Improvement of the distribution network state estimation with increase of accurate information and using a two-step method

Karim AMIRI, Rasool KAZEMZADEH
Renewable Energies Research Center, Faculty of Electrical Engineering, Sahand University of Technology, Tabriz, Iran

Abstract: Distribution networks (DNs) are gradually changing and this makes their control and utilization complicated. State estimation (SE) plays a significant role in active distribution networks. The performance of the energy management center in modern distribution networks is highly dependent on the results obtained from the SE. In the present study, considering the shortage of measurements in the DN, a two-step state estimation method with a new network reduction process (NRP) is proposed. In the proposed method, a new NRP is used in a two-step state estimation method to improve the performance of SE in a DN. Obtaining accurate initial information on the network condition improves the performance of SE. The initial SE is performed using a new NRP process to obtain accurate initial data. This information is used as the measurement to improve the performance of secondary SE. This method resolves the shortage of accurate measurements and redundancy measurements, and it improves network SE accuracy without adding any real-time measurement. Moreover, the proposed method is economically affordable. Simulations are performed on the 18-bus UK radial feeder and the IEEE 69-bus distribution network in MATLAB software to guarantee valid operation of the proposed style.

Key words: Distribution network, meters placement process, network reduction process, state estimation, two-step state estimation, weighted least squares error

1. Introduction

The presence of renewable energy resources, distributed generations (DGs), energy storages, and controllable nonlinear loads in active distribution networks (ADNs) is increasing. Electrical power transmission in the feeders of ADNs is double-sided. Energy management centers (EMCs) require complete and accurate information on network status [1]. Considering the development of measurement devices and communication problems, state estimation (SE) is a proper tool for providing real-time accurate information [1, 2]. SE is a relationship between the state variables of the system and measurement parameters [1]. SE provides the state variables of the system with a limited number of measurements. SE has been used in transmission networks since 1970 and it has been studied extensively since then [3–5]. Distribution system state estimation (DSSE) is designed to obtain operational states of the distribution network (DN) [3–5]. EMCs in smart networks and ADNs require SE for proper control and exploitation [4]. Considering DN with a weak radial or circular structure, high R/X ratio, unbalance, high number of buses, and load change, the weighted least squares error (WLSE) algorithm is chosen as a suitable SE method in DNs [1, 6–8].

*Correspondence: r.kazemzadeh@sut.ac.ir

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DSSE was developed in the 1990s [1, 6, 9, 10], in which the fundamentals of SE in a balanced or an unbalanced DN were described using the WLSE method. The number of measurements in the DN is limited, and they measure only the voltage amplitude and the consumption load of main station of the DN. Therefore, load prediction information is used as a pseudo-measurement, which helps to make the network observable, but its accuracy is low.

The authors of [1] used branch currents as state variables in DSSE and described advantages of this method over SE in terms of voltage of buses. The authors of [9] used zero injection buses (buses in which the sum of generated and consumed active and reactive power is zero) as high-accuracy measurements. They also described SE formulation in the three-phase DN as a nonlinear and complicated relationship regarding powers and voltages. The work in [10] investigated a distribution network imbalance in SE equations, the probabilistic concept of measurement information, and the effect of uncertainties, such as location and error of measurements, on the estimation accuracy. Studies conducted on DSSE have been initiated since the 1990s; the works presented in [9, 11] investigated DSSE cases. Since the 2000s, researchers started complementary studies on DSSE. In [12], the authors resolved the disadvantages of [1].

In [12], the measurement function and elements of the Jacobin matrix were obtained as linear and constant by normalizing the current equation. The authors of [13, 14] used the phasor measurement unit (PMU) in DSSE. Since the information of PMU is evaluated regarding coordinated universal time, a reference bus is not considered for calculating the angle of other buses of the DN. The voltage phasor of the reference bus is added to the state vector as a state variable.

The work in [15] presented the real modeling of a load as another uncertainty of the DN in the SE process of unbalanced three-phase distribution networks, which makes estimation conditions close to a real network. The authors of [2, 16] proposed a DSSE formulation based on the current of branches in the presence of PMU and DG in polar and Cartesian spaces, in which uncertainty of correlation between measured information is considered. Considering simulation results [2], SE based on branch current in Cartesian space is linear and the gain matrix is constant, which improves SE speed and accuracy. Between 2000 and 2013, authors conducted extensive studies on DSSE, as a result of which branch current-based state estimation (BCSE) was improved gradually, but the shortage of measurements and accuracy of DSSE have not been resolved completely yet. In 2014 and 2015, many studies were proposed that investigated various uncertainties of a DN and measurements [17–19].

The work in [17] studied the effect of measurement information correlation on DSSE results. Different types of measurements with various correlations have been considered, and this has improved the estimation accuracy. The work in [18] investigated the effect of different measurements on SE of the unbalanced standard IEEE-123 bus three-phase network. Under static conditions, the SE accuracy with nonsynchronized measurement (NSM) is obtained equal to the SE accuracy using PMU [18]. Comparison of NSM and PMU under dynamic conditions shows that PMU outperforms NSM due to its synchronized performance and the fast sampling rate, especially under significant load changes. Finally, [19] investigated NSM; this type of measurement provides nonsynchronized information with a sampling rate of about 15 min. Network conditions and loads might change between two sampling times, and these changes are added as an error with the normal distribution to the nonsynchronized measurement error [19].

Considering the studies conducted on DSSE, there are still problems, such as the shortage of accurate measurements with the proper sampling rate and real-time estimation of state variables. Considering cost
measurement, PMU is not widely used in a DN like a power system. DNs have a few real measurements. Extensive use of measurements, like PMU, increases costs, which is unfavorable [2, 7, 16, 18]. To make ADNs observable, previous information on a load and a generation are used as low-accuracy pseudo-measurements, and extensive use of these measurements decreases the estimation accuracy. Therefore, a method should be proposed by which accurate information on the network conditions is provided for the state estimator to replace pseudo-measurements.

Generally, in transmission system state estimation and DSSE, bad data detection (BDD) is based on the measurement residual [20]. At the transmission system level, Liu et al. proposed that an attacker can launch false data injection (FDI) attacks against state estimation to avoid being detected by the commonly used residual-based BDD [21]. The injected false data in a malicious attempt can destabilize the distribution network and can lead to serious financial loss and safety issues and can deteriorate the control performance of the system. This result has motivated researchers to do extensive study on investigating the construction of FDI attacks, the impact on the operation of power systems, and possible protective countermeasures [22]. For example, Liu and Li showed that an attacker can attack the real-time topology using the local network information of a power grid [23]. Cyberattacks are based on the information of actual meters to sabotage electrical networks, the most important of which are PMUs [24]. Therefore, reducing transient measurement information by reducing actual measurements or optimal locating processes will make the network more robust to FDI [24]. Isozaki et al. showed how data integrity attacks in the DN can certainly affect the power quality at the consumer’s side, damage power devices in DNs, and cause economic impacts on the revenue of DG owners [25]. Parvania et al. showed that cyberattacks in the DN can disrupt the performance of different processes in future automated distribution systems such as fault location, isolation, and service restoration, leading to reliability problems [26]. Moreover, an attacker can manipulate measurements in such a way that will lead to opening several circuit breakers to cause customers to lose power [24].

Therefore, considering the research outlined above, an appropriate method is needed to limit the use of pseudo-measurements in DSSE. In this paper, a two-step method with a NRP is proposed, by which accurate information of the network condition is provided for the state estimator to replace pseudo-measurements. Novelties of the proposed method can be listed as follows:

- Presenting a new network reduction formulation.
- Increasing accurate information on a network in the SE process and replacing pseudo-measurements with it.
- Considering the reduced number of pseudo-measurements, improving the SE operation in a DN with low number of real-time measurements.
- Improving the observability of network and making the SE robust against bad data.
- Accurate information on the initial SE not being available to cyberattackers.

The rest of this paper is organized as follows: Section 2 describes WLSE briefly. Section 3 describes a new network reduction formulation and a two-step-method DN. Section 4 shows the problem formulation. Section 5 presents the simulation results of the two-step state estimation based on the network reduction for the 18-bus UK radial feeder and the IEEE 69-bus distribution network in MATLAB software and the results.
are investigated. Simulation results are given and features and superiorities of the proposed method are represented. Finally, the paper is concluded.

2. Weighted least square error

The WLSE has been widely studied in the literature [1–4, 6–12, 16–19]. WLSE is a probabilistic method that uses the square error between measured and estimated values. Eq. (1) represents the relationship between measured values and state variables. 

\[ Y_e = Y + e = F(\hat{x}). \] (1)

The error of the \( i \)th measurement is assumed to have normal distribution with zero mean and a specified standard deviation of \( \sigma_i \). The WLSE method takes the gradient of the objective function regarding state variables and sets it equal to zero. Eq. (2) gives an estimation of the state vector in each iteration of the Newton method [3].

\[ x^{k+1} = x^k - [G(x^k)]^{-1} g(x^k). \] (2)

Eq. (3) and Eq. (4) represent the details of \( G(x^k) \), \( g(x^k) \), \( H(x^k) \), \( F(x^k) \), \( Y \), and \( R \). \( H(x^k) \) is obtained from the relative differentiation of the measurement function concerning the state variables, which is called the Jacobian matrix [3].

\[ H(\hat{x}) = \frac{\partial F(\hat{x})}{\partial \hat{x}}; \quad g(x^k) = H^T(x^k) R^{-1} (Y - F(x^k)); \quad G(x^k) = \frac{\partial g(x^k)}{\partial x^k} = H^T(x^k) R^{-1} H(x^k). \] (3)

\[ F(\hat{x}) = \begin{pmatrix} f(x_1, \ldots, x_n) \\ \vdots \\ f(x_1, \ldots, x_n) \end{pmatrix}, \quad Y = [y_1 \ y_2 \ \ldots \ y_m]^T, \quad R = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \ldots \\ \sigma_m \end{pmatrix}. \] (4)

\( W = R^{-1} \) is called the weighting matrix, which determines the effect of meters on the convergence and the accuracy of the SE [3].

The next section of this paper describes NRPs and the two-step estimation method based on WLSE.

3. Network reduction process and the two-step estimation

The original reason for the implementation of the two-step SE method is to increase the number of accurate data without adding the actual measurement to the DN measurement system. High numbers of accurate meters will increase the accuracy of SE.

3.1. New formulation of reduction network process

The main objective of the two-step estimation method is to improve SE by increasing accurate data on network conditions. The DN has a large number of buses and installing measurements on each bus of the network is not cost-effective. Therefore, load prediction information is used as a pseudo-measurement in the estimation process. The standard deviation of these measurements is high, and large numbers of measurements are required for
To solve this problem, the network reduction idea is proposed. It reduces the number of network buses while preserving important and critical buses of the network. Reduction methods such as Ward reduction, Kron reduction, Dimo’s reduction, Zhukov’s reduction, sparsity of equivalents, market-based reduction techniques, the PTDF-based reduction method, and LMP-based reduction have been proposed in power systems [27, 28].

In [27, 28], reduction methods were compared with each other, and the results were extracted: (1) the Ward and Kron methods can be applied to both small and large systems, and they are less complicated than the methods used for aggregating nodes, but the accuracy of the Kron method is lower compared to other methods. Ward’s reduction technique gives the best results. (2) Zhukov’s method operates well with smaller power systems with greater accuracy but it is not useful for large power systems, which might be due to the large r/x ratio of lines in the equivalent network. (3) Dimo’s method has obtained better results compared to Zhukov’s for both small and large power systems. However, it is helpful only with little load changes; for major load changes, the reduced system does not follow the original system. (4) Market-based methods focus more on congestion and cost characteristics; however, the equivalent network depends on the actual configuration of the system. For large-scale systems, it is almost impossible to know the marginal costs of production in all control areas. (5) LMP and PTDFs are mainly based on actual generation and consumption levels. Considering the results presented in [27, 28], Ward’s reduction is the simplest and most appropriate method for the network reduction in power systems. Considering unique features of a DN, Ward’s reduction method is extended to DNs. Since most DNs are radial, node reduction equations of the radial DN can be obtained.

Figure 1 shows part of a radial DN in which nodes 2 and 3 are eliminated thoroughly using NRP. Node 4 in Figure 1a is the node assumed to be fixed during network reduction. Kirchhoff’s current law (KCL) is applied to Figure 1a to obtain Eq. (5). Eq. (6) gives V₂ in terms of other parameters of the original network.

$$\begin{align*}
- Y_{12}V_1 + (Y_{12} + Y_{23})V_2 - Y_{23}V_3 &= I_2 \\ KCL \ Node2: &- Y_{12}V_1 + (Y_{12} + Y_{23})V_2 - Y_{23}V_3 = I_2 \\
KCL \ Node3: &- Y_{23}V_2 + (Y_{23} + Y_{34})V_3 - Y_{34}V_4 = I_3 \\
KCL \ Node4: &- Y_{34}V_3 + (Y_{34} + Y_{45})V_4 - Y_{45}V_5 = I_4.
\end{align*}$$

If Eq. (6) is substituted in the KCL written for node 3 in Eq. (5) and mathematical simplifications are applied, Eq. (7) is obtained. Node 2 is eliminated from the radial DN of Figure 1a as shown in Figure 1b. Eq. (8) shows the KCL governing node 3 of Figure 1b.

$$\begin{align*}
- Y_{12}V_1 + (Y_{12} + Y_{23})^{-1}V_1 + (Y_{12}Y_{23}(Y_{12} + Y_{23})^{-1} + Y_{34})V_3 - Y_{34}V_4 &= Y_{23}(Y_{12} + Y_{23})^{-1}I_2.
\end{align*}$$

Figure 1. (a) Part of a radial DN. (b) Reduced network with eliminated node 2. (c) Decreased network with retrenched nodes 2 and 3.
\[ KCL \text{ Node}3 : -Y_{eq13}V_1 + (Y_{eq13} + Y_{34})V_3 - Y_{34}V_4 = I_{eq3}. \] (8)

If Eq. (7) and Eq. (8) are set to be equal, the equivalent current of node 3 and equivalent admittance \( Y_{eq13} \) can be obtained by eliminating node 2, which is shown in Eq. (9).

\[
Y_{eq13} = \frac{Y_{12}Y_{23}}{Y_{12} + Y_{23}}, \quad I_{eq3} = I_3 + \frac{Y_{23}}{Y_{12} + Y_{23}}I_2. \tag{9}
\]

Considering Eq. (9), a node of the radial DN can be eliminated so that the voltage of other nodes remains constant. The equivalent admittance \( Y_{eq13} \) is obtained using the admittance of feeders connected to node 2 (eliminated node), and the equivalent current of node 3 \( (I_{eq3}) \) is obtained with respect to the consumed current of node 2 and 3 and the admittance of feeders connected to node 2 (eliminated node). Figure 1c shows the network after node elimination, in which node 2 is eliminated from Figure 1b and equations describing this process are represented in the following. KCLs governing the network shown in Figure 1b are given in Eq. (10).

\[
\begin{align*}
KCL & \text{ Node3} : -Y_{eq13}V_1 + (Y_{eq13} + Y_{34})V_3 - Y_{34}V_4 = I_{eq3} \\
KCL & \text{ Node4} : -Y_{34}V_3 + (Y_{34} + Y_{eq13})V_4 - Y_{45}V_5 = I_4.
\end{align*}
\tag{10}
\]

If \( V_3 \) is obtained from the KCL governing node 3 in Eq. (10) regarding other parameters of the network and substituted in the KCL governing node 4 in Eq. (10), Eq. (11) is obtained. The KCL equation governing node 4 shown in Figure 1c is written in Eq. (12).

\[
-Y_{34}Y_{eq13}(Y_{34} + Y_{eq13})^{-1}V_1 + (Y_{34}Y_{eq13}(Y_{34} + Y_{eq13})^{-1} + Y_{45})V_4 - Y_{45}V_5 = I_4 + Y_{34}(Y_{34} + Y_{eq13})^{-1} I_{eq3}. \tag{11}
\]

\[
KCL \text{ Node4} : -Y_{eq14}V_1 + (Y_{eq14} + Y_{35})V_4 - Y_{45}V_5 = I_{eq4}. \tag{12}
\]

By setting Eq. (11) equal to Eq. (12), the equivalent admittance and the injected current are obtained by eliminating nodes 2 and 3 from the main network, as represented in Eq. (13).

\[
Y_{eq14} = Y_{34}Y_{eq13}(Y_{34} + Y_{eq13})^{-1}, \quad I_{eq3} = I_4 + Y_{34}(Y_{34} + Y_{eq13})^{-1} I_{eq3}. \tag{13}
\]

If the value obtained from Eq. (9) is substituted in Eq. (13), Eq. (14) is obtained.

\[
Y_{eq14} = \frac{Y_{12}Y_{23}Y_{34}}{Y_{12}Y_{23} + Y_{12}Y_{34} + Y_{23}Y_{34}}, \quad I_{eq4} = I_4 + \left( \frac{Y_{12}Y_{34}}{Y_{12}Y_{23} + Y_{12}Y_{34} + Y_{23}Y_{34}} \right) I_3 + \left( \frac{Y_{23}Y_{34}}{Y_{12}Y_{23} + Y_{12}Y_{34} + Y_{23}Y_{34}} \right) I_2. \tag{14}
\]

By continuing node elimination, the required number of nodes can be eliminated from a red DN and consequently the equivalent admittance and the injected current can be obtained. Considering Eq. (14), the admittance of feeders of the DN plays a significant role in the NRP.

After presenting a new formulation of the NRP, the important point is how to determine the fixed nodes in the NRP. The nodes and branches obtained from the metering location process MLP are candidates for fixed nodes in the NRP. Therefore, in the next section, the MLP is briefly represented.
3.2. The process of locating meters

A MLP can find the best place and type of measurement [29–31]. A locating method is used to determine the appropriate location of meters. Locations obtained from the optimal locating problem remain constant in the NRP. An optimal locating method was used in [30] to determine the appropriate type and location of the measurement. In [30], a complete formulation of the proposed location method was presented. The MLP includes the following steps [30]:

Step 1: The WLSE state estimation is applied to the number of Monte Carlo simulations ($N_{mc}$) in the main network. The relative errors of the magnitude and the angle of voltage are extracted in each Monte Carlo simulation for all buses.

Step 2: If in more than 95% of cases, the relative errors of the voltage magnitude and the voltage angle are below the threshold (1% for voltage ($\epsilon_1$) and 5% for the angle ($\epsilon_2$)), stop the process; otherwise, go to step 3.

Step 3: If only the relative errors related to the voltage magnitude indicated in step 2 are below the threshold, go to step 6; otherwise, go to step 4.

Locating voltage measurements:

Step 4: The error mean covariance matrix of the node voltage is calculated in the Monte Carlo simulation ($N_{mc}$), and the submatrix corresponding to the voltage magnitude and the voltage angle (the real and imaginary part of each node or branch) of each bus is extracted.

Step 5: The error ellipse area (EEA) is calculated from the submatrix determinant in each node, the node with the largest EEA is identified, and the voltage measurement in this node is determined. If the measurement already exists in the desired node, the largest EEA is selected as the next. Go to step 1.

Locating current feeders or flow power measurements:

Step 6: The error mean covariance matrix corresponding to branch currents in $N_{mc}$ simulations is calculated. The submatrix corresponding to the amplitude and angle of each branch current (the real and the imaginary part of the branch current) is extracted. Step 7: The EEA of each branch of the network is calculated, and the flow measurement is placed in the branch with the largest area. If there is already a current measurement in this branch, the flow measurement in the next branch will be placed with the largest EEA. Go to step 1.

Now, after presenting the issues needed for the two-step state estimation, in the next section, the two-step state estimation method with the NRP will be thoroughly presented.

4. The two-step state estimation method with reduced network process

For clarity, the flowchart of the two-step estimation method is shown in Figure 2. It should be noted that transmission information from one block to another is given in ellipses. In the proposed method, there are two basic steps. Step 1 is about the NRP and the initial SE in the reduced network and step 2 is about the secondary SE in the original network. According to Figure 2, the two-step estimation method has substantial points, which are investigated in the following:

First of all, the original network is reduced using Eq. (9) to Eq. (14). Eq. (9) to Eq. (14) include feeder admittances of the main DN, which are the admittances used in power flow. The number and the location of network nodes kept fixed during the NRP are determined. Nodes of the network are fixed and the general structure and sensitive nodes of the network are preserved. Therefore, nodes including large loads (greater than 100 KVA), DGs, nodes including real measurement, nodes or branches identified as measuring locations, and nodes connected to several sideways branches are preserved during the NRP. If there is a loop inside the network, it should remain unchanged during the NRP. After the NRP, the initial SE is applied using WLSE to estimate the current of feeders of the reduced network. Since nodes and feeders including a real measurement are kept unchanged during the NRP, they can be used for the initial SE. It should be noted that information on actual measurements that can be used for the initial SE applied to the reduced network includes the following:

(a) According to Eq. (5) to Eq. (14), the voltage phasor of the fixed node (for example, node 4 in Figure 1) does not change during the NRP. Therefore, if an actual measurement of the voltage phasor exists, the amplitude and the
(b) Power measurement, consumption, and generation current measurements of the main network can be used as a measurement in the reduced network with the difference that this information should be used in Eq. (9) to Eq. (14) so that it can be used for the initial SE.

It should be noted that when the load consumption is converted to the equivalent current \((P + jQ)\), the node voltage should be used. Since the equivalent current is used in initial and secondary SEs, estimated values of the node voltage in the secondary SE should be selected as input information of the initial SE. Indeed, at the beginning of the flowchart in Figure 2, i.e. the flowchart of the two-step estimation method, estimated values of all nodes of the main network can be replaced by pseudo-measurements in the secondary SE. Since the values of the initial SE have low standard deviation compared to pseudo-measurements, the accuracy of the secondary SE of the main network can be estimated and provided for the EMC in a short time with the proper accuracy.

After the initial SE, the main estimation (secondary SE) is implemented on the main network using the output information of the initial SE. The output information of the initial SE, which is used as a measurement in the secondary SE, includes the following information: voltage phasor of nodes that are kept unchanged during the NRP and the current phasor of the output feeders connected to these nodes that do not include real measurements. The initial SE values of the reduced network can be replaced by pseudo-measurements in the secondary SE. Since the values of the initial SE have low standard deviation compared to pseudo-measurements, the accuracy of the secondary SE of the main network can be estimated and provided for the EMC in a short time with the proper accuracy.
is improved. The amount of accurate information is increased without increasing the cost. The output information of the secondary SE includes the voltage phasor of all nodes of the main network. In the following, mathematical formulas governing the secondary SE are written for clarification.

5. Results and discussion

Simulations are performed in MATLAB 2016 using a PC with Intel Core i5 and 2.3GHz. The following issues are considered in the simulation process: (1) The number of Monte Carlo experiments to validate estimation results is assumed to be $N_{mc} = 50000$ [2]. (2) The distribution function of measurements is a Gaussian function, whose standard deviation is one-third of the maximum error compared to real values obtained using Eq. (15) [2]. Furthermore, 99.7% of the Gaussian region is the interval $(\mu_i - 3 \times \sigma_i, \mu_i + 3 \times \sigma_i)$ [2].

\[
\sigma_i = \frac{\mu_i \times \max \text{ Error}_i}{3 \times 100}.
\]

(3) The maximum error of different measurements includes the following: pseudo-measurement: 20%; current and voltage amplitude measurement of PMU: 1%; current and voltage phase angle measurement of PMU: 0.01 radians; nonsynchronized voltage amplitude in static and dynamic modes: 1% and 5%; nonsynchronized current amplitude measurement and load flowing through feeders: 3% [2]. (4) The real values of state variables obtained from the load flow are applied to the network [23]. The mean value of measurement parameters in the Gaussian distribution is assumed to be the real value. In addition, the feeder admittance of the DN used in the power flow is applied in the NRP. (5) The two-step estimation method with the NRP is implemented on the 18-bus UK radial feeder and the IEEE 69-bus distribution network.

A conventional SE method is applied to the 18-bus UK radial feeder and the IEEE 69-bus distribution network based on the current of branches [2] (BCSE method) and results are compared with the two-step state estimation using the NRP (proposed method). Information about the 18-bus UK radial feeder and the IEEE 69-bus distribution network can be found in [2] and [12], respectively.

5.1. 18-Bus UK radial feeder

The 18-bus network has two PMU measurements at buses 4 and 11 where the voltage phasors of nodes and current phasors of branches connected to the node are measured. A voltage amplitude measurement in the slack bus, 5-nodes as zero injection node measurements, and 10-nodes as pseudo-measurements are other measurements in the 18-bus network. The 18-bus network is reduced to an 8-bus network using the network reduction process. The sensitive buses of the network are preserved during the NRP.

5.2. Meter placement and the initial state estimation for the 18-bus UK radial feeder

Before implementing the two-step state estimation, the MLP is implemented on the 18-bus UK radial feeder to determine the best locations and types of measurements. The threshold values for the voltage relative error and the voltage angle relative error are 1% and 5%, respectively. Suitable locations of the current phasor measurement are obtained to be branches 1, 2, 6, 7, 9, and 16 of the 18-bus UK radial feeder. Therefore, considering the output of the MLP, i.e. the nodes including PMU, large load, and network structure, the 18-bus UK radial feeder is reduced to an 8-bus network using NRP. The initial SE is applied to the 8-bus network. The impact of the measurements obtained from the MLP on the relative error of SE of each Monte Carlo simulation is presented in Figures 3a and 3b. Figure 3a illustrates the voltage magnitude estimation and the relative error of the voltage angle estimation ignoring the MLP in the measurement system of SE. Figure 3b shows the relative error of the voltage magnitude estimation and the relative error of the voltage angle estimation considering the MLP in the measurement system of the SE. Adding the measurements obtained from the
MLP to the SE measurement system allows 95% of the simulation results to be specified below the thresholds (\( \epsilon_1 = 1\% \) and \( \epsilon_2 = 5\% \)) in the Monte Carlo test, which are compared in Figures 3a and 3b.

**Figure 3.** The voltage magnitude and angle estimation relative error: (a) without considering locating measurements, (b) with considering locating measurements.

According to the locations specified by the MLP, the number of suitable locations for measurements is high, and installing real-time measurements at these locations is not economically feasible. Therefore, a solution is needed to solve this problem. The two-step state estimation can obtain the desired performance in Figure 3b without adding real-time measurements to these locations. In the two-step state estimation method, the locations of measurements
are kept constant in the NRP so that the parameters can be estimated precisely in the initial SE implemented on the reduced network and applied in the second SE. In other words, it can be said that the two-step state estimation method applies the precise information obtained from the initial SE to the secondary SE as meters. Therefore, the two-step state estimation can be an appropriate method for solving the redundancy measurement problem. The modification of state estimation will be displayed in the following.

The MLP is based on the ellipse error maximum area (EEMA) for the estimation error covariance matrix of network branches [30]. Details of the ellipse error calculation are presented in [30]. The ellipse error is calculated for every branch of the network that includes real and imaginary parts in the estimation error covariance matrix and the EEMA is selected as a candidate place for installing a real-time measurement.

Figure 4 shows the EEMA for the estimation error covariance matrix of the branches of the DN before and after the MLP. Figure 4a shows the EEMA before applying measurements, which is related to the current of the 7th branch of the 18-bus network, while Figure 4b shows the EEMA after applying meters. It is observed that the current of the first branch of the 18-bus network has EEMA. Comparing Figure 4a with Figure 4b, it is found that the meter MLP has led to a significant decrease in the EEMA.

Now, after performing the MLP and determining the location and type of measurements, the two-step estimation method is applied with the help of the NRP on the 18-bus UK radial feeder. In the first step, the two-step estimation method reduces the balanced three-phase DN of 18 nodes through the NRP.

Therefore, a limited number of nodes can be integrated with neighboring nodes. A group of 10 nodes of the 18-bus network are combined with their neighboring nodes, and a reduced 8-bus network is obtained. Since 10 nodes of the 18-bus network are eliminated, 20 state variables will be removed from the state vector of the 18-bus distribution network. In the initial SE, three PMUs, a voltage measurement in the slack bus, and several nodes as pseudo-measurements have been used. The initial SE is performed on an 8-bus-balanced three-phase network using the WLSE method with 50000 Monte Carlo experiments.

Figure 5 shows the initial SE of the 8-bus network. In Figure 5, the amplitude and the angle of the voltage of the 8-bus network are shown, which have desirable accuracy. According to Figure 5, the NRP (Eq. (13) and Eq. (14)) is carried out on the reduced 8-bus network with desirable accuracy and the extracted results are reliable. Therefore, the estimated values in the initial SE can be used as accurate information in the second SE (the second step).

Table 1 shows important parameters of the initial SE including the mean runtime of the SE in millisecond (ms), the mean Gaussian–Newton repetition in the WLSE method, and the maximum root mean square error (RMSE) value of the voltage magnitude and the phase angle, which indicate the suitable performance of the initial SE, especially in estimating the voltage magnitude on the reduced 8-bus DN in terms of accuracy and speed. The relationship used to
calculate the RMSE of the parameter $x_i$ is given in Eq. (16) [2].

$$RMSE(\hat{x}_i) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}$$ (16)

The standard deviation of the estimated parameters in the initial SE through 50000 Monte Carlo experiments is obtained from Eq. (16) and used in the secondary SE as a measurement with the specific standard deviation. It should be noted that the estimated values in the initial SE can be sent to the main control center of the network to monitor the main network nodes that remain constant in the network reduction process. The main network nodes are monitored in a shorter time (4.6 milliseconds) due to the reduction of network dimensions. The estimated values obtained from the initial SE are used as meters with specific standard deviations along with actual measurements in the secondary SE.

**Table 1.** Important parameters of initial SE of the reduction 8-bus DN.

| Min RMSE (|V|) | Max RMSE (|V|) | Min RMSE ($\theta_V$) | Max RMSE ($\theta_V$) | Average Runtime (ms) | mean repetition |
|-------|-------|-----------|----------------|----------------|------------------|---------------|
| 0.0072 (p.u.) | 0.0112 (p.u.) | 0.0013 (rad) | 0.0076 (rad) | 4.6 | 2.2 |

**5.3. Comparison of the BCSE method and the two-step state estimation method on the 18-bus UK radial feeder**

The branch current state estimation (BCSE) [2] is implemented on the balanced 18-bus UK radial feeder using the branch current phasor as state variables in the Cartesian chart. Results of the secondary SE are compared with BCSE (an effective and common state estimation method applied in the balanced DN) to observe the obtained enhancement. The BCSE method mainly uses virtual measurements (pseudo-measurements, zero injection nodes). In order to compare the proposed method with the BCSE method, important performance parameters are presented in Table 2. Due to increased redundancy of the accurate measurements in the secondary SE of the proposed method, the convergence mean
iteration in the Monte Carlo test and the average implementation time of the secondary SE in the 18-bus network have been decreased. The maximum RMSE of the voltage amplitude and phase angle of the 18-bus network have been improved compared to the conventional method.

Table 2. Comparing different parameters of the BCSE and the proposed SE method on the 18-bus UK radial feeder.

| Methods        | Min RMSE (|V|)     | Max RMSE (|V|)     | Min RMSE (θ_V) | Max RMSE (θ_V) | Average runtime (ms) | Mean repetition |
|----------------|-------------|------------------|-----------------|----------------|---------------------|-------------------|
| Proposed method| 0.0185 (p.u.) | 0.0895 (p.u.)   | 0.0018 (rad)    | 0.0101 (rad)   | 11.4                | 1.55              |
| BCSE method    | 0.00215 (p.u.) | 0.00512 (p.u.)  | 0.00021 (rad)   | 0.0098 (rad)   | 20.3                | 5.36              |

5.4. The IEEE standard 69-bus distribution network
The 69-bus network has three PMU measurements on buses 3, 9, and 12 where the voltage phasor of node and current phasor of branches connected to the node are measured. A voltage amplitude measurement in the slack bus, 21 nodes as zero injection node measurements, and 46 nodes as pseudo-measurements are other measurements in the 69-bus network. The 69-bus network is reduced to a 21-bus network using the NRP.

5.5. Meter placement and the initial state estimation on the IEEE standard 69-bus distribution network
Before implementing the two-step state estimation, the MLP is implemented on the 69-bus network to determine the best locations and types of measurements. Suitable locations of the current phasor measurement are obtained to be branches 6, 15, 23, 26, 33, 39, 50, 59, 62, 64, and 65 of the standard IEEE 69-bus radial network and a suitable location of the voltage phasor measurement is node 27. The 69-bus network is reduced to a 21-bus network using NRP. The initial SE is applied to the 21-bus network.

The impact of the measurements obtained from the MLP on the relative error of SE of each Monte Carlo simulation is presented in Figures 6a and 6b. Figure 6a and Figure 6b demonstrate the relative error of the magnitude and angle of node voltage ignoring the MLP and considering the MLP in the measurement system of the SE, respectively. Adding the measurements obtained from the MLP to the SE measurement system allows 95% of the simulation results to be specified below the thresholds (ε_1 = 1% and ε_2 = 5%) in the Monte Carlo test, which are compared in Figures 6a and 6b. The two-step state estimation can obtain the desired performance in Figure 6b without adding real-time measurements to these locations.

The ellipse error is calculated for every branch of the network that includes real and imaginary parts in the estimation error covariance matrix and the EEMA is selected as a candidate place for installing a real-time measurement.

Figure 7 shows the EEMA for the estimation error covariance matrix of the branches of the DN before and after the MLP. Figure 7a shows the EEMA before applying measurements, which is related to the current of the 19th branch of the 69-bus network, while Figure 7b shows the EEMA after applying meters. It is observed that the current of the 50th branch of the 69-bus network has EEMA. Comparing Figure 7a with Figure 7b, it is found that the meter MLP has led to a significant decrease in the EEMA.

Now, after performing the MLP and determining the location and type of measurements, the two-step estimation method is applied with the help of the NRP on the 69-bus network. In the first step, the two-step estimation method reduces the balanced three-phase DN of 69 nodes through the NRP.

Important nodes and branches obtained from the 69-bus DN are kept in the NRP, according to the following:

- Nodes and branches obtained from the MLP (6, 15, 23, 39, 58, 62).
- Split nodes maintaining the overall network structure (4, 8, 11).
The voltage magnitude and angle estimation relative error: (a) without considering locating measurements, (b) with considering locating measurements.

- Nodes including real measurements (1, 3, 9, 12).
- End nodes of the feeder or nodes with large loads (27, 35, 46, 50, 52, 65, 67, 69).

A group of 48 nodes of the 69-bus network are combined with their neighboring nodes, and a reduced 21-bus network is obtained. Since 48 nodes of the 69-bus network are eliminated, 96 state variables will be removed from the state vector of the 69-bus distribution network. In the initial SE, three PMUs, a voltage measurement in the slack bus, and several nodes as pseudo-measurements have been used. The initial SE is performed on a 21-bus-balanced three-phase network using the WLSE method with 50000 Monte Carlo experiments.

Figure 8 shows the initial SE of the 21-bus network. In Figure 8, the amplitude and the angle of the voltage of the 21-bus network are shown, which have desirable accuracy. According to Figure 8, the NRP is carried out on the reduced 21-bus network with desirable accuracy and the extracted results are reliable. Therefore, the estimated values
in the initial SE can be used as accurate information in the second SE (the second step). To evaluate the initial SE operation for 50000 Monte Carlo experiments, the estimation error for the voltage magnitude and angle are given in Figure 9. Figure 9 shows that the initial SE operation is appropriate for each Monte Carlo experiment.

Table 3 shows important parameters of the initial SE, which show the suitable operation of the initial SE, especially in estimating the voltage magnitude on the reduced 21-bus DN in terms of accuracy and speed.

| Min RMSE ($|V|$) | Max RMSE ($|V|$) | Min RMSE ($\theta_V$) | Max RMSE ($\theta_V$) | Average runtime (ms) | Mean repetition |
|-----------------|-----------------|-----------------------|-----------------------|----------------------|-------------------|
| 0.01 (p.u.)     | 0.078 (p.u.)    | 0.0011 (rad)          | 0.012 (rad)           | 18.7                 | 2.18              |

The main network nodes are monitored in a shorter time (18.7 milliseconds) due to the reduction of network dimensions. The estimated values obtained from the initial SE are used as meters with specific standard deviations.
along with actual measurements in the secondary SE.

5.6. Comparison of the BCSE method and the two-step state estimation method on the IEEE standard 69-bus DN

The branch current state estimation (BCSE) \cite{2} is implemented on the balanced 69-bus distribution network using the branch current phasors as state variables in the Cartesian chart. Results of the secondary SE are compared with BCSE (an effective and common state estimation method applied in the balanced DN) to observe the obtained enhancement. In order to compare the proposed method with the BCSE method, Table 4 shows important performance parameters. Due to increased redundancy of the accurate measurements in the secondary SE of the proposed method, the convergence mean iteration in the Monte Carlo test as well as the average implementation time of the secondary SE in the 69-bus network have been decreased. The maximum RMSE of the voltage amplitude and phase angle of the 69-bus network have been improved compared to the conventional method.

| Methods          | Min RMSE (|V|) | Max RMSE (|V|) | Min RMSE (θ_V) | Max RMSE (θ_V) | Average Runtime (ms) | Mean Repetition |
|------------------|--------|------------|--------------|---------------|-------------------|---------------------|
| Proposed method  | 0.0016 (p.u.) 0.004 (p.u.) | 0.00013 (rad) | 0.00456 (rad) | 37.3 | 3.006 |
| BCSE method      | 0.0058 (p.u.) 0.0075 (p.u.) | 0.00021 (rad) | 0.00878 (rad) | 51.2 | 4.051 |

In order to further evaluate the performance of the proposed method and compare it with the BCSE method, the estimation error of the voltage magnitude and the voltage angle are displayed for any iteration of the Monte Carlo simulation ($N_{mc} = 50000$) in Figures 10 and 11. Figures 10 and 11 show the performance of the BCSE method and the proposed method for 50000 iterations of the Monte Carlo simulation, respectively. Comparing Figures 10 and 11, it can be expressed that the estimation performance of the proposed method is better than that of the BCSE method. The proposed method employs more accurate data in the secondary SE with the help of the initial SE, whereas the BCSE method uses more pseudo-measurements with standard deviation of 20%. In order to decrease the presence of
pseudo-measurements in the proposed method, the maximum error of magnitude and angle voltage estimation are less than 0.04 and 0.03, while these are less than 0.1 in the BCSE method.

![Figure 10](image1.png)

**Figure 10.** Error estimation of node voltage for $N_{mc} = 500000$ in the BCSE method.

![Figure 11](image2.png)

**Figure 11.** Error estimation of node voltage for $N_{mc} = 50000$ in the proposed method.

6. Conclusion

Widespread changes in DNs have led to the development of novel control, exploitation, and protection methods. There are few high-accuracy real measurements in the DN. Most previous studies have used pseudo-measurement with low accuracy to solve this problem and obtained results with reduced estimation accuracy and speed. In the present paper, a two-step estimation method using the NRP was presented to improve the performance of the SE. Considering the SE results of the 69-bus IEEE DN, it is obvious that the two-step state estimation outperforms BCSE methods in terms of accuracy. A MLP was used to specify the candidate locations that are fixed in the NRP; their values are estimated in
the initial SE and then are used as measurements in the secondary SE. Advantages of the two-step estimation method using the NRP compared to the BCSE method include (1) increasing the number of high-accuracy measurements due to the SE applied to the reduced network, (2) accessing the voltage phasor information of important and main nodes of the network considering the results extracted from the initial SE with low runtime, (3) improving the voltage phasor estimation accuracy using the secondary SE and information of the initial SE, and (4) achieving the redundancy of the accurate information using the proposed method. The future of work in this field can be:

- Implementation of two-stage state estimation by network reduction considering distribution network uncertainties such as distributed generations and reconfiguration.
- Investigation of cyberattacks considering the network reduction issue in the SE and improving the detection problem in cyberattacks.
- Implementation of the SE considering the process of network reduction in unbalanced and asymmetric distribution networks.
- Using the optimal locating problem considering the process of network reduction in the SE.

Research on the SE of modern distribution networks is one of the interesting issues to make the most of the capacity created in active distribution networks.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_e$</td>
<td>Measurement vector with error</td>
</tr>
<tr>
<td>$Y$</td>
<td>Measurement vector without error</td>
</tr>
<tr>
<td>$e$</td>
<td>Measurement vector error</td>
</tr>
<tr>
<td>$F(\mathbf{x})$</td>
<td>Measurement function vector</td>
</tr>
<tr>
<td>$\mathbf{x}^k$</td>
<td>The estimation of state variable vector in $k$th iteration</td>
</tr>
<tr>
<td>$G(\mathbf{x}^k)$</td>
<td>The gain matrix in $k$th iteration of the Gauss–Newton method</td>
</tr>
<tr>
<td>$\mathbf{H}(\mathbf{x})$</td>
<td>The Jacobian matrix of WLSE</td>
</tr>
<tr>
<td>$R$</td>
<td>The diagonal matrix of measurements</td>
</tr>
<tr>
<td>$W$</td>
<td>The weighting matrix of measurements</td>
</tr>
<tr>
<td>$Y_{ij}$</td>
<td>The feeder admittance between $i$th and $j$th nodes</td>
</tr>
<tr>
<td>$V_i$</td>
<td>The $i$th node voltage phasor</td>
</tr>
<tr>
<td>$I_i$</td>
<td>The consumed current of $i$th node</td>
</tr>
<tr>
<td>$I_{eq,i}$</td>
<td>The consumed equivalent current of $i$th node in NRP</td>
</tr>
<tr>
<td>$Y_{eq,ij}$</td>
<td>The equivalent feeder admittance between $i$th and $j$th nodes in NRP</td>
</tr>
<tr>
<td>$I_{eq,ij}$</td>
<td>The equivalent feeder current between $i$th and $j$th nodes in NRP</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>The standard deviation of the $i$th measurement</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>The mean value of the $i$th measurement</td>
</tr>
<tr>
<td>$MaxError%$</td>
<td>The maximum error percentage of a measurement</td>
</tr>
<tr>
<td>$RMSE(\tilde{x}_i)$</td>
<td>The root mean square error of the $i$th estimation state variable</td>
</tr>
<tr>
<td>$n$</td>
<td>The number of state variables</td>
</tr>
<tr>
<td>$m$</td>
<td>The total number of network measurements</td>
</tr>
<tr>
<td>$k$</td>
<td>The counter of the Gauss–Newton method</td>
</tr>
</tbody>
</table>

**References**


2002


