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A fuzzy model of directional relationships from the phi-descriptor

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Abstract: Directional spatial relationships are a category of spatial relationships and have applications in the fields of image processing, geographic information systems, natural language processing, and robot navigation. They can be directly extracted from images or can be interpreted from a type of image descriptors called the relative position descriptors. Examples of relative position descriptors are the angle histogram, the force histogram, and the recently proposed phi-descriptor. So far, fuzzy models of directional spatial relationships from the angle histogram and force histograms have been proposed in the literature. These include the compatibility method, the aggregation method, and the method of effective forces. However, extraction of directional spatial relationships from the phi-descriptor has not been investigated. In this work, the first fuzzy model of directional spatial relationships from the phi-descriptor is presented. The model calculates the truth degree of a directional spatial proposition (e.g., "object A is to the right of object B") about two objects from the average angle between the objects. Furthermore, it takes into account the angle which the argument object subtends the reference object when calculating the truth-degree. A novelty of the proposed model is the use of the cone concept in the calculation of the average direction and the handling of boundary cases. The model was tested on standard image data and the results were compared with those of the existing models. The performance is found to be satisfactory and the results meet users' perceptions and expectations.

Key words: Directional spatial relationships, topological spatial relationships, distance spatial relationships, spatial relationships, relative position descriptors, the phi-descriptor

1. Introduction

Directional spatial relationships provide directional information about the location of one object in space relative to another and include cardinal directions, "right", "left", "above", and "below" (correspondingly, east, west, north, and south) [1]. In daily life, information about directional spatial relationships is conveyed through linguistic expressions such as "object A is to the right of object B" or "object A is to the left but slightly above object B". Directional and other spatial relationships such as topological and distance relationships have applications in many areas, e.g., in image processing and computer vision in tasks like scene description and interpretation, in the field of GIS (geographic information systems) in spatial queries, and in the case of robotic motion in navigational tasks [2, 3]. Significant attention has been paid in the literature to the modeling of directional spatial relationships and both qualitative and quantitative models of directional relationships have been proposed. Qualitative models, common in the field of artificial intelligence, use crisp true–false evaluation to assess directional relationships between objects. However, due to the inherently ambiguous nature of directional

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spatial relationships, crisp true–false evaluation does not yield satisfactory results. Thus, quantitative models based on fuzzy approaches that assess the existence of directional relationship between objects in terms of a validity degree are more popular in the domain of image processing and computer vision. Another aspect of the modeling of spatial relationships is that they can either be derived directly from images or extracted from a relative position descriptor. A relative position descriptor is an image descriptor, such as color, texture, and shape descriptors, that describes the position of one object in space relative to another. Apart from describing the relative position of objects, a relative position descriptor serves as a repository of spatial relationship information and provides a basis from which models of spatial relationships can be derived. Some relative position descriptors proposed in the literature are the force histogram [4], angle histogram [5, 6], R-histogram [7], and visual area histogram [8]. Methods for extracting directional relationships from relative position descriptors include the compatibility method [3, 9], the aggregation method [5, 6, 10], and the method of effective forces [11].

The work in this paper presents a fuzzy model of directional spatial relationships based on the phi-descriptor (the Φ-descriptor). The model assesses the truth-degree of a spatial preposition (e.g., "object A is to the right of object B") by obtaining the average angle between the given objects using relative position information of the objects from the phi-descriptor and feeding it to a fuzzification function. The phi-descriptor [12] is a recently proposed relative position descriptor with many useful properties. Compared to its predecessors, the phi-descriptor encapsulates a richer variety of spatial relationships and allows the extraction of directional as well as topological and distance relationships. Additionally, it has a known behavior to affine transformations, and therefore the calculation of descriptor after the transformation from the original descriptor is easy. Lastly, the normalization procedure for the descriptor is straightforward. Some of these properties make the derivation of invariant models of spatial relationships from the descriptor possible.

The rest of the paper is organized as follows. Section 2 gives an overview of the existing literature and the phi-descriptor. Section 3 describes in detail the proposed fuzzy model of directional relational relationships and the theoretical explanation underlying the model. Section 4 presents experimental results and discussion and Section 5 presents the conclusion and plans for future work.

2. Literature review

2.1. Related work

Many models of directional spatial relationships have been proposed in the literature. The approach used in [3] is a direct method that extracts directional spatial information from images by constructing a fuzzy landscape around the reference object and evaluating the compatibility degree of the landscape with the given direction. A number of methods for extracting directional relations from relative position descriptors have also been proposed. The compatibility method [3, 9] treats the normalized descriptor (e.g., the angle histogram) as an unlabeled fuzzy set and computes its compatibility with the given fuzzy directional relation (e.g., "RIGHT"). The center of gravity of the compatibility set is then regarded as the acceptability degree of the proposition (e.g., "A is to the RIGHT of B"). Similar to the compatibility method, the descriptor is normalized (e.g., angle histogram) in the aggregation method [5, 6, 10] and treated as an unlabeled fuzzy set. The normalized histogram values are then used to compute the weighted average of the degrees of alignment between the directions in the histogram and the given direction. The weighted average represents the acceptability degree of the proposition being assessed. The method of effective forces assesses the validity degree of the proposition from the degree of alignment between the average direction of the histogram and the given direction [11, 13]. Besides, a method based on machine learning has been proposed in [7]. It uses fuzzy k-NN classifier to extract directional relations from R-histogram.

2.2. The phi-descriptor

A phi-descriptor, denoted as Φ^{AB}, where B is the reference object and A, the argument is an n-tuple of F-histograms together with other quantitative measures (see Eq. (1)) [12]. Each F-histogram corresponds to a region of interaction of objects A and B in some direction θ, $\theta \in [0, 2\pi]$ (see [4]). More specifically, an F-histogram represents the area (when objects A and B are 2D) or volume (when objects are 3D) of the region of interaction of objects A and B in direction θ. The interaction is a spatial relationship between A and B. For example, F_o^{AB} (θ) gives the area of the region where A and B *overlap* in the direction θ. The intended purpose of the Φ-descriptor is to give a quantitative description of the position of object A in space with respect to object B.

$$\begin{aligned}\Phi^{AB}(\theta) = (&F_a^{AB}(\theta), F_c^{AB}(\theta), F_d^{AB}(\theta), F_e^{AB}(\theta), F_f^{AB}(\theta), F_l^{AB}(\theta), F_o^{AB}(\theta), F_r^{AB} \\ &(\theta), F_s^{AB}(\theta), F_t^{AB}(\theta), F_u^{AB}(\theta), F_v^{AB}(\theta), F_i^{AB}(\theta), W_i^{AB}(\theta), |A|, |B|).\end{aligned} \tag{1}$$

Each F element in the definition of the Φ-descriptor corresponds to an interaction type that exists between A and B in direction θ. The explanation is given in Figure 1. The direction (i.e. θ) assumed is the rightward direction. In the figure, the object in the light gray area labeled with the letter A represents the argument object. The argument object is the object whose position with respect to the other object in the object pair is described by the descriptor. The other object in the dark gray area labeled with B is the reference object with respect to which the position of the argument object is given by the descriptor. Each entry in the figure includes an interaction type between objects A and B and the associated F meanings. For example, the interaction type illustrated in Figure 1a is "A trails B" (i.e. object A lags behind object B with some distance). The extent of this interaction type is given by F_t^{AB}, which is the measure of the area of the dotted region between A and B. Likewise, F_o^{AB} (θ) in Figure 1b gives the area of the region where objects A and B overlap in direction (θ). Similar notation has been used to explain the meanings of the other F values. For example, F_f^{AB} (θ) ("A follows B", Figure 1c) corresponds to the region of object B, which is followed by object A in direction (θ) (object A is behind object B in the rightward direction and touches object B). In the same way, F_l^{AB} (θ) corresponds to the region of object A (the dotted region in Figure 1d), which leads B in direction (θ). In the physical sense, this is the region of A that is in front of object B but touches object B. The idea of Φ-descriptor is inspired by the concept of Allen relations. Proposed in 1983 [14], Allen relations are 13 binary relations that can be used to model topological relationships between spatio-temporal phenomena. The 13 Allen relations include relations such as "overlaps" and "starts" and their semantic inverse, e.g., "overlapped by" and "started by". The spatio-temporal phenomena in the Allen relations are represented by 1D spatial entities. In Figure 1, 2D objects are assumed instead of 1D entities for ease of explanation. However, the evaluation of the Φ-descriptor is based on 1D modeling of objects (objects are considered to be composed of directional 1D segments. For detailed definition of the interaction types and the Φ-descriptor, see [12].

The idea of the Φ-descriptor is illustrated in Figure 2. Objects A and B (Figure 2a) are two nonempty bounded subsets of the Euclidean space S. Object A is the argument object whereas object B is the reference object. The region in Figure 2b bound by the lines L_1, L_2, L_3, and L_4 gives the region of the interaction of A and B along the direction θ (e.g., $\theta = \pi/6$). The average width of this region is W_i^{AB} (θ), which is one of

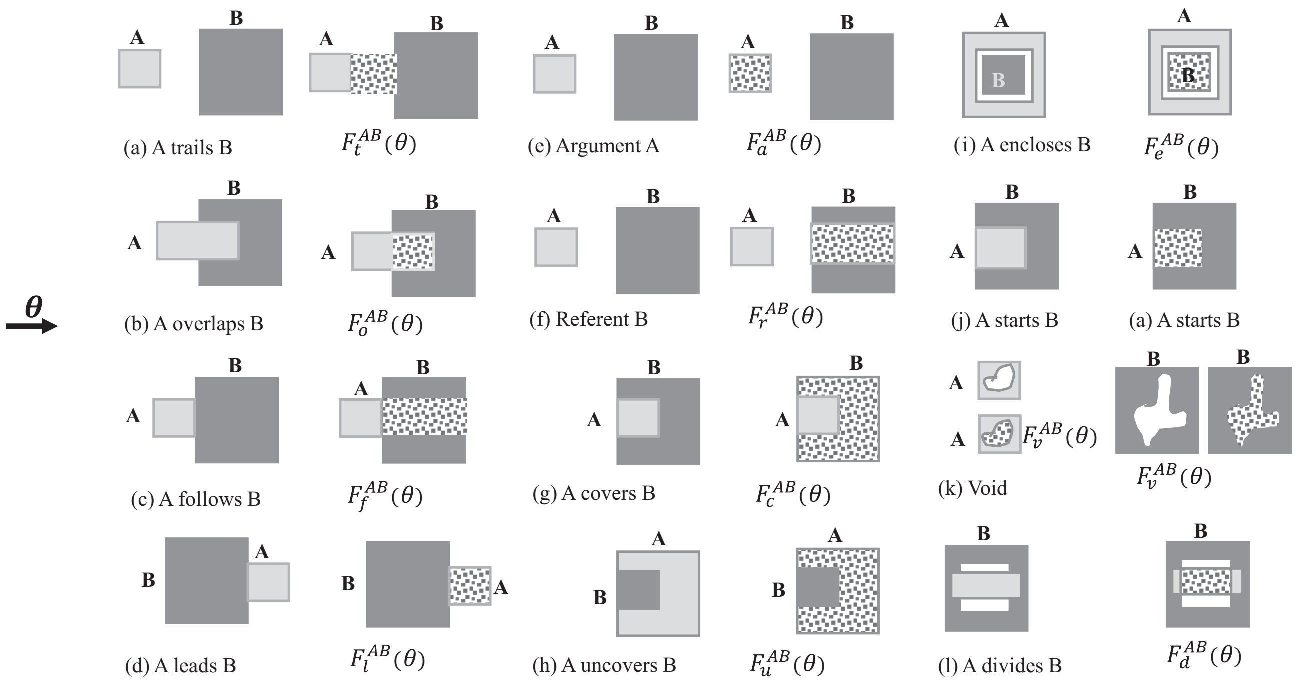


Figure 1. Meaning of the interaction types in the phi-descriptor.

the tuple-values in Φ-descriptor (see Eq. (1) above). The interaction types that exist between A and B in the direction (θ) are "A leads B", "argument A", and "referent B", see Figure 2c. The regions involved in these interaction types are labeled with the letters "l" (leads), "a" (argument), and "r" (the referent). The sizes of these regions given by the area measures $F_l^{AB}\ (\theta)$, $F_a^{AB}\ (\theta)$, and $F_r^{AB}\ (\theta)$, respectively, represent the extent to which the interaction types "A leads B", "Argument A", and "referent A" hold between objects A and B. The region l indicating the interaction type "A leads B" between A and B is the part of A that touches B and is in the direction (θ) of B. Regions labeled with a and r are parts of A and B, respectively, that are in the direction (θ) and are disconnected from each other (i.e. they do not touch). Thus, if the Φ-descriptor was computed for the direction (θ); $F_l^{AB}\ (\theta)$, $F_a^{AB}\ (\theta)$,$F_r^{AB}\ (\theta)$, $F_i^{AB}\ (\theta)$, and $W_i^{AB}\ (\theta)$ (in Eq. (1)) would have nonzero values whereas the rest of the F measures would have zero values as the remaining interaction types do not hold between A and B in direction (θ). $F_i^{AB}\ (\theta)$ would give the area of the region of interaction. Practically, an F measure, e.g., $F_l^{AB}\ (\theta)$, represents the number of pixels in the corresponding region because of the discrete nature of digital images. Evaluation of the Φ-descriptor in the opposite direction, i.e. in the $-\theta = \theta + \pi = \pi/6 + \pi$ direction is illustrated in Figure 2d. The interaction types that hold between A and B in this direction are "A trails B", "A follows B", "argument A", and "referent R" indicated by the regions labeled with f, t, a, and r. Consequently, $F_f^{AB}\ (-\theta)$, $F_t^{AB}\ (-\theta)$, $F_a^{AB}\ (-\theta)$, $F_r^{AB}\ (-\theta)$, $F_i^{AB}\ (-\theta)$, and $W_i^{AB}\ (-\theta)$ in Eq. (1) would have nonzero values whereas the rest of the F measures would have zero values. The extended notation $F_{(|lfar|)}^{AB}\ (\theta)$ means areas $F_l^{AB}\ (\theta) + F_f^{AB}\ (\theta) + F_a^{AB}\ (\theta) + F_r^{AB}\ (\theta)$. $|A|$ and $|B|$ denote the areas of objects A and B (given in terms of number of pixels).

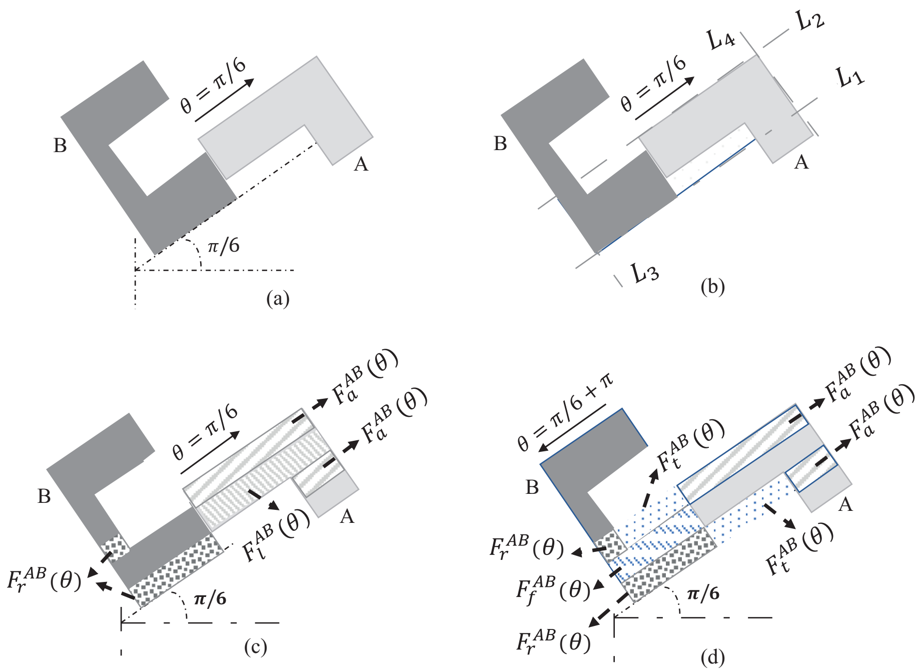


Figure 2. Principle of evaluation of the phi-descriptor.

3. The proposed model

Properties commonly sought in the models of directional relationships are computational efficiency and quality of results in terms of the satisfaction of user perception and expectations. Additionally, preservation of the property of symmetry (i.e. object B is the same degree to the left of object A as the degree to which object A is to the right of object B), sensitivity to distance between objects and object shapes, and satisfaction of the semantic inverse property (i.e. if object A is to the right of object B, object B is to the left of object A) are considered useful. Likewise, the behavior of boundary cases (where the relationship cannot be, e. g., just "right" but both "right and above") is also an important consideration. A goal in the model of directional relationships presented in this paper (here called the model M) is the satisfaction of these requirements. The model basically associates a validity degree with the spatial proposition of the form, "object A is in direction α of object B" and works by taking the direction α and the Φ-descriptor as inputs and returning the acceptability degree of the proposition "object A is in direction α of object B". For the purpose of illustration, here the direction $\alpha = 0$ (α is the angle) or "A is to the right of B" is considered. To find the truth degree of the proposition for other directions, the same computation can be repeated. It should be noted that the directional relations ("right", "left", "above" etc.) here should be interpreted from the standpoint of an extrinsic reference frame [15], i.e. from the view-point of an external observer. Other reference frames such as deictic (or egocentric) or intrinsic [8] are also possible but are not considered here.

3.1. The model M

Generally, the perception of "A is to the right of B" (angle α) is formed by the perceiver by locating the object A in a field subtending π degrees from angle $\alpha - \pi/2$ to $\alpha + \pi/2$ [1] around B. Here, this field is called

α-viewfield or the right view field of B. The model of directional relationship presented, therefore, is based on the calculation and fuzzification of two angles; the average angle $\widehat{\theta}$ and the around angle $\breve{\theta}$. The average angle $\widehat{\theta}$ gives the average direction of the location of object A (argument object) in the α-viewfield of B (reference object). The around angle $\breve{\theta}$ represents the angle by which A extends around B. The fuzzification principle for each angle is illustrated in Figures 3a–3d and Figures 3e–3h. Explanation is as below.

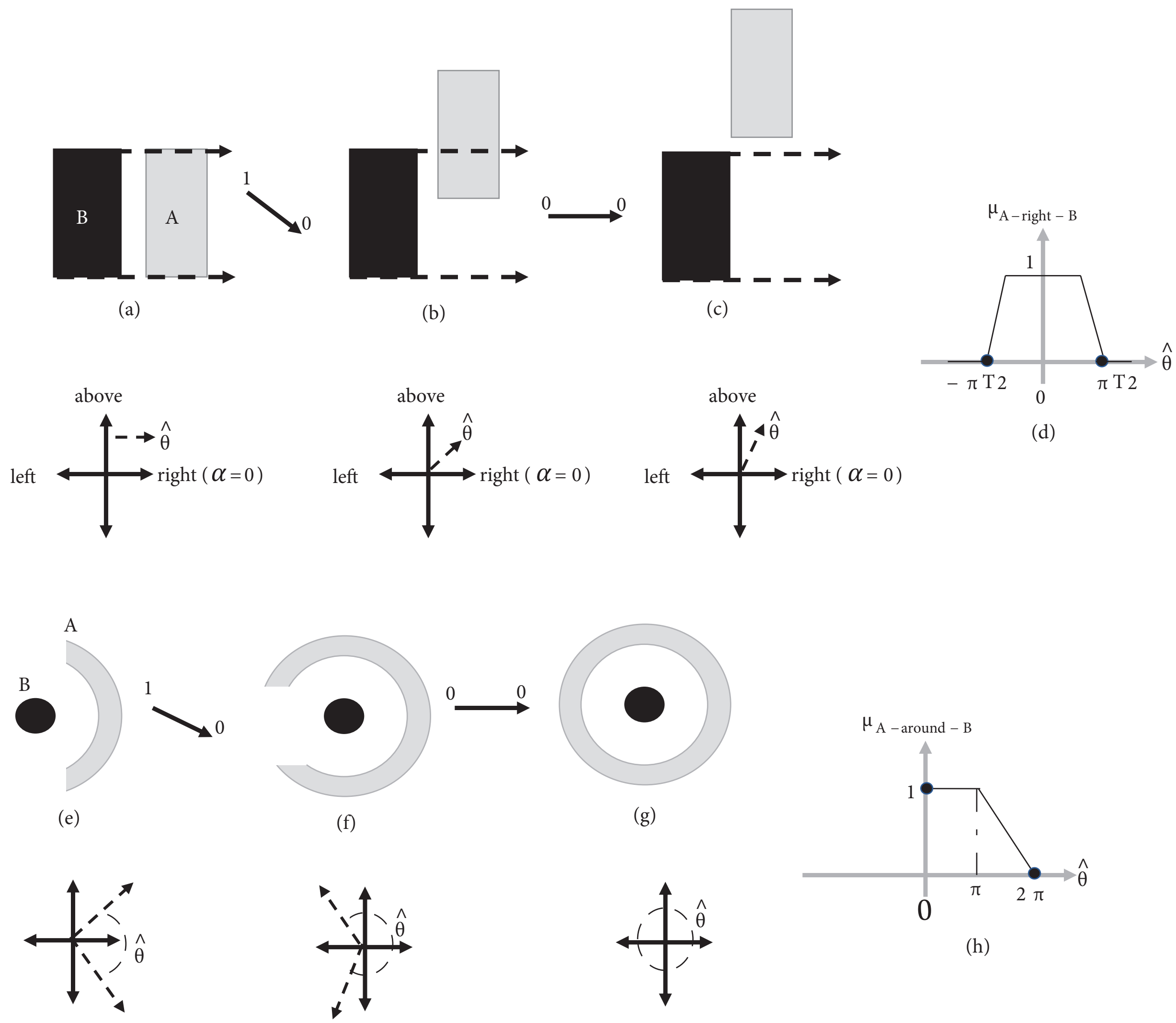


Figure 3. Fuzzification principle for $\widehat{\theta}$ and $\breve{\theta}$.

In Figure 3a, where object A lies entirely to the right of object B (i.e. in direction α), the acceptability degree of "A is to the right of B" can be considered maximum. In Figure 3b, as only a part of object A is located in the direction α of B, the acceptability degree can be considered lower. In Figure 3c, no parts of A lie to the right of B but A is still somewhat to the right of A. It may be argued that the relationship in the case of the configuration in Figure 3c belongs more to the category "above and to the right" rather than the category "to the right". Thus, the truth degree of "A is to the right of B" is the lowest in this case.

Generally, if the average direction of A is more aligned with the given direction α, the truth degree of

"A is to the direction α of B" is higher but as the average direction deviates from α, the truth degree gets lower until A is outside the viewfield when it is the lowest. A similar principle is used to take the around factor into account when directional relationships between objects are assessed. When the extent of object A is wholly located within the α-viewfield of B, the acceptability degree of A being within the viewfield of B is the highest but as A extends beyond the viewfield, the truth degree becomes lower.

A question arises as to why it is important to consider the around factor in assessing the validity degree of "A is to the right of B"? The case in Figure 3g explains the reason. An assessment of the relationship between objects in the configuration in Figure 3g may suggest that all the directional relationships, such as "above", below", "left", "right", hold for the objects. However, such an assessment may be unsatisfactory because people generally do not combine more than two propositions when communicating visual information (in language) [16, 17]. The purpose in considering the around condition, thus, is to identify two dominant directional relationships between the objects and to distinguish between the "surround" and the directional cases.

The truth degree of "A is to the right of B" is derived from the combination of the above two factors, i.e. the directional and around factors, as follows. Let *right*(A,B) be the acceptability degree of the proposition "A is to the right of B", then *right*(A,B) is defined by:

$$right(A,B) = \mu_{A-right-B}(\widehat{\theta}) \times \mu_{A-around-B}(\breve{\theta}), \tag{2}$$

where $\mu_{A-right-B}\colon[-\pi/2,\pi/2] \longrightarrow [0,1]$ is a membership function that assigns a truth degree to "A is to the right of B" in A-right-B. As can be seen, $\mu_{A-right-B}$ is a trapezoidal function that takes the average angle $\widehat{\theta} \in [-\pi/2,\pi/2]$ as an argument and map it to a truth degree in [0,1]. The set A-right-B is the fuzzy set that represents the "right" relationship. The function $\mu_{A-right-B}\colon[0,2\pi] \longrightarrow [0,1]$ tackles the around condition and maps the around angle $\breve{\theta}$ to a truth degree. Both these functions are defined in the following sections.

3.2. Definition of $\mu_{\mathbf{A-right-B}}$ $(\widehat{\theta})$

The overall direction of A with respect to B is represented by the average angle $\bar{\theta}$. When objects are closer to each other in the direction "right", the closer parts of A and B have more influence on the acceptability level of the directional spatial proposition, "A is to the right of B". To take this influence into account, an adjusted angle $\widehat{\theta}$ is then derived from $\bar{\theta}$. Finally, $\mu_{A-right-B}$ is calculated from the adjusted average angle $\widehat{\theta}$. The average angle $\bar{\theta}$ is defined by:

$$\bar{\theta} = \frac{\int_{-\pi/2}^{\pi/2} \theta \times F^{AB}_{|lafr|}(\theta) \times \varepsilon d\theta}{\int_{-\pi/2}^{\pi/2} F^{AB}_{|lafr|}(\theta) \times \varepsilon d\theta}, \tag{3}$$

where
$F^{AB}_{|lafr|}(\theta) = F^{AB}_{lafr}(\theta) + F^{AB}_{lafr}(-\theta)$ and
$F^{AB}_{lafr}(\theta) = F_l^{AB}(\theta) + F_a^{AB}(\theta) + F_f^{AB}(\theta) + F_r^{AB}(\theta)$ and
$F^{AB}_{lafr}(-\theta) = F_l^{AB}(-\theta) + F_a^{AB}(-\theta) + F_f^{AB}(-\theta) + F_r^{AB}(-\theta)$.
The quantity $F^{AB}_{|lafr|}(\theta)$ represents the part of the region of interaction between A and B in direction θ that is occupied by A or B (here nonintersecting objects are considered). Furthermore, $-\theta = \theta + \pi$. It serves as a weight or evidence for the proposition that "A is in direction θ of B". The symbol ε is a devaluation factor. It

represents the devaluation of the weight $F^{AB}_{|lafr|}(\theta)$ of the angle θ according to whether θ is closer to $\alpha = 0$, i.e., "A is to the right of B", or to $\alpha = \pi/2$ (resp. $\alpha = -\pi/2$), i.e. "A is above B" or (resp. "A is below B") (see Figure 4). The choice to devaluate the directions that drift away from the given direction is inspired by the cone model of directional relations [18]. In order to calculate the adjusted average angle $\bar{\theta}$ an average $\bar{F}^{AB}_{|lafr|}$ is also calculated as follows.

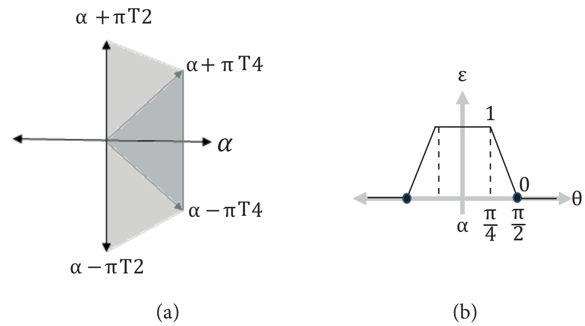


Figure 4. Definition of the devaluation factor ε.

$$\bar{F}^{AB}_{|lafr|} = \frac{\int_{-\pi/2}^{\pi/2} F^{AB}_{|lafr|}(\theta) d\theta}{\int_{-\pi/2}^{\pi/2} d\theta}, \tag{4}$$

Next, the adjusted average angle $\widehat{\theta}$ is defined as:

$$\widehat{\theta} = \frac{\int_{-\pi/2}^{\pi/2} (\bar{\theta}\bar{F}^{AB}_{|lafr|} + \theta\lambda(\theta)F^{AB}_{|lafr|}(\theta)) \times \varepsilon d\theta}{\int_{-\pi/2}^{\pi/2} (\bar{F}^{AB}_{|lafr|} + \lambda(\theta)F^{AB}_{|lafr|}(\theta)) \times \varepsilon d\theta}, \tag{5}$$

where $\lambda \in [0,1]$ is a factor that represents the effect of the closer parts of A to B on the acceptability level of the proposition. It can be defined in many ways, e.g., using an inverse square law such as the gravitational law. Here, it is defined by:

$$\lambda(\theta) = \frac{F^{AB}_{|lafr|}(\theta)}{F^{AB}_{|lafr|}(\theta) + F^{AB}_{t}(-\theta)}, \tag{6}$$

where $F^{AB}_{t}\ (-\theta)$ is an element of Φ-descriptor that represents the area of the region between A and B. As can be seen, parts of the objects A and B are given importance according to their distance from each other with closer parts receiving more importance than the farther parts. Thus $\widehat{\theta}$ is adjusted according to distance such that when the objects move far from each other, the adjusted average angle $\widehat{\theta}$ approaches the original average angle $\bar{\theta}$. Finally, the membership degree $\mu_{A-right-B}$ is calculated from $\widehat{\theta}$ as follows:

$$\mu_{A-right-B}(\widehat{\theta}) = \frac{\frac{\pi}{2} - \max(k, |\widehat{\theta}|)}{\frac{\pi}{2} - k}. \tag{7}$$

As can be seen, the core of $\mu_{A-right-B}(\widehat{\theta})$ is defined over $[-k, k]$ where k is a constant such that $0 \leq k \leq \frac{\pi}{2}$. For pessimistic evaluation of the proposition, k can be set to a small value and for optimistic evaluation, it can be set to a large value.

3.3. Definition of $\mu_{A-right-B}$ $(\breve{\theta})$

The angle $\breve{\theta}$ is the angle by which object A extends around object B. If $\breve{\theta} > |\alpha + \pi/2| - |\alpha - \pi/2|$, i.e. parts of object A lie outside the right viewfield, the proposition "A is to the right of B" is weakened by a corresponding amount, otherwise it receives full weight. $\breve{\theta}$ is calculated as follows:

$$\breve{\theta} = \frac{1}{2\pi \times |B|} \int_0^{2\pi} F_e^{AB}(\theta) d\theta. \tag{8}$$

Finally, $\mu_{A-around-B}$ $(\breve{\theta})$ is defined as:

$$\mu_{A-around-B}(\breve{\theta}) = \frac{2\pi - \max(\breve{\theta}, \pi)}{\pi}. \tag{9}$$

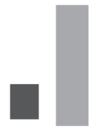
4. Experimental results

The model (M) was implemented in C and tested on synthetic images used in a previous study [11] (see Figure 5). Results are presented in Figure 5. Comparison with the models K, F0, and F2 is given. The model K represents the aggregation method and is based on the construction of the histogram of angles [5, 6] and models F0 and F1 are two variants of the method of effective forces [11] defined on the histogram of forces [4]. The descriptors (i.e. the Φ-descriptor) were generated using 72 reference directions (more reference directions gave similar results). As can be seen, the models produce comparable results for most of the configurations. For example, for configuration in Figure 5a, all the models agree that A is to the right of B. The models M, F0, and F2, however, assert it with more confidence by reporting the highest truth degree of 1.00 for the proposition, similar explanation applies to Figure 5b. This is a further confirmation of the conclusion of an earlier work [19] on a similar topic that the phi-descriptor allows the extraction of different types of spatial relationship and that the extraction is straight forward. In fact, the extraction can be done in many ways.

Likewise, in the case of the configurations in Figure 5c and 5d, all models agree on the directional position of A relative to B. The divergence occurs in Figure 5e where there is a disagreement between the models as to whether A is more to the right of B or above B. In this case, whereas M and F2 assert that A is more to the right of B, K and F0 strongly assert that A is above B (truth degrees assigned to "A is above B" by K and F0 are 0.62 and 0.72, respectively). The model M is more cautious and assigns a higher acceptability degree (i.e. 0.68) to the proposition "A is to the right of B" but also gives sufficient weight (i.e. 0.41) to the possibility of "A is above B". F2 strongly asserts that A is to the right of B. The reason for this behavior is that, models K and F0 are based on the aggregation method, which gives the same weight to all parts of object A when evaluating a proposition about a directional relationship between A and B, and therefore, the massive part of object A is assigned more weight. Thus, K and F0 assign exceptionally high truth degrees to the proposition "A is above B" instead of the proposition "A is to right of B"(the less massive part of object A is located in the right direction). Method F2 gives more importance to the parts of object A that are closer to object B. So, when object A is closer to object B in the target direction, the truth-degree assigned to the given proposition is high. The model M, on the other hand, uses the alignment of the average direction of the location of A with respect to B with the target direction to assign truth-degree to the directional proposition. The average direction of the location of object A with respect to B is somewhere between the high-point and low-point of object A. Furthermore, the average direction is calculated over the entire viewfield (see Section 3.1). That is

(a)


	M	K'	F0'	F2'
Right	1.00	0.60	1.00	1.00
Above	0.22	0.20	0.00	0.00
Left	0.00	0.00	0.00	0.00
Below	0.23	0.20	0.00	0.00

(b)


M	K'	F0'	F2'
0.92	0.55	0.92	1.00
0.28	0.42	0.29	0.13
0.00	0.00	0.00	0.00
0.16	0.04	0.00	0.00

(c)


M	K'	F0'	F2'
0.25	0.24	0.24	0.24
0.81	0.76	0.87	0.87
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00

(d)


M	K'	F0'	F2'
0.98	0.63	1.00	1.00
0.23	0.18	0.00	0.00
0.00	0.00	0.00	0.00
0.25	0.19	0.00	0.00

(e)

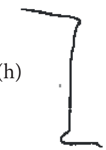

	M	K'	F0'	F2'
Right	0.68	0.38	0.55	0.86
Above	0.41	0.62	0.72	0.41
Left	0.00	0.00	0.00	0.00
Below	0.10	0.10	0.00	0.00

(f)

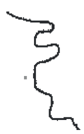

M	K'	F0'	F2'
0.16	0.10	0.00	0.00
0.00	0.00	0.00	0.00
0.16	0.10	0.00	0.00
1.00	0.79	1.00	1.00

(g)


M	K'	F0'	F2'
0.33	0.61	0.53	0.21
0.00	0.00	0.00	0.00
0.15	0.20	0.00	0.00
0.58	0.37	0.76	0.99

(h)


M	K'	F0'	F2'
0.18	0.30	0.00	0.00
0.27	0.20	0.00	0.10
0.68	0.26	0.74	0.98
0.40	0.52	0.20	0.00

(i)


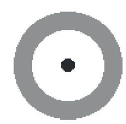
	M	K'	F0'	F2'
Right	0.05	0.10	0.00	0.00
Above	0.25	0.15	0.00	0.10
Left	0.64	0.36	0.87	0.98
Below	0.36	0.48	0.19	0.70

(j)


M	K'	F0'	F2'
0.00	0.00	0.00	0.00
0.25	0.24	0.00	1.00
0.76	0.51	0.96	0.96
0.25	0.25	0.00	0.00

(k)


M	K'	F0'	F2
0.42	0.50	0.00	0.00
0.29	0.25	0.00	0.10
0.42	0.46	0.56	0.52
0.29	0.25	0.00	0.00

(l)


M	K'	F0'	F2'
0.00	0.25	0.00	0.00
0.00	0.25	0.00	0.10
0.00	0.25	0.00	0.00
0.00	0.25	0.00	0.00

Figure 5. Experimental results for directional relationships.

why model M associates the higher truth-degree of 0.68 to the proposition "A is to the right of B" and also gives some credence to the proposition "A is above B" by assigning it the truth degree of 0.41.

This is more evident in the case of Figure 5g where object A is elongated (extends to the right) relative to B. For this case, F0 and F2 report the dominant relation between A and B to be the "below" relationship whereas K reports it to be the "right" relationship. M, on the other hand, takes a moderate position and whereas it identifies the "BELOW" relationship to be the dominant relationship between A and B, it also gives some credibility to the proposition "A is to the right of B".

The results for the remaining configurations can be interpreted in a similar way. For Figures 5h, M says that A is predominantly to the left of B but also is, to a degree, below and above it. F2 makes a very strong assertion that A is to the left of B but negligibly below B. K reports A to be predominantly below B but also to a degree in other directions. The surround and partially surround cases in Figure 5j–5l are interpreted comparably by M, F0, and F2 but differently by K.

Which model is better is difficult to decide and depends on the context in which the models are used. If the requirement is to find the dominant relationship between objects A and B, then F2 may be better but if the aim is to have the full picture, then the remaining models may be useful.

5. Conclusion and future work

A fuzzy model of directional relationships based on the Φ-descriptor was proposed in this paper. The performance of the model on images used in previous studies was shown in the experimental section. A comparison with the results for the aggregation method and the method of effective forces was also presented. As can be seen, the model gives an accurate assessment of the directional relationships between objects. The extraction is easier and the results meet expectations. Moreover, the model preserves the semantic inverse and other properties. In future, this work will be extended to fuzzy objects. Furthermore, 3D objects will be considered.

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