Semiactive Control of Earthquake-Excited Structures

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Abstract

A new semiactive closed loop control algorithm is developed using an actuator with variable stiffness and damping to suppress structural vibrations caused by unknown disturbances such as earthquakes. It is assumed that the stiffness and the damping characteristics of the actuator can be changed independently from each other. The closed loop control algorithm is obtained by forcing the rate of change of the system energy to be as negative as possible. Optimal active control algorithm, which is an application of the linear regulator problem to structures, is used for the purpose of comparison. To demonstrate the efficiency of the proposed method, controlled and uncontrolled behaviour of a single story damped structure subjected to earthquake forces is investigated. It is shown numerically that the proposed semiactive control can be used effectively to reduce the earthquake induced structural vibrations.

Key Words: Semiactive control, Structural control, Optimal control

1. Introduction

In recent decades, many advances have been made in modern control theory and its application to engineering systems. The new civil engineering struc-

237
A possible alternative method other than the passive and active control methods may be semiactive control. Semiactive control systems are a class of active control systems for which the external energy requirements are orders of magnitude smaller than typical active control systems, thus contributing to energy saving. In semiactive control, the counteractive control forces are generated by reactive devices with variable damping and/or stiffness characteristics (Onodo and Watanabe, 1991). These systems can operate during large earthquakes since they do not require much energy (Laï and Wang, 1996). Semiactive control devices do not add mechanical energy to the structural system (including the system and control actuators), but control the states of the system such that the damping performance is maximized. Semiactive control devices are often viewed as controllable passive devices.

In this paper, a semiactive device with variable stiffness and damping is used to suppress the structural vibration of a single story damped structure subjected to El Centro ground motion. It is assumed that the variable stiffness and damping coefficients can be changed independently from each other. The control policy is developed by forcing the rate of change of system energy to be as negative as possible. To demonstrate the efficiency of the new closed loop control algorithm, controlled and uncontrolled behavior of a single story damped structure is investigated and the results are discussed.

2. Control Algorithm Derivation

For the purpose of discussion, a single story damped structure with a dynamic absorber having adaptable stiffness and damping is used, as shown in Figure 1, to illustrate the control law. \( m \) is the mass of the structure; \( k \) and \( c \) are the stiffness and the damping coefficients of the structure, respectively. \( k_s(u) \) and \( c_s(u) \) are adaptable stiffness and damping coefficients of the dynamic absorber which depend on the control parameter \( u \). The control parameter \( u \) is different for different control materials.

In an electro-rheological type material, the control parameter \( u \) denotes voltage. \( k_s(u) \) and \( c_s(u) \) increase monotonically with \( u \). When the voltage is off, both \( k_s(u) \) and \( c_s(u) \) take their minimum values. When the voltage is on, \( k_s(u) \) and \( c_s(u) \) take their maximum values. But, it must be noted that very complicated nonlinear models are proposed for the electro-rheological materials in the literature (Gavin...
In this paper, for the sake of simplicity, it is assumed that the dynamic absorber has adaptable stiffness $k_s(u)$ and adaptable damping $c_s(u)$ and these can be changed independently of each other. They can only provide two different values of stiffness, $(k_s)_\text{min}$ and $(k_s)_\text{max}$, and damping, $(c_s)_\text{min}$ and $(c_s)_\text{max}$. The equation of motion of a base-excited single story damped structure given in Figure 1 can be presented as

$$m\ddot{x} + c\dot{x} + kx + f_u = -\ddot{x}_g$$ (1)

where $\ddot{x}_g$ is the earthquake acceleration and $f_u$ is the control force which occurs in the dynamic absorber and is given by

$$f_u = k_s(u)x + c_s(u)\dot{x}$$ (2)

For the system given by equation (1), the mechanical energy of the main structure can be expressed as:

$$V = E_r + E_s$$ (3)

where $E_r$ and $E_s$ are the relative kinetic energy and the elastic strain energy of the main structure, respectively and are given by the equations

$$E_r = \frac{1}{2}m\dot{x}^2$$ (4)

$$E_s = \frac{1}{2}kx^2$$ (5)

It must be noted here that if the absolute kinetic energy is used instead of the relative kinetic energy this will result in a closed-open loop control algorithm. From equation (3), the energy rate of the system is obtained as

$$\frac{dV}{dt} = m\ddot{x}\dot{x} + kx\dot{x}$$ (6)

Using equation (1), the rate of change of energy can be rewritten as

$$\frac{dV}{dt} = m\ddot{x} \left( -\frac{k}{m}x - \frac{c}{m}\dot{x} - \frac{f_u}{m} - \ddot{x}_g \right) + kx\dot{x}$$ (7)

Upon substituting the control force $f_u$ given by equation (2) into equation (7), equation (7) becomes

$$\frac{dV}{dt} = -c(\dot{x})^2 - m\ddot{x}\ddot{x}_g - [k_a + c_s b]$$ (8)

where $a = x\dot{x}$ and $b = (\dot{x})^2$. To make the time derivative of system energy given by equation (8) as negative as possible, the last term in parentheses must be forced to be as positive as possible by changing $k_a$ and $c_s$. Control policy for changing the active stiffness $k_a$ and $c_s$ is obtained as follows:

$$k_a = \frac{1}{2}[(k_a)_\text{max}(1 + \text{sgn}(a)) + (k_a)_\text{min}(1 - \text{sgn}(a))]$$ (9)

$$c_s = \frac{1}{2}[(c_s)_\text{max}(1 + \text{sgn}(b)) + (c_s)_\text{min}(1 - \text{sgn}(b))]$$ (10)

Since the term $b$ is always positive, the adaptive damping coefficient $c_s$ must be set to its maximum value during the control process. Therefore the variable damping part of the dynamic absorber serves as a passive damping device. This will result in a variable stiffness control. For the application of this algorithm, it is assumed that the quantities $x$ and $\dot{x}$ can be measured in real time.

3. Optimal Active Control

An idealized model of a single story damped structure under optimal active control is given in Figure 2.
Using the state vector concept, the equation of motion of this structure can be expressed as:

$$\ddot{Z}(t) = A \dot{Z}(t) + B \nu_c(t) + H\ddot{x}_g(t), Z(0) = 0$$  \hspace{1cm} (11)$$

where $Z(t) = [x(t) \dot{x}(t)]^T$, and the parameter matrices for the system, $A$, for the control location, $B$, and for force application, $H$, are

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$  \hspace{1cm} (12)$$

in which $\nu_c(t)$ is the active control force. If the control forces are assumed to be linear, then for the sake of simplicity

$$\nu_c(t, Z) = -K Z$$  \hspace{1cm} (13)$$

where the gain matrix $K$ includes the constant coefficients for the structural control devices.

Substituting equation (13) into equation (11) results in

$$\ddot{Z}(t) = A_e \dot{Z}(t) + H\ddot{x}_g(t), Z(0) = 0$$  \hspace{1cm} (14)$$

where the controlled system matrix $A_e$ is

$$A_e = A - B K$$  \hspace{1cm} (15)$$

In obtaining the optimal gain matrix $K$, the classical quadratic performance measure

$$J = \int_{0}^{t_1} (Z^T Q Z + r \nu_c^2) dt$$  \hspace{1cm} (16)$$

is minimized under the constraints imposed by equation (11), where $[0, t_1]$ time interval is the control time and is defined to be longer than that of the external excitation duration; $Q$ is a $(2 \times 2)$ dimensional positive semidefinite symmetric weighting matrix and $r$ is a positive coefficient. Numerical values for the elements of $Q$ and $r$ are assigned according to the relative importance of the state variables and the control force in the minimization procedure in order to adjust the power requirements in the actuators. Using Pontryagin’s (1962) maximum principle, the optimal gain matrix $K$ is obtained as:

$$K = \frac{1}{2r} B^T P$$  \hspace{1cm} (17)$$

where the Riccati matrix $P$ is the solution of the following equation:

$$\dot{P} + PA + A^T P - \frac{1}{2r} PBB^T P + 2Q = 0; \quad P(t_1) = 0$$  \hspace{1cm} (18)$$

The matrix Riccati equation given by equation (18) is a nonlinear matrix differential equation and is likely to cause computational difficulties. Matrix transformation can be used to avoid these difficulties. To overcome these difficulties, upon substituting the matrix transformation $P(t) = E(t)F(t)^{-1}$ into equation (18) and making the necessary manipulations, one gets the following linear matrix differential equation (Meirovitch, 1990).

$$\begin{bmatrix} \dot{E} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} -A^T & Q \\ BR^{-1}B^T & A \end{bmatrix} \begin{bmatrix} E(t) \\ F(t) \end{bmatrix}$$  \hspace{1cm} (19)$$

Since the solution of the Riccati equation in structural control applications approaches a constant value after a short time (Soong, 1990), it is generally solved as an algebraic equation by neglecting the derivative term. But, this approximation will not be used in this paper.

4. Numerical Solution

The equations of the control algorithms defined in this study have a form similar to that of the first order matrix differential equation which is given below.

$$\ddot{S}(t) = DS(t) + p(t)$$  \hspace{1cm} (20)$$

where $p(t)$ denotes all known terms including the excitation. Solution of this equation for the initial condition $S(0)$ given by Gantmacher (1960 ) is

$$S(t) = \Lambda_0(D)S(0) + \int_{0}^{t} \Lambda_0(D)[\Lambda_0(D)]^{-1} p(\tau)d\tau$$  \hspace{1cm} (21)$$

Here

$$\Lambda_0(D) = e^{\int_{0}^{t} D(\tau)d\tau}$$  \hspace{1cm} (22)$$

Equation (21) has the following form for the constant $D$ matrix.

$$S(t) = e^{Dt}S(0) + \int_{0}^{t} e^{D(t-\tau)} p(\tau)d\tau$$  \hspace{1cm} (23)$$

In many cases, it is not possible to obtain the closed form solution of $\Lambda_0$ and $e^{Dt}$. Therefore, numerical methods are employed for the solution of the
problem. When the matrix $D$ is constant, $e^{Dt}$ can be expressed as a power of $\Theta$, $(\Theta = Dt)$, using the Taylor’s expansion as follows

$$e^\Theta = I + \Theta + \frac{1}{2!}\Theta^2 + \cdots = \sum_{k=0}^{\infty} \frac{\Theta^k}{k!}$$  \hspace{1cm} (24)

To find the numerical solution to the homogeneous part of differential equations given by equation (20) with the aid of the Runge-Kutta method, let us divide the control length $[0, t_1]$ into $m$ equal intervals. There are $(m+1)$ points $(0, 1, 2, ..., m)$ and $m$ intervals $(1, 2, ..., m)$. By using the fourth order Runge-Kutta method, $S$ vector at point $(j+1), S_{j+1}$, can be written in terms of $S_j$ at point $j$ as follows:

$$S_{j+1} = S_j + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$ \hspace{1cm} (25)

Here, $h$ is the interval length, $k_1, k_2, k_3$ and $k_4$ values are given by

$$k_1 = D_{j+1}S_j$$
$$k_2 = D_{j+1}\left(S_j + \frac{h}{2}k_1\right)$$
$$k_3 = D_{j+1}\left(S_j + \frac{h}{2}k_2\right)$$
$$k_4 = D_{j+1}(S_j + hk_3)$$

where the matrix $D_{j+1}$ is the average value of the $D$ matrix in the $(j+1)th$ interval. Upon substituting the $k_1, k_2, k_3$ and $k_4$ given by equation (26) into equation (25), $S_{j+1}$ can be expressed as

$$S_{j+1} = G_{j+1}S_j$$ \hspace{1cm} (27)

Here, $G_{j+1}$ is given by

$$G_{j+1} = I + hD_{j+1} + \frac{1}{2!}h^2D^2_{j+1} + \frac{1}{3!}h^3D^3_{j+1} +$$
$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \frac{1}{4!}h^4D^4_{j+1}$$ \hspace{1cm} (28)

It is seen in equation (27) and equation (28) that the fourth order Runge-Kutta method is equivalent to the Taylor series method of fourth order except for the roundoff errors. For a fixed step size $h$ and a constant matrix $D$, the matrix $G$ is constant. So, in the Taylor series method, the matrix $G_{j+1}$ is computed only once. The Runge-Kutta method would require the calculations of $k_1, k_2, k_3$ and $k_4$ at each step. High order terms can be easily added to equation (28) when it is needed. In this study, in the numerical solution of the problem, the Taylor series method is used step by step because of its advantages over the Runge-Kutta method.

To calculate the integral which appears in equation (23), let us define the term $c_{j+1}$ in the $(j+1)th$ interval as in the following form.

$$c_{j+1} = \int_0^h e^{D_{j+1}(h-\tau)}p(\tau)d\tau$$ \hspace{1cm} (29)

If it is assumed that $p(\tau)$ varies linearly in each interval, $c_{j+1}$ can be calculated by expanding the power function in equation (29) to the Taylor series as follows:

$$c_{j+1} = b_{j+1}p_j + d_{j+1}(p_{j+1} - p_j)$$ \hspace{1cm} (30)

Here, $p_j$ and $p_{j+1}$ are the values of $p(t)$ at point $j$ and $j+1$, respectively; $b_{j+1}$ and $d_{j+1}$ are given by

$$b_{j+1} = h \left( I + \frac{1}{2}hD_{j+1} + \frac{1}{6}h^2D^2_{j+1} + \frac{1}{24}h^3D^3_{j+1} + \frac{1}{120}h^4D^4_{j+1} \right)$$ \hspace{1cm} (31)
$$d_{j+1} = h \left( \frac{1}{2}I + \frac{1}{6}hD_{j+1} + \frac{1}{24}h^2D^2_{j+1} + \frac{1}{120}h^3D^3_{j+1} + \frac{1}{720}h^4D^4_{j+1} \right)$$

The $S$ vector at the $kth$ point, $S_k$, can be calculated in terms of $S_0$ by the repeated applications of equation (27) from the first interval to the $kth$ interval by taking into account $c_i$. When $G$ is constant, the expression of $S_k$ can be given by the following simple form:

$$S_k = G^kS_0 + \sum_{i=1}^{k} G^{k-i}c_i$$ \hspace{1cm} (32)

5. Numerical Example

For the purpose of illustration, the behavior of a damped single story structure is investigated under El Centro ground motion shown in Figure 3. Selected structural parameters are given in Table 1.

The behavior of the investigated structure is analyzed for the cases of uncontrolled, optimal active control, proposed semiactive control and passive control. Relative displacements of the structure are given in Figure 4 for the cases of uncontrolled and the proposed semiactive control.
Table 1. Parameters of the idealized structure.

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>50 tons</td>
</tr>
<tr>
<td>( k )</td>
<td>47000 kN/m</td>
</tr>
<tr>
<td>( c )</td>
<td>90 kN sec/m</td>
</tr>
<tr>
<td>( (k_s)_{\text{max}} )</td>
<td>15000 kN/m</td>
</tr>
<tr>
<td>( (k_s)_{\text{min}} )</td>
<td>2 kN/m</td>
</tr>
<tr>
<td>( (c_s)_{\text{max}} )</td>
<td>70 kN sec/m</td>
</tr>
<tr>
<td>( (c_s)_{\text{min}} )</td>
<td>5 kN sec/m</td>
</tr>
</tbody>
</table>

Figure 3. El Centro ground motion

Figure 4. Uncontrolled and controlled displacements.

Even though all calculations are performed for the entire earthquake duration, only the first 4 second-parts of the graphs are presented since the maximum values occur in these parts. As shown in Figure 4, earthquake induced uncontrolled response of the structure can be reduced effectively via the proposed semiactive control. The reduction in the peak response of the uncontrolled structure is 42%. It is obvious that this reduction is a result of the control scheme given by equation (9) and equation (10) under the given assumptions. This significant decrease in the response can also be seen in Figure 5 in terms of the reduction in the system energy. As shown in Figure 5, the energy of the uncontrolled structure is reduced drastically due to the applied semiactive control.

The change in the force in the variable stiffness element \( k_s \) versus story relative displacement \( x \) is shown in Figure 6.

Figure 5. Effect of the semiactive control on the uncontrolled system energy.
As clear from Figure 6, changing the variable stiffness coefficient according to the control scheme given by equation (9) and equation (10) generates the hysteresis loops, which result in energy dissipation as shown in Figure 5.

The passive case, in which the variable stiffness and damping coefficients are set to their maximum values during the control process, is also investigated in order to see the effect of semiactive control. In the passive case, the structure has a damping ratio of 0.045 while the uncontrolled main structure has a damping ratio of 0.029. Relative displacements and the control forces which occur in the control element are shown in Figures 7 and 8. It is obvious from these figures that the proposed semiactive control is better than the passive case in terms of the peak response and control force reduction. Reductions in peak response and force in the semiactive case compared with the passive case are 18.6% and 17.7%, respectively.
However, if the variable stiffness $k_s$ is changed in reverse order, then it is expected that this will cause an energy increase in the structure, which results in increasing in the displacements. The effect of the reverse changing of $k_s$ on the uncontrolled system response is shown in Figure 9.

**Figure 9.** Response of the structure for reverse changing of active stiffness element

In the optimal active control case, which is an application of the linear regulator problem, numerical values for the elements of the weighting matrix $Q$ and the positive coefficient $r$ are selected as

$$Q = \begin{bmatrix} 600000 & 0 \\ 0 & 0 \end{bmatrix} ; \quad r = 0.0001 \quad (33)$$

By increasing the numerical values of the weighting matrix $Q$ compared with $r$, it is possible to suppress the structural response completely. But, this will of course require very large control forces. Time histories of the controlled and uncontrolled displacements and the optimal active control force for this case are given in Figures 10 and 11, respectively.

**Figure 10.** Displacement for optimal active control case

**Figure 11.** Optimal active control force
It is obvious that optimal active control is more effective than semiactive control in terms of the peak response reduction, if the necessary optimal active control force can be provided. Structural vibrations can even be suppressed completely via active control systems if there is enough external energy. Maximum displacements and control forces for the investigated cases are also summarized in Table 2.

**Table 2.** Maximum displacements and control forces.

<table>
<thead>
<tr>
<th></th>
<th>Displacement x (cm)</th>
<th>Control force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>0.83</td>
<td>-</td>
</tr>
<tr>
<td>Semiactive</td>
<td>0.48</td>
<td>73.50*</td>
</tr>
<tr>
<td>Passive</td>
<td>0.59</td>
<td>89.30*</td>
</tr>
<tr>
<td>Reverse</td>
<td>1.04</td>
<td>155.10</td>
</tr>
<tr>
<td>Optimal active control</td>
<td>0.18</td>
<td>127.60</td>
</tr>
</tbody>
</table>

*These forces are not the actual active control forces, but the forces occur in the control element.

It must be noted here that the energy need for a semiactive device is always much less than the energy need of an active control system. This is valid even if the force which occurs in the semiactive device is larger than the active control force generated by an actuator, because a semiactive device needs a very small amount of energy which can be supplied even by a battery just to change the variable stiffness or damping coefficient, not to generate an actual active control force.

6. Conclusions

This paper presents a new semiactive closed loop control algorithm, by using a semiactive device with variable stiffness and damping characteristics to suppress the vibrations of earthquake-excited structures. The performance of the proposed algorithm is demonstrated by numerical simulations on a single story damped structure subjected to El Centro ground motion. It is shown that the energy level and the earthquake induced vibration amplitude of the uncontrolled structure can be suppressed effectively. This result is also confirmed by the hysteresis loops occurring in the structure. It is illustrated by numerical simulation that changing the variable stiffness in reverse order causes an energy increase in the uncontrolled structure. It is also shown that the proposed method provides better performance than the passive control in which the variable stiffness and damping coefficients are set to their maximum values in terms of the reduction in peak response and force. Among all the investigated cases studied herein, except the optimal active control case, the proposed algorithm delivers the most damping to the system response. But, it should be noted here that the energy requirements for high damping of the fully active control systems are very large and these systems might not operate during large earthquakes due to power failure. But, semiactive systems can operate even during large earthquakes since their energy requirement can be supplied even by a battery.

References


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