

Steady Flow of an Oldroyd 4-Constant Fluid in a Corner Region Formed by Two Planes

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Abstract

The analysis of the flow of a viscoelastic fluid near a corner point finds its application in the design of extrusion dies of technological importance. This paper discusses a problem of this type corresponding to the steady flow of a viscoelastic fluid simulated by the Oldroyd 4-constant fluid model in a corner region formed by two planes. The aim of this study is to investigate theoretically whether or not fluid elasticity is responsible for the formation of circulating cells near the corner, which has been observed experimentally in various polymer processes. Since such circulating cells are detrimental in equipment for polymer processing, it is important to understand and be able to predict the conditions under which circulating cells appear. Using series expansions proposed by Strauss (1974) for the stream function and stress components, the governing equations of the problem are reduced to ordinary differential equations. These equations have been solved by employing a numerical technique. The effects of the viscoelastic parameters on the flow pattern are carefully delineated. There is, unlike the case of Newtonian fluid, a secondary flow near the corner point.

Key Words: Non-radial flow, Oldroyd 4-constant fluid, viscoelasticity, circulating cells.

4 Sabitli Oldroyd Akışkanının İki Düzlem Tarafından Oluşturulan Bir Köşe İçindeki Daimi Akımı

Özet

Viskoelastik akışkanların bir köşe noktası civarındaki akışımın analizi teknolojik bir öneme sahip olan ekstrüzyon kalıplarının tasarımında uygulama alanı bulmaktadır. Bu makale, 4 sabitli Oldroyd akışkan modeliyle temsil edilen viskoelastik bir akışkanın iki düzlem tarafından oluşturulan bir köşe içindeki daimi akımıyla ilgili olan böyle bir problemi tartışmaktadır. Çalışmanın amacı ; çeşitli polimer proseslerinde köşe noktası civarında oluştukları deneysel olarak gözlemlenmiş olan sirkülasyon halkalarına akışkanın elastisitesinin neden olup olmadığını teorik olarak araştırmaktır. Sözü edilen sirkülasyon halkaları polimer prosesinde kullanılan teçhizata zarar verebileceği için bunların hangi koşullar altında meydana gelebileceğini anlamak ve tahmin edebilmek önem kazanmaktadır. Akım fonksiyonu ve gerilme bileşenleri için Strauss (1974) tarafından önerilen seri açılımları kullanılarak problemi yöneten denklemler adi diferansiyel denklemlere indirgenmiştir. Bu diferansiyel denklemler nümerik bir metot kullanılarak çözülmüştür. Viskoelastik parametrelerin akım çizgileri üzerindeki etkisi dikkatli bir şekilde incelenmiştir. Newtonian akışkandan farklı olarak , köşe noktası civarında ikincil akımların oluştuğu görülmüştür.

Anahtar Sözcükler: Radyal olmayan akım, 4 sabitli Oldroyd akışkanı, viskoelastisite, sirkülasyon halkaları.

Introduction

Theoretical investigations on steady converging flows of viscoelastic fluids were initiated by Langlois and Rivlin (1959), who carried out perturbation analyses about the slow flow of a Newtonian fluid through a wedge and cone. They found that the stresses developed by the viscoelastic properties of the fluid were incompatible with radial flow, and vortices were predicted in the perturbation solutions. Strauss (1974) was the first to present the solution for the steady, two-dimensional, and inertial flow of an incompressible Maxwell fluid between intersecting planes by using a series expansion in terms of decreasing powers of r . In his subsequent study (1975), he considered the stability of the same flow problem. Yoo and Han (1981) carried out experiments on the converging slow flow of a polymer between intersecting planes and tried to explain the data in terms of the second-grade fluid model. They found that the theoretical analysis corroborates qualitatively determined experimental stress distributions.

In the case of most non-Newtonian fluids a purely radial flow is not possible if inertial terms are to be retained in the equations of motion. Kaloni and Kamel (1980) have shown that there cannot be a purely radial flow of Cosserat fluids in convergent channels. Later, Hull (1981) studied the non-inertial flow of a general linear viscoelastic fluid in this geometry. He showed that radial flow is obtained for a wedge of 90° and no others. Similar results are valid for Rivlin-Ericksen fluids (Mansutti and Rajagopal, 1991).

Mansutti and Rajagopal (1991) studied the non-inertial flow of a shear thinning fluid between intersecting planes. They showed that sharp and pronounced boundary layers develop adjacent to the solid boundaries, even at zero Reynolds number. Recently, Bhatnagar et al. (1993) have extended the analysis of Strauss (1974) to an Oldroyd-B fluid which is characterized by viscosity and two material constants with units of time. In their work, the effects of the much higher values of the Reynolds number than in Strauss' work (1974) and the elastic parameter on the streamline patterns was discussed.

In this paper, the flow of an Oldroyd 4-constant fluid in a convergent channel is examined using series expansions in terms of decreasing powers of r given by Strauss (1974) for stream function and stress components. It is shown that the non-Newtonian parameter τ_3 , which does not appear in previous studies,

affects the streamlines of the secondary flow near the corner point in a significant way. Our results are similar to those of Strauss (1974) and Bhatnagar et al. (1993), but differ in some details. Also, it is thought possible to establish a relationship with their works.

Formulation of the problem and its solution

The steady, two-dimensional, incompressible, laminar flow of the Oldroyd 4-constant fluid through a converging channel bounded by two non-parallel planes is schematically illustrated in Fig. 1.

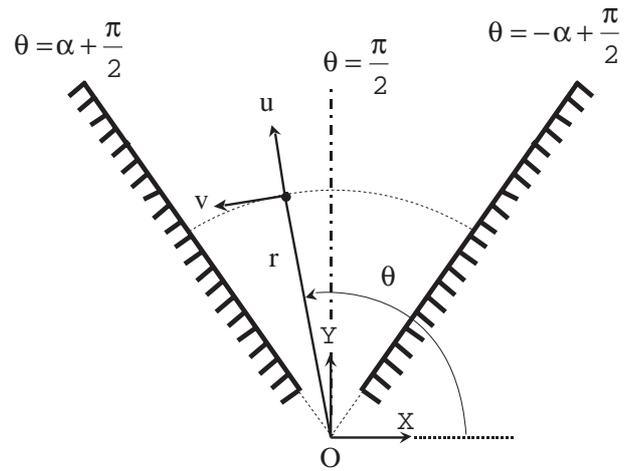


Figure 1. Basic geometry of the problem.

The viscoelastic fluid model used here is the Oldroyd 4-constant model, constitutive equation of which is given as follows (Bird et al.,1987):

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{1}$$

$$\mathbf{S} + \Lambda_1 \frac{\delta \mathbf{S}}{\delta t} + \Lambda_3 (tr \mathbf{S}) \mathbf{A}_1 = \mu (\mathbf{A}_1 + \Lambda_2 \frac{\delta \mathbf{A}_1}{\delta t}) \tag{2}$$

where \mathbf{T} is the Cauchy stress tensor, p is the pressure, \mathbf{I} is the identity tensor, \mathbf{S} is the extra stress tensor, and μ is the coefficient of viscosity, while Λ_1 , Λ_2 , and Λ_3 are the material constants. \mathbf{A}_1 is the first Rivlin-Ericksen tensor and $\delta / \delta t$ the contravariant convected derivative defined as follows, respectively

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{L} + \mathbf{L}^T \\ \mathbf{L} &= \nabla \mathbf{v}, L_{ij} = v_{j;i} \end{aligned} \tag{3}$$

$$\frac{\delta \mathbf{S}}{\delta t} = \frac{\partial \mathbf{S}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{S} - \mathbf{S} \cdot \mathbf{L} - \mathbf{L}^T \cdot \mathbf{S} \quad (4)$$

where \mathbf{v} is the velocity vector, ∇ is the gradient operator, the superscript T denotes a transpose operation, and the semicolon stands for covariant differentiation.

When $\Lambda_1 = \Lambda_2 = \Lambda_3 = 0$, the model (2) reduces to the classical linearly viscous Navier Stokes fluid (Newtonian fluid). Also, it should be noted that this model includes the Maxwell fluid for $\Lambda_1 \neq 0, \Lambda_2 = \Lambda_3 = 0$ and the Oldroyd-B fluid for $\Lambda_1 \neq 0, \Lambda_2 \neq 0, \Lambda_3 = 0$.

In addition to Eqs. (1) and (2), the field equations consist of the equations of motion and the continuity equation. In the case of steady flow, the former equations in the absence of body forces take the form

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla \cdot \mathbf{T} \quad (5)$$

where ρ is the (constant) density. The continuity equation is

$$tr \mathbf{A}_1 = 0. \quad (6)$$

We shall assume a velocity field in a plane polar coordinate system (r, θ) of the form

$$\mathbf{v}(r, \theta) = u(r, \theta) \mathbf{e}_r + v(r, \theta) \mathbf{e}_\theta \quad (7)$$

where u and v denote the velocity components in the directions of r and θ respectively.

We shall now write the field equations in terms of a set of dimensionless variables and, for this purpose, we shall choose Λ_1, μ and Q as characteristic units. If \bar{f} is used to denote the dimensionless form of a quantity f , it follows that

$$\begin{aligned} \bar{r} &= \frac{r}{\sqrt{Q\Lambda_1}}, \bar{u} = \sqrt{\frac{\Lambda_1}{Q}}u, \bar{v} = \sqrt{\frac{\Lambda_1}{Q}}v, \\ \bar{p} &= \frac{\Lambda_1}{\mu} p, \bar{S}^{ij} = \frac{\Lambda_1}{\mu} S^{ij} \end{aligned} \quad (8)$$

where Q is the volume flux per unit distance normal to the plane of flow and has dimensions of $L^2 T^{-1}$. We suppose that the flow is driven steadily with flux Q . Thus, Eqs. (1), (2), (5), and (6) in non-dimensional form become

$$\bar{\mathbf{T}} = -\bar{p}\mathbf{I} + \bar{\mathbf{S}} \quad (9)$$

$$\bar{\mathbf{S}} + \frac{\delta \bar{\mathbf{S}}}{\delta t} + \tau_3 (tr \bar{\mathbf{S}}) \bar{\mathbf{A}}_1 = \bar{\mathbf{A}}_1 + \tau_2 \frac{\delta \bar{\mathbf{A}}_1}{\delta t} \quad (10)$$

$$Re(\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}) = \nabla \cdot \bar{\mathbf{T}} \quad (11)$$

$$tr \bar{\mathbf{A}}_1 = 0 \quad (12)$$

where

$$Re = \frac{\rho Q}{\mu}, \tau_2 = \frac{\Lambda_2}{\Lambda_1}, \tau_3 = \frac{\Lambda_3}{\Lambda_1}, \quad (13)$$

where $\bar{\mathbf{A}}_1$ satisfies the above dimensionless equations obtained from Eq. (3) by replacing \mathbf{A}_1 by $\bar{\mathbf{A}}_1$. In this section, henceforth for convenience, we shall drop the bars that appear over the dimensionless quantities.

We now turn our attention to the equations of motion (11). After the pressure is eliminated by cross-differentiating Eqs. (11), one obtains the following governing equation:

$$\begin{aligned} Re \left\{ \frac{\partial u}{\partial r} \left(\frac{\partial u}{\partial \theta} - r \frac{\partial v}{\partial r} - v \right) + \frac{1}{r} \frac{\partial v}{\partial \theta} \left(\frac{\partial u}{\partial \theta} - 2v - r \frac{\partial v}{\partial r} \right) + u \left(\frac{\partial^2 u}{\partial r \partial \theta} - 2 \frac{\partial v}{\partial r} - r \frac{\partial^2 v}{\partial r^2} \right) \right. \\ \left. + v \left(\frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^2 v}{\partial r \partial \theta} \right) \right\} = \frac{1}{r} \frac{\partial^2}{\partial r \partial \theta} \{ r (S^{rr} - S^{\theta\theta}) \} - \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r^2 S^{r\theta}) \right\} + \frac{1}{r} \frac{\partial^2 S^{r\theta}}{\partial \theta^2}. \end{aligned} \quad (14)$$

We shall express stream function and extra stress components as a series expansion of the following form (Strauss, 1974):

$$\psi(r, \theta) = \sum_{n=0}^{\infty} \frac{\psi_n(\theta)}{r^n} \quad (15)$$

$$\begin{aligned} S^{rr}(r, \theta) &= \sum_{n=0}^{\infty} \frac{a_n(\theta)}{r^n}, \\ S^{r\theta}(r, \theta) &= \sum_{n=0}^{\infty} \frac{b_n(\theta)}{r^n}, \\ S^{\theta\theta}(r, \theta) &= \sum_{n=0}^{\infty} \frac{c_n(\theta)}{r^n} \end{aligned} \quad (16)$$

Here, we take into account the first five terms of the series expansion (15).

By defining a stream function $\psi(r, \theta)$, such that

$$u = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, v = -\frac{\partial \psi}{\partial r} \quad (17)$$

the continuity equation is satisfied automatically.

The adherence boundary conditions of the problem are as follows:

$$u(r, \pm\alpha) = 0, v(r, \pm\alpha) = 0 \quad (18)$$

which by virtue of Eqs. (15) and (17) implies that

$$\psi'_n(\pm\alpha) = 0, (n = 0, 1, 2, \dots) \quad (19)$$

$$\psi_n(\pm\alpha) = 0, (n = 1, 2, 3, \dots) \quad (20)$$

Furthermore we use two additional boundary conditions. For this reason, the volumetric flow rate through the channel used is:

$$\int_{-\alpha}^{+\alpha} u r d\theta = \psi(r, +\alpha) - \psi(r, -\alpha) = -1 \quad (21)$$

Here the minus sign denotes the flow of a convergent channel. Assuming the flow is symmetric about $\theta = 0$, then

$$\psi(r, \pm\alpha) = \mp \frac{1}{2} \quad (22)$$

and using the series expansion (15), we have

$$\psi_0(\pm\alpha) = \mp \frac{1}{2}. \quad (23)$$

The functions $a_n(\theta)$, $b_n(\theta)$, and $c_n(\theta)$ may be expressed in terms of the $\psi_n(\theta)$ functions in the series expansion (15) by substituting Eqs. (3) and (16) into Eqs. (10). Next, inserting a_n 's, b_n 's, and c_n 's (upto $n = 6$) into Eq. (16) and substituting these expressions for S^{rr} , $S^{r\theta}$ and $S^{\theta\theta}$ into Eq. (14), a very long and tedious calculation yields the equations at various orders of r^{-n} . Here, we carry out our analysis up to order $n=4$.

The zeroth-order problem is governed by

$$\psi_0^{IV} + 4\psi_0'' + 2Re\psi_0'\psi_0'' = 0, \quad (24)$$

$$\psi_0(\pm\alpha) = \mp \frac{1}{2}, \psi_0'(\pm\alpha) = 0. \quad (25)$$

The differential equation of the zeroth-order problem is non-linear, and the only parameter is the Reynolds number. This equation, subject to (25), is solved numerically.

The differential equation and boundary conditions governing $\psi_1(\theta)$ are as follows:

$$\begin{aligned} \psi_1^{IV} + (10 + 3Re\psi_0')\psi_1'' + (2Re\psi_0'')\psi_1' + \\ (9 + Re\{3\psi_0' - \psi_0'''\})\psi_1 = 0, \end{aligned} \quad (26)$$

$$\psi_1(\pm\alpha) = 0, \psi_1'(\pm\alpha) = 0. \quad (27)$$

Eq. (26) is a linear homogeneous ordinary differential equation subject to boundary conditions (27), and its solution is

$$\psi_1(\theta) \equiv 0. \quad (28)$$

The differential equation and boundary conditions governing $\psi_2(\theta)$ are as follows:

$$\psi_2^{IV} + (20 + 4Re\psi_0')\psi_2'' + (2Re\psi_0'')\psi_2' +$$

$$(64 + Re \{ 16 \psi_0' - 2\psi_0''' \}) \psi_2 = -4(1 - \tau_2) (\psi_0^{IV} + 4 \psi_0'') \psi_0', \quad (29)$$

$$\psi_2(\pm\alpha) = 0, \psi_2'(\pm\alpha) = 0. \quad (30)$$

The right hand side of Eq. (29) represents the non-Newtonian character of the fluid. In the case of slow motion ($Re \ll 1$) the right hand side of Eq. (29) is equal to zero due to Eq. (24). Then it becomes a linear homogeneous ordinary differential equation and gives a zero solution under boundary conditions (30), i.e. $\psi_2(\theta) \equiv 0$. For inertial flow Eq. (29) subject to (30) is solved numerically.

Differential equation and boundary conditions governing $\psi_3(\theta)$ are as follows:

$$\psi_3^{IV} + (34 + 5 Re \psi_0') \psi_3'' + (2 Re \psi_0'') \psi_3' + (225 + Re \{ 45 \psi_0' - 3 \psi_0''' \}) \psi_3 = 0, \quad (31)$$

$$\psi_3(\pm\alpha) = 0, \psi_3'(\pm\alpha) = 0. \quad (32)$$

Eq. (31) is a linear homogeneous ordinary differential equation subject to boundary conditions (32), and its solution is

$$\psi_3(\theta) \equiv 0. \quad (33)$$

The differential equation and boundary conditions governing $\psi_4(\theta)$ are as follows:

$$\begin{aligned} &\psi_4^{IV} + (52 + 6 Re \psi_0') \psi_4'' + (2 Re \psi_0'') \psi_4' + (576 + Re \{ 96 \psi_0' - 4 \psi_0''' \}) \psi_4 \\ &= Re (-8 \psi_2 \psi_2' - 4 \psi_2' \psi_2'' + 2 \psi_2 \psi_2''') + (1 - \tau_2) (-384 \psi_0' \psi_2 - 144 \psi_0'' \psi_0'^2 \\ &- 16 \psi_0'' \psi_2' + 12 \psi_0''' \psi_2 - 120 \psi_0' \psi_2'' + 8 \psi_0'' \psi_2' - 24 \psi_0' \psi_0'' \psi_0''' - 24 \psi_0^{IV} \psi_0'^2 \\ &- 4 \psi_0^{IV} \psi_2' - 6 \psi_0' \psi_2^{IV} + 2 \psi_0^V \psi_2) + 2 \tau_3 (1 - \tau_2) (144 \psi_0'' \psi_0'^2 + 4 \psi_0'''^3 \\ &+ 64 \psi_0' \psi_0'' \psi_0''' + 6 \psi_0'' \psi_0'''^2 + 4 \psi_0^{IV} \psi_0'^2 + 3 \psi_0^{IV} \psi_0''^2), \end{aligned} \quad (34)$$

$$\psi_4(\pm\alpha) = 0, \psi_4'(\pm\alpha) = 0. \quad (35)$$

Eq. (34) subject to (35) is solved numerically.

and ours.

Results and Discussion

The same problem as that investigated in the present paper has been solved previously by Strauss (1974) for Maxwell fluid, and Bhatnagar et al. (1993) for Oldroyd-B fluid. In those special cases corresponding to Maxwell fluid ($\tau_2 = \tau_3 = 0$) and Oldroyd-B fluid ($\tau_2 \neq 0, \tau_3 = 0$), there is, as expected, an overlap between their governing equations

We solved the governing differential equations as a system of first-order differential equations by using the shooting method. For given values of parameters, the conditions $\psi_n''(-\alpha)$ and $\psi_n'''(-\alpha)$ are roughly estimated and differential equations are processed by using the fourth-order Runge-Kutta procedure. The mathematical problem is to find the correct values of $\psi_n''(-\alpha)$ and $\psi_n'''(-\alpha)$ which yield the known values of $\psi_n(+\alpha)$ and $\psi_n'(+\alpha)$ at the terminal point. Since for $Re=0$ the analytic solution provides exact ini-

tial values for $\psi_n''(-\alpha)$ and $\psi_n'''(-\alpha)$, then a successive numerical solution can be generated as Re is increased. The systematic way used here to find the values of the missing initial conditions is equivalent to a modified Newton's method for finding the roots of equations in several variables. The accuracy of the missing initial conditions at $\theta = -\alpha$ which yield the known values at the terminal point ($\theta = +\alpha$) is 10^{-5} at least.

The results presented here are in complete agreement with those given by the present authors (cf. Strauss (1974) , Bhatnagar et. al (1993)) for the specific values of the Reynolds number Re and the elastic parameter τ_2 for which they have given results. This gives us confidence regarding the numerical work.

The predictions based on the foregoing analysis are displayed graphically in Figs. 2 to 9. To draw the streamlines presented in these figures, the first thing to do is to give a constant value to the dimensionless stream function of the form

$$\bar{\psi}(\bar{r}, \theta) = \bar{\psi}_0(\theta) + \frac{\bar{\psi}_2(\theta)}{\bar{r}^2} + \frac{\bar{\psi}_4(\theta)}{\bar{r}^4}.$$

$$\bar{\psi} = \frac{\psi}{Q}, \bar{\psi}_0 = \frac{\psi_0}{Q},$$

$$\bar{\psi}_2 = \frac{\psi_2}{Q^2 \Lambda_1}, \bar{\psi}_4 = \frac{\psi_4}{Q^3 \Lambda_1^2}. \tag{36}$$

For this constant value, the proper values of \bar{r} are calculated from Eq. (36) for various values of θ in the interval $-\alpha + \pi/2 \leq \theta \leq \alpha + \pi/2$. After this, the non-dimensional cartesian coordinates (\bar{X}, \bar{Y}) can be found from the non-dimensional polar coordinates (\bar{r}, θ) by using the relations $\bar{X} = \bar{r} \cos \theta$ and $\bar{Y} = \bar{r} \sin \theta$. If this process is repeated for different values of constants given the dimensionless stream function, the streamlines in Figs. 2 to 9 are obtained. We would prefer (\bar{X}, \bar{Y}) coordinates to (\bar{r}, θ) coordinates in order to depict the streamline patterns more easily.

Our main purpose is to delineate the effect of the parameter τ_3 , which does not appear in previous studies, on flow patterns. In Figs. 3 and 5, we have plotted the streamlines related to the Oldroyd-4 constant fluid with the intention of investigating the contribution of this new parameter to the flow field. From these figures, it is evident that the parameter τ_3 does affect the streamlines of the secondary flow near the corner in a significant way.

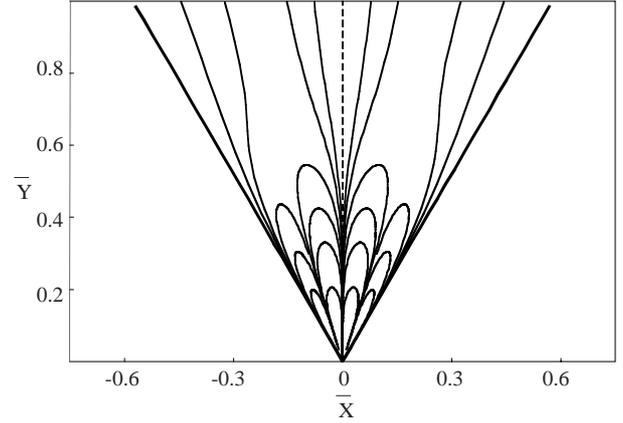


Figure 2. Streamline patterns for $Re = 11.42, \alpha = 30^\circ, \tau_2 = 0.5, \tau_3 = 0$.

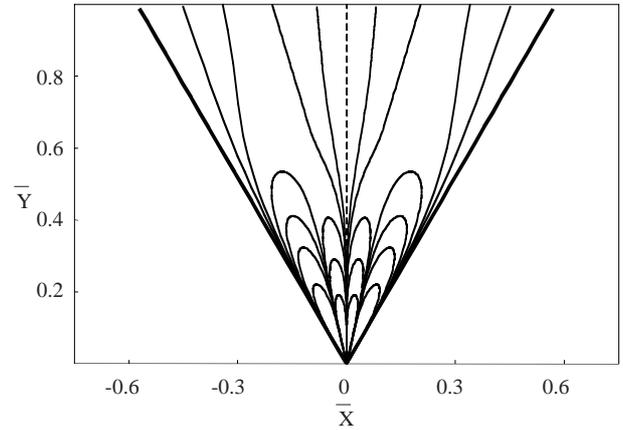


Figure 3. Streamline patterns for $Re = 11.42, \alpha = 30^\circ, \tau_2 = 0.5, \tau_3 = 0.02$.

As the parameter τ_3 is increased from 0 to 0.02, while keeping Re fixed at 11.42 and $\tau_2 = 0.5$, the flow still has a four-cell structure as in Oldroyd-B fluid but the size of the circulating cells adjacent to the stationary plates are bigger and the others smaller (see Figs. 2 and 3).

On comparing Fig. 5 with Fig. 4, we arrive at the conclusion that the streamlines of the secondary flow near the corner corresponding to $\tau_3 \neq 0$ changes from a two-cell structure into a four-cell structure.

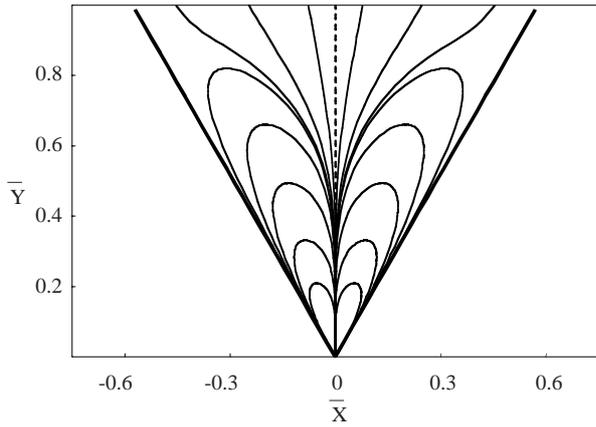


Figure 4. Streamline patterns for $Re = 100, \alpha = 30^\circ$, $\tau_2 = 0.5, \tau_3 = 0$,

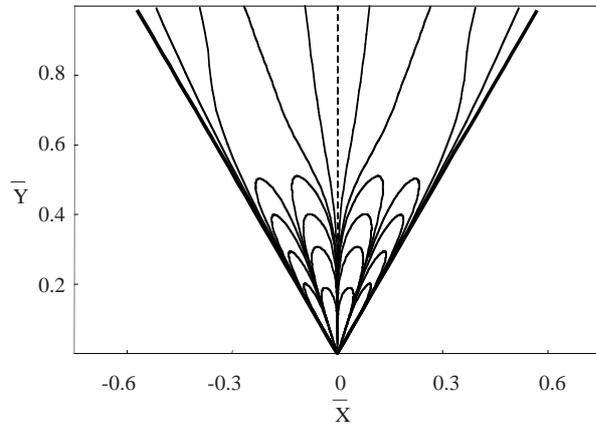


Figure 5. Streamline patterns for $Re = 100, \alpha = 30^\circ$, $\tau_2 = 0.5, \tau_3 = 0.036$.

Finally, we shall discuss the reliability of the solutions near the apex of the wedge. The series expansions used for the stream function and stress components are not appropriate for a perturbation for $\bar{\tau} < 1$. This is why the solutions based upon this approximation cannot be reliable when $\bar{\tau} < 1$. Of course, this also depends on the nature of functions $\bar{\psi}_n(\theta)$, that is, if $\bar{\psi}_n(\theta)$ are not identically zero for large n , the solution cannot be trusted as being meaningful for $\bar{\tau} < 1$. In addition, as a result of singularity at $\bar{\tau} = 0$, the solutions are expected to be valid only in converging (or diverging) nozzles rather than between two intersecting planes having a source (or sink) at $\bar{\tau} = 0$.

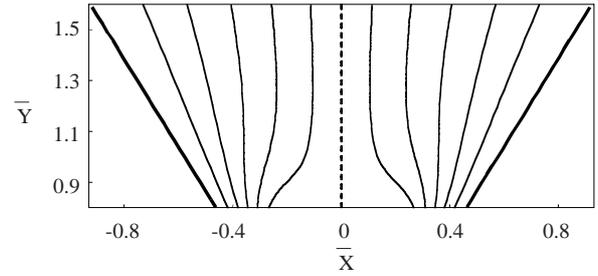


Figure 6. Streamline patterns for $Re = 11.42, \alpha = 30^\circ$, $\tau_2 = \tau_3 = 0$ for $0.8 < \bar{Y} < 1.6$.

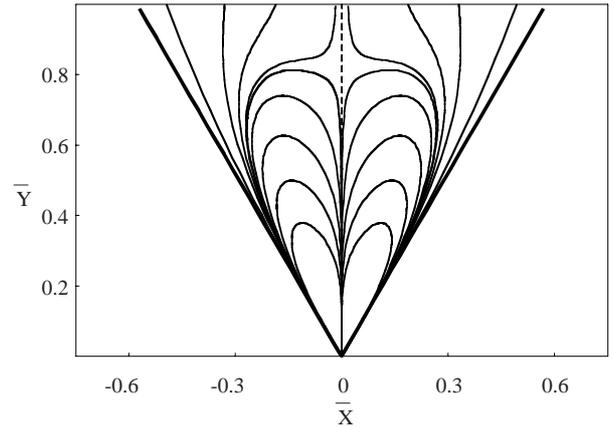


Figure 7. Streamline patterns for $Re = 11.42, \alpha = 30^\circ$, $\tau_2 = \tau_3 = 0$ for $0 < \bar{Y} < 1$.

On the other hand, for $\bar{\tau} \geq 1$, since the effect of successive terms are less significant, the solution is probably quite reliable as n increases. This gives us adequate information in the flow domain $\bar{\tau} < 1$ from the tendencies suggested by the results at $\bar{\tau} \geq 1$. For instance, the streamlines depicted in Fig. 6 indicate the presence of two-cells in Fig. 7, whereas Fig. 8 suggests a four-cell structure in Fig. 9. Of course, the streamlines of the secondary flow illustrated in the figures for $\bar{\tau} < 1$ may not have a precise structure.

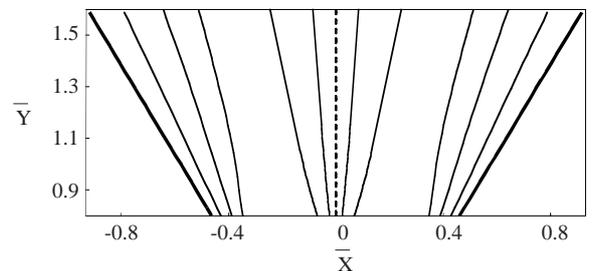


Figure 8. Streamline patterns for $Re = 19.67, \alpha = 30^\circ$, $\tau_2 = \tau_3 = 0$ for $0.8 < \bar{Y} < 1.6$.

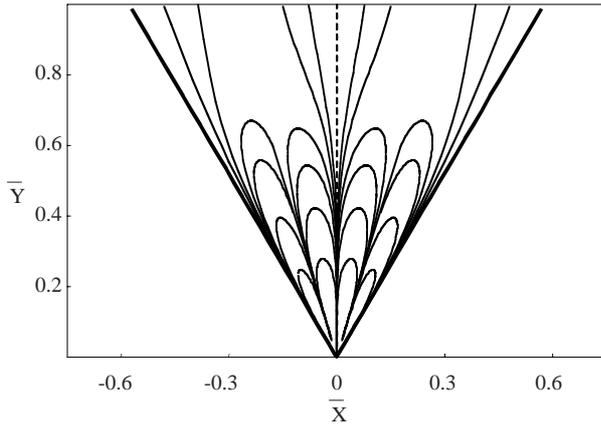


Figure 9. Streamline patterns for $Re = 19.67, \alpha = 30^\circ$, $\tau_2 = \tau_3 = 0$ for $0 < \bar{Y} < 1$.

Nomenclature

\mathbf{A}_1 Rivlin-Ericksen tensor of rank one, T^{-1}

\mathbf{I}	Identity tensor, dimensionless
p	Pressure, $M L^{-1} T^{-2}$
Q	Volumetric flow rate, $L^2 T^{-1}$
\bar{r}, θ	Polar coordinates, dimensionless
Re	Reynolds number, dimensionless
\mathbf{S}	Extra stress tensor, $M L^{-1} T^{-2}$
\mathbf{T}	Cauchy stress tensor, $M L^{-1} T^{-2}$
u, v	Components of the velocity vector, $L T^{-1}$
\mathbf{v}	Velocity vector, $L T^{-1}$
\bar{X}, \bar{Y}	Cartesian coordinates, dimensionless
α	Half angle of corner, dimensionless
μ	Coefficient of viscosity, $M L^{-1} T^{-1}$
Λ_i	Material constants, T
ρ	Density, $M L^{-3}$
τ_i	Ratio of two material constants, dimensionless
ψ	Stream function, $L^2 T^{-1}$
$\bar{\psi}_0, \bar{\psi}_2, \bar{\psi}_4$	Stream function components, dimensionless

References

- Bhatnagar, R.K., Rajagopal, K.R. and Gupta, G., "Flow of an Oldroyd-B Fluid between Intersecting Planes", *J. Non-Newt. Fluid Mech.* 46, 49-67, 1993.
- Bird, R.B., Armstrong, R.C. and Hassager, O., *Dynamics of Polymeric Liquids*, New York: Wiley, 354, 1987.
- Hull, A.M., "An Exact Solution for the Slow Flow of a General Linear Viscoelastic Fluid through a Slit", *J. Non-Newt. Fluid Mech.* 8, 327-336, 1981.
- Kaloni, P.N. and Kamel, M.T., "A note on the Hamel Flow of Cosserat Fluids", *Zamp* 31, 293-296, 1980.
- Langlois, W.E. and Rivlin, R.S., "Steady Flow of Slightly Viscoelastic Fluids", Technical Report No. DA - 4725 / 3, Division of Applied Mathematics, Brown University, 1959.
- Mansutti, D. and Rajagopal, K.R., "Flow of a Shear Thinning Fluid between Intersecting Planes", *Int. J. Non-Linear Mech.* 26, 769-775, 1991.
- Strauss, K., "Die Strömung einer Einfachen Viscoelastischen Flüssigkeit in einem Konvergenten Kanal. Teil I : Die stationäre Strömung", *Acta Mech.* 20, 233-246, 1974.
- Strauss, K., "Die Strömung einer Einfachen Viscoelastischen Flüssigkeit in einem Konvergenten Kanal. Teil II : Die Stabilität der Strömung", *Acta Mech.* 21, 141-152, 1975.
- Yoo, H.J. and Han, C.D., "Stress Distribution of Polymers in Extrusion through a Converging Die", *J. Rheol.* 25, 115-137, 1981.