+M)\,\theta_2-2T] \tag{42}$$

where $\theta_1:=r^{-1+M}\int Tr^{-M}\,dr$, $\theta_2:=r^{-1-M}\int Tr^{M}\,dr$ and $M^2:=2\,(1-\nu)$.

Using the equilibrium equation, the radial stress is found to be

$$\sigma_r=\frac{C_9}{M}r^{-1+M}-\frac{C_{10}}{M}r^{-1-M}+\frac{1}{1-2\nu}\,(\sigma_0+E\varepsilon_z)+\frac{E\alpha}{2M}[(2-M)\,\theta_1-(2+M)\,\theta_2] \tag{43}$$

and the axial stress

$$\sigma_z=\sigma_r-\sigma_0. \tag{44}$$

Once the stress components are known, there follows:

$$\begin{aligned}Eu\;=\;&\left(1-\frac{2\nu}{M}\right)\left[C_9r^{M}+\frac{2-M}{2}E\alpha r\theta_1\right]+\left(1+\frac{2\nu}{M}\right)\left[C_{10}r^{-M}+\frac{2+M}{2}E\alpha r\theta_2\right]\\&+(1+\nu)\,\sigma_0r+E\varepsilon_zr,\end{aligned} \tag{45}$$

and the plastic strains are found as follows:

$$\begin{aligned}E\varepsilon_r^p\;=\;&-E\varepsilon_z^p=\frac{(1-\nu-\nu M)}{M}\left[C_9r^{-1+M}+\frac{(2-M)}{2}E\alpha\theta_1\right]\\&-\frac{(1-\nu+\nu M)}{M}\left[C_{10}r^{-1-M}+\frac{(2+M)}{2}E\alpha\theta_2\right]+(1+\nu)\,E\alpha T\end{aligned} \tag{46}$$

Conditions and solution procedure

In the second stage of plastic deformation, the tube is composed of plastic region III for $a<r<r_1$, plastic region IV for $r_1<r<r_2$, the elastic region for $r_2<r<r_3$, plastic region II for $r_3<r<r_4$ and plastic region I for $r_4<r<b$. The total number of unknowns is 15, namely $C_1,\ C_2,\ \ldots,\ C_{10}$, $r_1,\ r_2,\ r_3,\ r_4$ and ε_z.

The condition that $\sigma_r^I=0$ at $r=b$, and the three continuity conditions at $r=r_3$ and $r=r_4$ remain unchanged. As a result of this, the integration constants $C_3,\ C_4, C_5,\ C_6$ and ε_z given by equations (23)-(26) are maintained. For the determination of the remaining unknowns we have the following boundary conditions: $\sigma_r^{III}=0$ at $r=a$, $\sigma_r^{III}=\sigma_r^{IV}$, $\sigma_\theta^{III}=\sigma_\theta^{IV}$ and $(\varepsilon_\theta^p)^{III}=0$ at $r=r_1$, $\sigma_r^{IV}=\sigma_r^{el}$, $\sigma_\theta^{IV}=\sigma_\theta^{el}$ and $(\varepsilon_r^p)^{IV}=0$ at $r=r_2$. The unknown integration constants in the second stage of plastic deformation can be determined as follows:

$$C_7 = \sigma_0 \ln a, \tag{47}$$

$$C_8 = r_1^2 \left\{ -\frac{1-2\nu}{2}\sigma_0 \ln a/r_1 - \frac{3-2\nu}{4}\sigma_0 + \frac{E\varepsilon_z}{2} + E\alpha[T(r_1) - 3\theta(a, r_1)] \right\}, \tag{48}$$

$$C_9 = \frac{r_1^{1-M}}{2} \left[(1+M)\sigma_0 \ln a/r_1 + \frac{M}{1-M}\sigma_0 + \frac{E}{1-M}\varepsilon_z + E\alpha T(r_1) \right], \tag{49}$$

$$C_{10} = \frac{r_1^{1+M}}{2} \left[(1-M)\sigma_0 \ln a/r_1 - \frac{M}{1+M}\sigma_0 + \frac{E}{1+M}\varepsilon_z + E\alpha T(r_1) \right], \tag{50}$$

$$C_1 = \frac{1+M}{M} C_9 r_2^{-1+M} - \frac{1-M}{M} C_{10} r_2^{-1-M} + \frac{2}{1-2\nu}(\sigma_0 + E\varepsilon_z)$$

$$+E\alpha \left\{ \frac{1}{2M}[(1+M)(2-M)\theta_1(r_1, r_2) - (1-M)(2+M)\theta_2(r_1, r_2)] + \frac{\nu}{1-\nu}T(r_2) \right\}, \tag{51}$$

$$C_2 = r_2^2 \left\{ \frac{C_1}{2} - C_9 r_2^{-1+M} - C_{10} r_2^{-1-M} - \frac{1}{1-2\nu}(\sigma_0 + E\varepsilon_z) \right.$$

$$\left. - \frac{E\alpha}{2}[(2-M)\theta_1(r_1, r_2) + (2+M)\theta_2(r_1, r_2)] \right\}. \tag{52}$$

In the determination of the above unknowns no use was made of the conditions $(\varepsilon_r^p)^{IV} = 0$ at $r = r_2$ and $\sigma_r^{II} = \sigma_r^{el}$, $\sigma_\theta^{el} - \sigma_r^{el} = \sigma_0$ at $r = r_3$. They constitute, together with the free end condition, a system of four nonlinear equations in the interface radii r_1, r_2, r_3 and r_4 with $\bar{q}'''$ as a parameter:

$$(1-\nu-\nu M)\, C_9 \frac{r_2^{-1+M}}{M} - (1-\nu+\nu M)\, C_{10} \frac{r_2^{-1-M}}{M} + (1+\nu) E\alpha T(r_2)$$

$$+E\alpha \left\{ \frac{1}{2M}\ [(2-M)(1-\nu-\nu M)\theta_1(r_1, r_2) - (2+M)(1-\nu+\nu M)\theta_2(r_1, r_2)] \right\} = 0 \tag{53}$$

$$\frac{E\alpha}{1-\nu}\theta(r_2, r_3) + \sigma_0 \ln r_3/b - \frac{C_1}{2} - \frac{C_2}{r_3^2} = 0, \tag{54}$$

$$\frac{E\alpha}{1-\nu}[2\theta(r_2, r_3) - T(r_3)] - 2\frac{C_2}{r_3^2} - \sigma_0 = 0, \tag{55}$$

$$I^{III}(a, r_1) + I^{IV}(r_1, r_2) + I^{el}(r_2, r_3) + I^{II}(r_3, r_4) + I^{I}(r_4, b) = 0, \tag{56}$$

where the integrals $I(s,t) = \int_s^t r\sigma_z \, dr$ are given in the Appendix.

Numerical Results

With the radius ratio taken to be Q=0.2 and the Poisson's ratio $\nu = 0.295$, Figure 1 shows the distribution of nondimensional stresses $\bar{\sigma}_{ij} := \sigma_{ij}/\sigma_0$ and nondimensional displacement $\bar{u} := Eu/\sigma_0 b$ at the load parameter $\bar{q}''' = \bar{q}_1''' = 6.3319$, which corresponds to the onset of plastic flow at the outer surface of the tube.

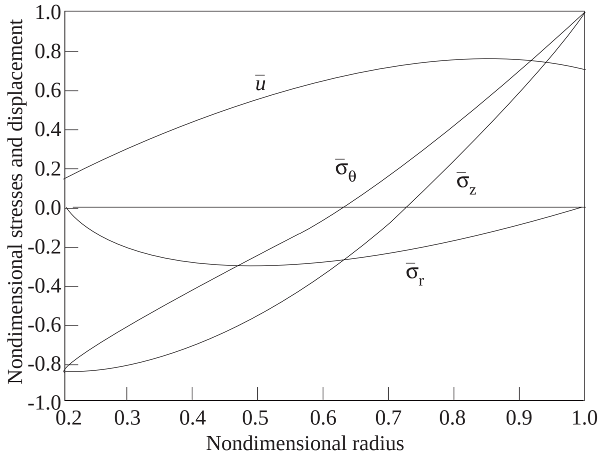

Figure 1. Stresses and displacement at $\bar{q}''' = \bar{q}_1''' = 6.3319$.

The evolution of the interface radii $\bar{r}_i := r_i/b$ with increasing load parameter $\bar{q}'''$ is depicted in Figure 2. It can be observed that the outer plastic region, composed of two different parts, sets in first at $\bar{q}''' = \bar{q}_1'''$ and the inner plastic region, which corresponds to another 'edge regime', emerges beyond $\bar{q}''' = \bar{q}_2''' = 7.3965$. The inner and outer plastic regions approach each other at a decreasing rate.

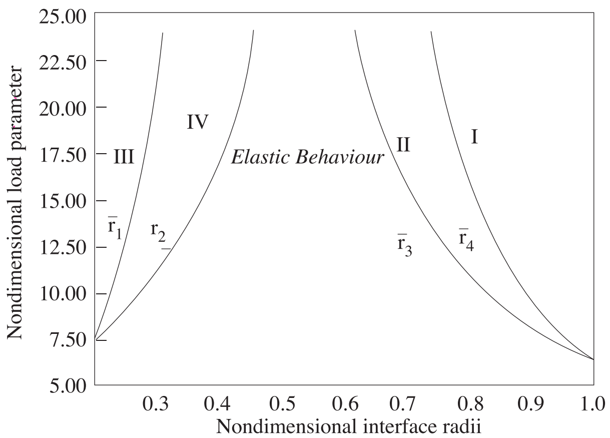

Figure 2. Evolution of plastic regions with increasing load parameter.

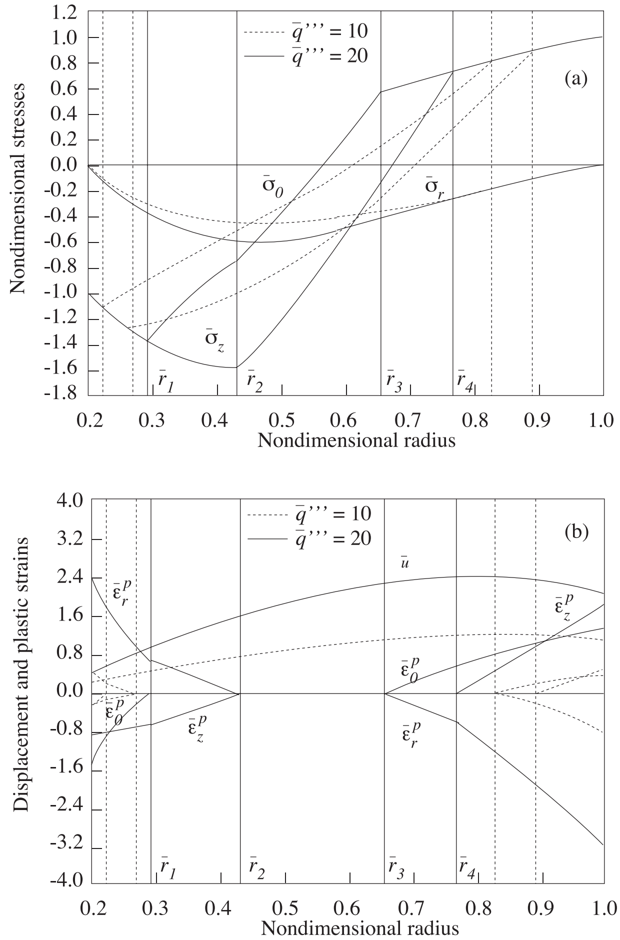

Figure 3. (a) Stresses, (b) displacement and plastic strains at $\bar{q}''' = 10.0$ and 20.0.

The distribution of stresses, displacement and plastic strains $\bar{\varepsilon}_{ij} := E\varepsilon_{ij}/\sigma_0$ for two different load parameters, $\bar{q}''' = 10.0$ and $\bar{q}''' = 20.0$, is given in Figure 3. They are plotted on the same graph for the purpose of comparison. The corresponding interface radii are $\bar{r}_1 = 0.22333$, $r_2 = 0.26719$, $\bar{r}_3 = 0.82752$, $\bar{r}_4 = 0.88851$ for $\bar{q}''' = 10.0$ and $\bar{r}_1 = 0.29071$, $\bar{r}_2 = 0.42840$, $\bar{r}_3 = 0.65304$, $\bar{r}_4 = 0.76658$ for $\bar{q}''' = 20.0$. With further increase in the thermal load, beyond $\bar{q}''' = \bar{q}_3''' = 24.602$, the tube enters the third stage of plastic deformation as soon as $\sigma_\theta = \sigma_r$ at the elastic-plastic interface $r = r_2$ and the validity of the results presented hitherto ceases. The corresponding interface radii are $\bar{r}_1 = 0.31109$, $\bar{r}_2 = 0.45966$, $\bar{r}_3 = 0.61797$ and $\bar{r}_4 = 0.74031$. Although the applicability of the geometrically linear theory can be justified for a wide range of load parameters, the authors believe that the analysis in the third stage of plastic deformation will be rewarding if the temperature dependence of the material properties (especially the yield stress) is considered.

References

Bengeri, M. and Mack, W., "The Influence of the Temperature Dependence of the Yield Stress on the Stress Distribution in a Thermally Assembled Elastic-Plastic Shrink Fit". Acta Mechanica 103, 243-257, 1994.

Bland, D. R., "Elastoplastic Thick-Walled Tubes of Work-Hardening Material Subject to Internal and External Pressure and to Temperature Gradients". J. Mech. Phys. Solids 4, 209-229, 1956.

Gamer, U., "On the Quasi-Analytical Solutions of Elastic-Plastic Problems with Nonlinear Hardening". In: Advances in Continuum Mechanics (Brüller, O., Mannl, V., Najor, J. eds.), 168-177. Berlin, Springer 1991.

Gülgeç M. and Orçan, Y., "Elastic-Plastic Deformation of a Heat Generating Tube with Temperature Dependent Yield Stress". Int. J. Engng. Sci. 38, 89-106, 2000.

Kammash, T., "Elastic-Plastic Thermal Stresses in Tubes Subjected to Uniform Heat Generation, Evaluation of Experimental Results Obtained Using Graphite Tubes". Nucl. Sci. Engng. 7, 425-434, 1960.

Orçan, Y., "Thermal Stresses in a Heat Generating Elastic-Plastic Cylinder with Free Ends". Int. J. Engng. Sci. 32, 883-897, 1994.

Orçan, Y. and Gamer, U., "Elastic-Plastic Deformation of a Centrally Heated Cylinder". Acta Mechanica 90, 61-90, 1991.

Orçan, Y. and Gülgeç M., "Influence of the Temperature Dependence of the Yield Stress on the Stress Distribution in a Heat Generating Tube with Free Ends". J. Thermal Stresses 23, 529-547, 2000.

Appendix

The integrals $I\,(s,t) = \int_s^t r\sigma_z\,dr$ occurring in equations (28) and (56) take the following forms:

$$I^{el}\,(s,t) = \frac{1}{2}\,(\nu C_1 + E\varepsilon_z)\,\left(t^2 - s^2\right) - \frac{E\alpha}{1-\nu}\int_s^t T\,r\,dr$$

$$I^{I}\,(s,t) = \frac{\sigma_0}{2}\left(t^2\,\ln t/b - s^2\,\ln s/b + \frac{1}{2}\left(t^2 - s^2\right)\right)$$

$$I^{II}\,(s,t) = \nu\sigma_0\left(t^2\,\ln t/b - s^2\,\ln s/b\right) + \frac{1}{2}\left(t^2 - s^2\right)E\varepsilon_z - E\alpha\int_s^t Tr\,dr$$

$$I^{III}\,(s,t) = -\frac{\sigma_0}{2}\left(t^2\,\ln t/a - s^2\,\ln s/a + \frac{1}{2}\left(t^2 - s^2\right)\right)$$

$$I^{IV}\,(s,t) = \frac{C_9}{M\,(1+M)}\left(t^{1+M} - s^{1+M}\right) - \frac{C_{10}}{M\,(1-M)}\left(t^{1-M} - s^{1-M}\right) + \frac{\nu\sigma_0}{1-2\nu}\left(t^2 - s^2\right)$$

$$+\frac{1}{2M}\left[(2-M)\int_s^t \theta_1\,r\,dr - (2+M)\int_s^t \theta_2\,r\,dr\right] + \left(t^2 - s^2\right)\frac{E\varepsilon_z}{2\,(1-2\nu)}$$