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A novel and unified approach for averaged channel capacity and averaged effective capacity analyses of diversity combining and multihop transmission schemes in flat fading environments

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Abstract: In this paper, we introduce an L_p-norm aggregation to present a signal-to-noise ratio expression unified not only for such diversity combining schemes as equal-gain combining, maximal-ratio combining, and selection combining, but also for such transmission techniques as multihop transmission. Accordingly, we propose two moment-generating function-based approaches that both respectively unify the exact analyses of the averaged channel capacity and averaged effective capacity over generalized fading channels with respect to the diversity combining and multihop transmission schemes. Finally, the mathematical formalism is illustrated by numerical special cases and verified by simulations.

Key words: Averaged channel capacity, averaged effective capacity, diversity combining schemes, multihop transmission schemes, unified signal-to-noise ratio

1. Introduction

Wireless systems continue to strive for higher data rates and better reliability while migrating to higher and higher frequency bands. Due to high data-rate and coverage requirements of current systems, the channel capacity (CC) analysis of diversity combining and transmission schemes becomes a fundamental issue that needs to be considered from theoretical, empirical, and practical viewpoints. In the literature, there exist several papers targeting the averaged symbol error performance (ASEP) of diversity combining schemes over fading channels (see, for example, [1] and the references therein). Advances over the last decade on the ASEP analysis of diversity combining schemes over fading channels emphasized the moment generating functions (MGF) as a powerful tool for simplifying the ASEP analysis of diversity combining schemes [1–3]. However, to the best of our knowledge, published papers concerning the exact averaged channel capacity (ACC) over fading channels have been scarce when compared to those concerning the exact analysis of the ASEP [1, and references therein]. For maximal-ratio combining (MRC) diversity combining schemes, some insightful and special-case results for ACC analysis are found in [4–8] and [9,10, and references therein]. Inspired by the landmark paper of Simon and Alouini [2], two MGF-based frameworks concerning the exact ACC analysis over fading channels were proposed in [10, Eq. (5)] and [10, Eq. (7)], respectively, each of which is targeting MRC but neither applicable nor extendible not only to the ACC analyses of equal-gain combining (EGC) and selection combining (SC) but also

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to those of amplify-and-forward (AF) and cascaded multihop transmission schemes. Later, Yilmaz and Alouini proposed in [11, Eq. (6)] the other MGF-based framework, which unifies the ACC analyses of EGC and MRC combining schemes. However, in this article, we introduce a unified signal-to-noise ratio (SNR) expression, and then we propose a novel MGF-based approach that remarkably unifies the ACC analyses of MRC, EGC, and SC combining schemes and AF and cascaded transmission schemes over correlated/uncorrelated generalized fading channels.

In wireless systems, transmission is achieved based on packet switching that introduces a variable packet delay as a quality-of-service (QoS) due to the fact that packets have to be buffered either after being received from or before being transmitted to the other wireless nodes. This buffering will become a serious problem for applications that are sensitive to delays, especially when the packet delay exceeds a certain threshold. Thus, QoS provisioning is required in order to guarantee the packet delay performance with the achievable maximum transmission rate. In this context, the averaged effective capacity (AEC) was introduced in [12] to quantify the performance of single-input single-output and multiple-input single-output communication systems under QoS limitations [13–22]. Particularly, the analysis of AEC was published in [13–15] considering MRC combining over mutually independent fading channels, and also in [16,17] considering the same combining over correlated fading channels. Specifically, presented in [13–15,17–19] are AEC analyses over mutually independent not necessarily identical well-known fading channels, while [16] presented systems with a number of antennas in fading environments with correlation and keyholes. Note that some analytical approaches are available in the literature, e.g., see [20–22], each of which essentially focuses on the AEC analysis of MRC combining over fading channels and is not only applicable to the AEC analyses of EGC and SC diversity receivers but also to those of AF and cascaded transmission schemes. However, we propose in this article a novel MGF-based framework to simultaneously analyze the ACC and AEC of diversity combining and transmissions schemes over correlated/uncorrelated fading channels.

The remainder of this paper is organized as follows. In Section 2, we introduce a unified SNR expression in terms of L_p-norm aggregation. In Section 3, we propose two novel MGF-based approaches for the ACC and AEC analyses, each of which is remarkably unified not only for a variety of diversity combining and transmission schemes but also for correlated and uncorrelated generalized fading channels. In Section 4, the schemes of diversity combining and transmission are outlined to show how to treat the ACC and AEC analyses simultaneously. Finally, conclusions are drawn in the last section.

2. System and fading environment models

Consider an L-branch diversity combining or L-hop transmission scheme over additive white Gaussian noise (AWGN) channels in flat fading environments. Accordingly, for $\ell \in \{1, 2, \ldots, L\}$, the instantaneous SNR of the ℓth branch/hop is defined as $\gamma_\ell = \alpha_l^2 E_s/N_0$, where $\alpha_l E_s$, and N_0 denote the fading amplitude, average symbol energy, and noise power, respectively. Let γ_{end} denote the instantaneous SNR at the output either of diversity combining or of multihop transmission. As a contribution of this paper, it is consistently written in a manner of unifying the instantaneous SNRs of diversity combining and transmission schemes by using the L_p-norm of $\gamma_1\gamma_2, \ldots, \gamma_L$, that is [23, Eq. (5)]:

$$\gamma_{end} = \gamma_{end}\left(\eta, p, q\right) = \eta \left(\frac{1}{L} \sum_{l=1}^{L} \gamma_l^p \right)^q \tag{1}$$

where $\eta \in \mathbb{R}^+$, $p \in \mathbb{R}$, and $q \in \mathbb{R}$ specify the type of the diversity receiver or transmission technique; see Table.

In wireless communications, the analysis of the highest achievable rate over the fading channels is corroborated by the ACC analysis and accordingly given by:

$$C_{avg} = E\left[\log\left(1+\gamma_{end}\right)\right] \tag{2}$$

where $E[\cdot]$ is the expectation operator and $\log(\cdot)$ is the natural logarithm [24, Eq. (4.1.1)]. By substituting Eq. (1) into Eq. (2), the ACC evaluation involves the L-fold integral, that is:

$$C_{avg} = \int_0^\infty \cdots \int_0^\infty \log_2\left(1+\eta\left(\frac{1}{L}\sum_{l=1}^{L}\gamma_l^p\right)^q\right) p_{\gamma_1,\gamma_2,\ldots,\gamma_L}\left(r_1,r_2,\ldots,r_L\right) dr_1 \ldots dr_L \tag{3}$$

where $p_{\gamma_1,\gamma_2,\ldots,\gamma_L}\left(r_1,r_2,\ldots,r_L\right)$ denotes the joint PDF of $\gamma_1\gamma_2,\ldots,\gamma_L$.

Table. Some special cases of the unified overall instantaneous SNR.

MRC receiver has the overall instantaneous SNR [1] $\gamma_{MRC} = \gamma_{end}\left(L,1,1\right) = \sum_{l=1}^{L}\gamma_l$	**Cascaded fading channel** has the overall instan-taneous SNR [25, and references therein] $\gamma_{CFC} = \gamma_{end}\left(1,pL,1/p\right) = \prod_{l=1}^{L}\gamma_l$
EGC receiver has the overall instantaneous SNR [1] $\gamma_{EGC} = \gamma_{end}\left(L,1/2,2\right) = \frac{1}{L}\left(\sum_{l=1}^{L}\sqrt{\gamma}_l\right)^2$	**Geometric-mean approximation** of the overall instan-taneous SNR can be written as [26, 27] $\gamma_{GMA} = \gamma_{end}\left(1,p,1/p\right) = \left(\prod_{l=1}^{L}\gamma_l\right)^{\frac{1}{L}}$
SC receiver has the overall instantaneous SNR [1] $\gamma_{SC} = \gamma_{end}\left(L,p,1/p\right) = \max\left(\gamma_1,\gamma_2,\ldots,\gamma_L\right)$	**AF Multihop Transmission** has the overall instant- taneous SNR [34, and references therein] $\gamma_{AFM} = \gamma_{end}\left(\frac{1}{L},-1,-1\right) = \frac{1}{\frac{1}{\gamma_1}+\frac{1}{\gamma_2}+\ldots+\frac{1}{\gamma_L}}$
RMSC receiver has the overall instantaneous SNR [28] $\gamma_{RMSC} = \gamma_{end}\left(\sqrt{L},2,1/2\right) = \sqrt{\sum_{l=1}^{L}\gamma_l^2}$	**Minimum-bound** of the overall instantaneous SNR can be written as $\gamma_{MIN} = \gamma_{end}\left(L,p,1/p\right) = \min\left(\gamma_1,\gamma_2,\ldots,\gamma_L\right)$

In addition to the ACC analysis in wireless communications, the analysis of the highest achievable rate with delay constraints and buffer limitation over fading channels is corroborated by the AEC analysis, and it is accordingly given by [12] and [16, Eq. (4)]:

$$C_{eff} = -\frac{1}{A}\log\left(E\left[\left(1+\gamma_{end}\right)^{-A}\right]\right) \tag{4}$$

where $A \in \mathbb{R}^+$ represents a metric of delay constraint and buffer limitation [16, Eq. (5)]. It appears evident, by substituting Eq. (1) into Eq. (4), that the AEC evaluation involves the L-fold integral, that is:

$$C_{eff} = -\frac{1}{A}\log\left(\int_0^\infty \cdots \int_0^\infty \left[1+\eta\left(\frac{1}{L}\sum_{l=1}^{L}\gamma_l^p\right)^q\right]^{-A} p_{\gamma_1,\gamma_2,\ldots,\gamma_L}\left(r_1,r_2,\ldots,r_L\right) dr_1 \ldots dr_L\right) \tag{5}$$

The L-fold integration in both Eq. (3) and Eq. (5) are numerically tedious and inefficient even for a small number of L in addition to being inseparable from the product of one-dimensional integrals. To overcome this numerical inefficiency, we propose in what follows two MGF-based ACC and AEC analyses over correlated and uncorrelated fading channels.

3. Main results

In this section, we present two MGF-based approaches for ACC and AEC analyses.

3.1. An MGF-based ACC analysis

A unified MGF-based approach for the evaluation of the ACC of diversity combining and transmission schemes is given in the following theorem.

Theorem 1 *(ACC analysis over correlated fading channels). The ACC analysis over correlated fading channels can be exactly evaluated by*

$$C_{avg} = \int_0^\infty C_{\eta,q}\left(s\right)\left[\frac{\partial}{\partial s}M_{\gamma_{end}}\left(s\,|\,p\right)\right]ds \tag{6}$$

where $\eta \in \mathbb{R}^+$*,* $p \in \mathbb{R}$*, and* $q \in \mathbb{R}$ *are chosen according to the type of the combining or transmission technique, and where* $C_{\eta,q}\left(s\right)$ *is the auxiliary function given by*

$$C_{\eta,q}\left(s\right) = -H_{3,3}^{1,2}\left[\frac{\eta}{L^q s^q}\left|\begin{array}{c}\left(1,1\right),\left(1,1\right),\left(1,\left|q\right|/2+q/2\right)\\ \left(1,1\right),\left(0,1\right),\left(0,\left|q\right|/2-q/2\right)\end{array}\right.\right] \tag{7}$$

and $M_{\gamma_{end}}\left(s\,|\,p\right) = E\left[e^{-s\left(\gamma_1^p+\gamma_2^p+\ldots+\gamma_L^p\right)}\right]$ *is the joint generalized MGF of* $\gamma_1\gamma_2,\ldots,\gamma_L$*. Furthermore,* $H_{p,q}^{m,n}\left[\cdot\right]$ *denotes Fox's H function* [29, Eq. (8.3.1/1)][1]*.*

Proof For a specific q (i.e. $q=1$ for MRC, $q=1/2$ for EGC, and $q=-1$ for AF multihop transmission), we can write

$$\frac{1}{X}\log\left(1+X^q\right) = \frac{1}{X}G_{2,2}^{1,2}\left[X^q\left|\begin{array}{c}1,1\\1,0\end{array}\right.\right] \tag{8}$$

by using [29, Eq. (8.4.6/5)] and [30, Eqs. (2.9.1) and (2.1.5)], where $G_{p,q}^{m,n}\left[\cdot\right]$ denotes Meijer's G function [29, Eq. (8.2.1/1)]. Accordingly, Eq. (8) can also be written as

$$\frac{1}{X}\log\left(1+X^q\right) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{\Gamma\left(-s\right)\Gamma\left(-s\right)\Gamma\left(1+s\right)}{\Gamma\left(1-s\right)}X^{-qs-1}ds \tag{9}$$

within the convergence region $-1<\sigma<0$, where $i=\sqrt{-1}$ is the imaginary number, and $\Gamma\left(\cdot\right)$ is the gamma function [32, Eq. (6.1.1)]. Further, herein, q could be negative, namely $q=-1$ for the AF multihop transmission. Keeping this in mind, the term X^{-qs-1} can be written in terms of Laplace transform:

$$X^{-qs-1} = \int_0^\infty \frac{u^{qs}}{\Gamma\left(1+qs\right)}\exp\left(-Xu\right)du \tag{10}$$

within the convergence condition $q\Re\left\{s\right\}>-1$ (i.e., $q\sigma>-1$). Accordingly, substituting Eq. (10) into Eq. (9) and using [29, Eq. (8.3.1/1)], we obtain

$$\log\left(1+X^q\right) = \int_0^\infty C_{\eta,q}\left(s\right)\left[\frac{\partial}{\partial s}\exp\left(-Xs\right)\right]ds \tag{11}$$

[1] For more information about Fox's H function, readers are referred to [28–30].

where the auxiliary function $C_{\eta,q}(s)$ is given in Eq. (7). Finally, substituting Eq. (11) into Eq. (3) and then using the joint generalized MGF $M_{\gamma_{end}}(s\,|\,p) = E\left[e^{-s\left(\gamma_1^p+\gamma_2^p+\ldots+\gamma_L^p\right)}\right]$, that is

$$M_{\gamma_{end}}(s\,|\,p) = \int_0^\infty \cdots \int_0^\infty \exp\left(-s\sum_{l=1}^{L}\gamma_l^p\right) p_{\gamma_1,\gamma_2,\ldots,\gamma_L}(r_1,r_2,\ldots,r_L)\,dr_1\ldots dr_L \tag{12}$$

we readily reduce Eq. (3) into Eq. (6), which proves Theorem 1. □

Theorem 2 *demonstrates how to use the MGF to carry out the ACC analysis simultaneously for a variety of diversity combining and transmission schemes. In the case of no correlation between* $\gamma_1\gamma_2,\ldots,\gamma_L$*, it is rewritten as shown in the following theorem.*

Theorem 2 (ACC analysis over uncorrelated fading channels). The ACC analysis over uncorrelated fading channels can be achieved by

$$C_{avg} = \int_0^\infty C_{\eta,q}(s)\sum_{l=1}^{L}\left[\frac{\partial}{\partial s}M_{\gamma_l}(s\,|\,p)\right]\prod_{\substack{k=1,\\ k\neq l}}^{L} M_{\gamma_k}(s\,|\,p)ds \tag{13}$$

where $M_{\gamma_l}(s\,|\,p) = E\left[\exp\left(-s\gamma_l^p\right)\right]$ *is the generalized MGF of the* l*th branch/hop.*

Proof Note that, assuming that all instantaneous SNRs $\gamma_1\gamma_2,\ldots,\gamma_L$ are uncorrelated, the joint PDF can be rewritten as the product of the PDF of $\gamma_1\gamma_2,\ldots,\gamma_L$ as follows:

$$p_{\gamma_1,\gamma_2,\ldots,\gamma_L}(r_1,r_2,\ldots,r_L) = \prod_{l=1}^{L} p_{\gamma_l}(r_l) \tag{14}$$

where $p_{\gamma_l}(r_l)$ denotes the PDF of γ_l. Accordingly, substituting Eq. (14) into Eq. (12), we have

$$M_{\gamma_{end}}(s\,|\,p) = \prod_{l=1}^{L}\int_0^\infty e^{-s\gamma_l^p}\,p_{\gamma_l}(r)\,dr = \prod_{l=1}^{L} M_{\gamma_l}(s\,|\,p) \tag{15}$$

With this result, Eq. (6) can be readily simplified to Eq. (13), which proves Theorem 1. □

Regarding the usage of Eq. (7) in both Eqs. (6) and (13), an implementation of Fox's H function is available in [25]. Using [29, Eq. (8.3.2/22)], the auxiliary function $C_{\eta,q}(s)$ can also be expressed as the more familiar Meijer's G function. As such, for $|q| = k/l$ with $\gcd(k,l) = 1$, where $k \in \mathbb{Z}^+$ and $l \in \mathbb{Z}^+$, the auxiliary function $C_{\eta,q}(s)$ is given by

$$C_{\eta,q}(s) = -\sqrt{\frac{(2\pi)^{k+1}}{(2\pi)^{2l}\,k}}\,G_{2l+k,2l}^{l,2l}\left[\frac{\eta^l k^k}{L^k s^k}\,\middle|\,\begin{matrix}\Xi_{(1)}^{(l)},\Xi_{(1)}^{(l)},\Xi_{(1)}^{(k)}\\ \Xi_{(1)}^{(l)}\Xi_{(0)}^{(l)}\end{matrix}\right],\quad \text{for } q \geq 0; \tag{16}$$

$$C_{\eta,q}(s) = -\sqrt{\frac{(2\pi)^{k+1}}{(2\pi)^{2l}\,k}}\,G_{2l,2l+k}^{l,2l}\left[\frac{\eta^l L^k s^k}{k^k}\,\middle|\,\begin{matrix}\Xi_{(1)}^{(l)}\Xi_{(0)}^{(l)}\\ \Xi_{(1)}^{(l)},\Xi_{(0)}^{(l)},\Xi_{(0)}^{(k)}\end{matrix}\right],\quad \text{for } q < 0; \tag{17}$$

where $\Xi_{(n)}^{(x)}$ is the coefficient list defined as

$$\Xi_{(n)}^{(x)} \frac{x}{n} \frac{x+1}{n} \frac{x+2}{n}, \cdots, \frac{x+n-1}{n} \tag{18}$$

with $x \in \mathbb{C}$ and $n \in \mathbb{Z}^+$. It is worth noting that the computational efficiency and latency of $G_{p,q}^{m,n}[\cdot]$ is primarily addressed by the total number of coefficients (i.e. $p+q$). In this context, regarding $|q| = k/l$, the numbers $k \in \mathbb{Z}^+$ and $l \in \mathbb{Z}^+$ have to be kept small while supporting $|q| \approx k/l$. Otherwise, Fox's H function is preferable.

In accordance with the Table, let us consider some special cases of Eq. (7) to illustrate the flexibility and analytical accuracy of the novel MGF-based approach presented above. For instance, for $p > 0$ and $q > 0$, it is shown that using [30, Eq. (2.1.12)], Eq. (7) simplifies to [11, Eq. (9)] as expected. Specifically, setting $\eta = L$ and $q = 1$ in Eq. (7) and using [29, Eqs. (8.3.2/21) and (8.2.2/9), (8.2.2/14), and (8.4.11/1)], the auxiliary function $C_{L,1}(s)$ for the L-branch MRC combining is given as

$$C_{L,1}(s) = -H_{3,2}^{1,2}\left[\frac{1}{s}\,\middle|\, \begin{matrix} (1,1),(1,1),(1,1) \\ (1,1),(0,1) \end{matrix}\right] = Ei(-s) \tag{19}$$

where $Ei(\cdot)$ is the exponential integral function [24, eq. (8.211/1)]. Accordingly, putting (19) into (6) yields the well-known MGF-based approach reported in [10, eq. (7)], i.e.

$$C_{avg} = \int_0^\infty Ei(-s)\left[\frac{\partial}{\partial s} M_{\gamma_{end}}(s\,|\,1)\right] ds \tag{20}$$

Further, putting (19) into (13) further yields to the ACC analysis of the L-branch MRC diversity receiver with uncorrelated diversity branches as expected.

For the L-branch EGC diversity receiver, setting $\eta = L$ and $q = 2$ in Eq. (7) and performing some algebraic manipulations using [29, Eqs. (8.3.2/21), (8.2.2/9), and (8.4.12/4)], the auxiliary function $C_{L,2}(s)$ for the L-branch EGC diversity receiver can be expressed as

$$C_{L,2}(s) = -H_{3,2}^{1,2}\left[\frac{1}{Ls^2}\,\middle|\, \begin{matrix} (1,1),(1,1),(1,2) \\ (1,1),(0,1) \end{matrix}\right] = 2Ci\left(\sqrt{L}s\right) \tag{21}$$

where $Ci(\cdot)$ is the cosine integral function [32, Eq. (5.2.27)]. Eventually, substituting Eq. (21) into Eq. (6) yields our result [33, Eq. (4)], that is

$$C_{avg} = \int_0^\infty 2Ci\left(\sqrt{L}s\right)\left[\frac{\partial}{\partial s} M_{\gamma_{end}}(s\,|\,1/2)\right] ds \tag{22}$$

For the case where there is no correlation among diversity branches, the ACC analysis is further simplified to our result [33, Eq. (5)], as expected.

For the L-hop AF multihop transmission scheme, the parameters η, p, and q are chosen to be $\eta = 1/L$, $p = -1$, and $q = -1$, respectively, in accordance with the Table. Using [34, Eqs. (07.34.03.0475.01) and (06.06.03.0003.01)], $C_{1/L,-1}(s)$ can be expressed as

$$C_{1/L,-1}(s) = -H_{2,3}^{1,2}\left[s\,\middle|\, \begin{matrix} (1,1),(1,1) \\ (1,1),(0,1),(0,1) \end{matrix}\right] = Ei(s) - \log(s) - \gamma \tag{23}$$

where $\gamma = 0.57721566\cdots$ is the Euler–Mascheroni constant [24, Eq. (8.367/1)]. Putting Eq. (23) into Eq. (13) yields the ACC analysis for the L-hop AF multihop transmission, that is

$$C_{avg} = \int_0^\infty \left(Ei\left(s\right) - \log\left(s\right) - \gamma\right) \left[\frac{\partial}{\partial s} M_{\gamma_{end}}\left(s\,|-1\right)\right] \, ds \tag{24}$$

which is different than [35, Eq. (10)], but it can be easily transformed to [35, Eq. (10)] by means of using the integration by parts rule [24, Eq. (2.02/5)].

Consequently, all these special cases presented above prove the analytical accuracy and validity of the proposed MGF-based approach simultaneously treating the ACC analysis for a variety of diversity combining and transmission schemes.

3.2. An MGF-based AEC analysis

A unified MGF-based approach for AEC analysis is proposed in the following.

Theorem 3 *(AEC analysis over correlated fading channels). The AEC analysis over correlated fading channels can be readily achieved by*

$$C_{eff} = -\frac{1}{A}\log\left(\int_0^\infty E_{A,\eta,q}\left(s\right) \left[\frac{\partial}{\partial s} M_{\gamma_{end}}\left(s\,|\,p\right)\right] ds\right) \tag{25}$$

where $\eta \in \mathbb{R}^+$, $p \in \mathbb{R}$, and $q \in \mathbb{R}$ are chosen according to the type of the combining or transmission technique, and where $E_{\eta,q}\left(s\right)$ is the auxiliary function given by

$$E_{A,\eta,q}\left(s\right) = -\frac{1}{\Gamma\left(A\right)} H_{2,2}^{1,1}\left[\frac{\eta}{L^q s^q} \,\middle|\, \begin{matrix} \left(1-A,1\right), \left(1, \left|q\right|/2 + q/2\right) \\ \left(0,1\right), \left(0, \left|q\right|/2 - q/2\right) \end{matrix}\right] \tag{26}$$

Proof Note that, using [29, Eq. (8.4.2/5)] and [30, Eqs. (2.9.1) and (2.1.5)] for a certain $q \in \mathbb{R}^+$, we can write

$$\frac{1}{X}\left(1 + X^q\right)^{-A} = \frac{1}{X\Gamma\left(A\right)} G_{1,1}^{1,1}\left[X^q \,\middle|\, \begin{matrix} 1-A \\ 0 \end{matrix}\right] \tag{27}$$

which is rewritten in terms of Mellin–Barnes contour integration [29, Eq. (8.2.1/1)] as $\frac{1}{X}\left(1+X^q\right)^{-A} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{\Gamma(A-s)\Gamma(s)}{\Gamma(A)} X^{-qs-1} ds$ within the converging region $0 < \sigma < A$, where substituting Eq. (10) and then utilizing Fox's H function [29, Eq. (8.3.1/1)] after changing the order of integrals, we obtain

$$\left(1 + X^q\right)^{-A} = \int_0^\infty E_{A,\eta,q}\left(s\right) \left[\frac{\partial}{\partial s}\exp\left(-Xs\right)\right] ds \tag{28}$$

where the auxiliary function $E_{A,\eta,q}\left(s\right)$ is given in Eq. (26). Finally, substituting Eq. (28) into Eq. (5) and then using Eq. (12), we readily reduce Eq. (5) into Eq. (25), which proves Theorem 3.

Similar to the idea that we established in Figure 1, Theorem 3 presents how to simultaneously carry out the AEC analysis of diversity combining and transmission schemes over correlated fading channels. In the case that there is no correlation among $\gamma_1\gamma_2, \ldots, \gamma_L$, it is also rewritten as shown in the following theorem. □

Theorem 4 *(AEC analysis over uncorrelated fading channels). The AEC analysis over uncorrelated fading channels can be achieved by*

$$C_{eff} = -\frac{1}{A}\log\left(\int_0^\infty E_{A,\eta,q}(s)\sum_{l=1}^{L}\left[\frac{\partial}{\partial s}M_{\gamma_l}(s\,|\,p)\right]\prod_{\substack{k=1\\k\neq l}}^{L}M_{\gamma_k}(s\,|\,p)ds\right) \tag{29}$$

Proof The proof is obvious by following the same steps as in the proof of Theorem 2. □

It is worth using Theorem 3 for certain special cases of diversity combining and transmission schemes in order to check the accuracy of the proposed MGF-based AEC analysis. For example, for $q \in \mathbb{R}^+$ such as $q = 1$ for MRC and $q = 2$ for EGC diversity receiver, the auxiliary function $E_{A,\eta,q}(s)$ is simplified to

$$E_{A,\eta,q}(s) = -\frac{1}{\Gamma(A)}H_{2,1}^{1,1}\left[\frac{\eta}{L^q s^q}\,\middle|\,\begin{matrix}(1-A,1),(1,q)\\(0,1)\end{matrix}\right] \tag{30}$$

with the aid of [30, Eq. (2.1.12)]. It can be readily shown that, using [30, Eq. (2.9.1)] and [29, Eq. (8.4.16/1)], Eq. (30) is further simplified for MRC combining as follows:

$$E_{A,L,1}(s) = -\frac{1}{\Gamma(A)}H_{2,1}^{1,1}\left[\frac{1}{s}\,\middle|\,\begin{matrix}(1-A,1),(1,1)\\(0,1)\end{matrix}\right] = \frac{\Gamma(A,s)}{\Gamma(A)} - 1, \tag{31}$$

where $\Gamma(\cdot,\cdot)$ denotes the complementary incomplete gamma function [32, Eq. (6.5.3)]. Accordingly, the AEC analysis of MRC combining is obtained as

$$C_{eff} = \log\left(\frac{1}{\Gamma(A)}\int_0^\infty (\Gamma(A,s)-1)\left[\frac{\partial}{\partial s}M_{\gamma_{end}}(s\,|\,p)\right]ds\right) \tag{32}$$

Applying the integral rule [24, Eq. (2.02/5)] to our result Eq. (32) results in perfect agreement with [20, Eq. (7)] and [22, Eq. (8)]. Furthermore, using [29, Eq. (8.3.2/22)], Eq. (30) is simplified for EGC combining as follows:

$$E_{A,L,2}(s) = -\frac{\sqrt{\pi}}{\Gamma(A)}G_{3,1}^{1,1}\left[\frac{4}{s^2}\,\middle|\,\begin{matrix}1-A,1,1/2\\0\end{matrix}\right] = \frac{-s^{2A}}{\Gamma(2A+1)}{}_1F_2\left[A;A+1/2;A+1;-s^2/4\right] \tag{33}$$

where ${}_pF_q[\cdot;\cdot;\cdot;\cdot]$ denotes the generalized hypergeometric function [29, Eq. (7.2.3/1)]. Then, with the aid of Eq. (33), the AEC analysis of EGC combining is readily achieved by

$$C_{eff} = -\frac{1}{A}\log\left(-\int_0^\infty \frac{s^{2A}}{\Gamma(2A+1)}{}_1F_2\left[A;A+\frac{1}{2};A+1;-\frac{s^2}{4}\right]\left[\frac{\partial}{\partial s}M_{\gamma_{end}}(s\,|\,p)\right]ds\right) \tag{34}$$

In addition to the AEC analyses of MRC and EGC combining, Theorem 3 can also be used for AF multihop transmission by just setting $\eta = 1/L$ and $p = q = -1$ in Eq. (25), such that the auxiliary function $E_{A,1/L,-1}(s)$ is readily simplified by utilizing both [29, Eq. (8.3.2/21)] and [29, Eq. (8.4.45/1)] as follows:

$$E_{A,1/L,-1}(s) = -\frac{1}{\Gamma(A)}H_{1,2}^{1,1}\left[s\,\middle|\,\begin{matrix}(1-A,1)\\(0,1),(0,1)\end{matrix}\right] = -{}_1F_1[A;1;-s] \tag{35}$$

where ${}_1F_1\left[\cdot;\cdot;\cdot\right]$ denotes the Kummer confluent hypergeometric function [29, Eq. (7.2.2/1)]. Accordingly, the AEC analysis of AF multihop transmission is obtained as

$$C_{eff} = -\frac{1}{A}\log\left(-\int_0^\infty {}_1F_1\left[A;1;-s\right]\left[\frac{\partial}{\partial s}M_{\gamma_{end}}\left(s\,|\,p\right)\right]ds\right) \tag{36}$$

which is in perfect agreement with [36, Eq. (3)], as expected.

4. Performance analysis results

In this section, the proposed MGF-based approaches presented in the previous sections are employed to exactly evaluate the ACC and AEC analyses. Accordingly, extended generalized K (EGK) distribution [23] is the model of the channel fading distributions that the branches/hops are subjected to. Due to its versatility in capturing different well-known fading conditions either as special or as limiting cases, it provides unification over fading distributions, e.g., see [23, Table I]. In EGK fading channels, the instantaneous SNR γ_l of the lth branch/hop follows the PDF [23, Eq. (3)]

$$p_{\gamma_l}\left(\gamma\right) = \frac{\xi}{\Gamma\left(m\right)\Gamma\left(m_s\right)}\left(\frac{\beta\beta_s}{\bar{\gamma}}\right)^{m\xi}\Gamma\left(m_s - m\frac{\xi}{\xi_s}, 0, \left(\frac{\beta\beta_s}{\bar{\gamma}}\right)^{m\xi}\gamma^{\xi}, \frac{\xi}{\xi_s}\right) \tag{37}$$

where $m\,(0.5 \leq m < \infty)$ and $\xi\,(0 \leq \xi < \infty)$ represent the fading figure (diversity severity/order) and the fading shaping factor, respectively, while $m_s\,(0.5 \leq m_s < \infty)$ and $\xi_s\,(0 \leq \xi < \infty)$ are the shadowing severity and the shadowing shaping factor, respectively. β and β_s are $\beta = \Gamma(m+1/\xi)/\Gamma(m)$ and $\beta_s = \Gamma(m_s{+}1/\xi_s)/\Gamma\left(m_s\right)$, respectively. Further, $\Gamma\left(\cdot,\cdot,\cdot,\cdot\right)$ denotes the extended incomplete gamma function defined as $\Gamma\left(\alpha,x,b,\beta\right) = \int_x^\infty r^{\alpha-1}\exp\left(-r - br^{-\beta}\right)dr$, where $\alpha\beta b \in \mathbb{C}$ and $x \in \mathbb{R}^+$. Referring to the previous section, the joint generalized MGF of $\gamma_1\gamma_2,\ldots,\gamma_L$, i.e. $M_{\gamma_{end}}\left(s\,|\,p\right) \triangleq E\left[\exp\left(-s\sum_{l=1}^{L}\gamma_l^p\right)\right]$, is required in closed form. Assuming the branches/hops are strictly uncorrelated, and referring to Theorem 2 and Theorem 4, the generalized MGF of γ_l, i.e. $M_{\gamma_l}\left(s\,|\,p\right)E\left[\exp\left(-s\gamma_l^p\right)\right]$, and its derivative $\frac{\partial}{\partial s}M_{\gamma_l}\left(s\,|\,p\right)\frac{\partial}{\partial s}E\left[\exp\left(-s\gamma_l^p\right)\right]$ are also required in closed form. Using [30, Eq. (2.1.5)], the PDF of γ_l is obtained as

$$p_{\gamma_l}\left(\gamma\right) = \frac{1}{\Gamma\left(m\right)\Gamma\left(m_s\right)\gamma}H_{1,3}^{3,1}\left[\frac{\beta\beta_s}{\bar{\gamma}}\gamma\,\middle|\,\begin{array}{c}---\\ \left(m_s,1/\xi_s\right),\left(m,1/\xi\right)\end{array}\right] \tag{38}$$

where $---$ denotes the empty arguments. Exercising [30, Eqs. (2.5.29) and (2.5.30)] on Eq. (38), the generalized MGF $M_{\gamma_l}\left(s\,|\,p\right)E\left[\exp\left(-s\gamma_l^p\right)\right]$ is readily obtained as[2]

$$M_{\gamma_l}\left(s\,|\,p\right) = \frac{H_{1,3}^{3,1}\left[\left(\frac{\beta\beta_s}{\bar{\gamma}}\right)^{|p|}\frac{1}{s^{|p|/p}}\,\middle|\,\begin{array}{c}\left(\theta\left(p\right),\theta\left(p\right)\right)\\ \left(m,|p|/\xi\right),\left(m_s,|p|/\xi_s\right),\left(\theta\left(p\right),\theta\left(-p\right)\right)\end{array}\right]}{\Gamma\left(m\right)\Gamma\left(m_s\right)} \tag{39}$$

where $\theta\left(x\right)$ denotes the unit step function, resulting in 1 if $x \geq 0$ and 0 otherwise. By means of using [30, Eqs. (2.1.5) and (2.2.2)], its derivative MGF, i.e. $\frac{\partial}{\partial s}M_{\gamma_l}\left(s\,|\,p\right)\frac{\partial}{\partial s}E\left[\exp\left(-s\gamma_l^p\right)\right] = -\int_0^\infty \gamma^p\exp\left(-s\gamma^p\right)p_{\gamma_l}\left(\gamma\right)d\gamma$, is

[2] Note that there exist some printing errors in [11, Eqs. (A.2) and (A.3)], whose corrected versions are given above in Eqs. (39) and (40), respectively.

derived as

$$\frac{\partial}{\partial s}M_{\gamma_l}\left(s\,|\,p\right)=\frac{H_{2,4}^{4,1}\left[\left(\frac{\beta\beta_s}{\bar{\gamma}}\right)^{|p|}\frac{1}{s^{|p|/p}}\left|\begin{array}{l}(\theta\left(p\right),\theta\left(p\right)),(0,|p|)\\(1,|p|),(m,|p|/\xi),(m_s,|p|/\xi_s),(\theta\left(p\right),\theta\left(-p\right))\end{array}\right.\right]}{\Gamma\left(m\right)\Gamma\left(m_s\right)ps} \tag{40}$$

Referring to [23, Table I], both $M_{\gamma_l}\left(s\,|\,p\right)$ and $\frac{\partial}{\partial s}M_{\gamma_l}\left(s\,|\,p\right)$ of many well-known fading distributions commonly used in the literature are found to be the special cases of Eqs. (39) and (40), respectively. Thus, inserting Eqs. (39) and (40) in both Eqs. (13) and (29), the ACC and AEC analyses can be evaluated for many well-known fading distributions. For example, Eqs. (39) and (40) reduce to the MGF and derivative MGF of the generalized-K distribution for $\xi\to 1$ and $\xi_s\to 1$, and to those of gamma distributions for $\xi\to 1$ and $\xi_s\to\infty$.

In Figure 1 and Figure 2, the ACC and AEC performances are respectively depicted for the MRC and EGC combining schemes over uncorrelated Nakagami-m fading channels while being depicted in Figure 3 for multihop transmission over the same fading channels. The results achieved by Monte Carlo simulations are shown to be in perfect match with the analytical ones obtained by our two MGF-based approaches.

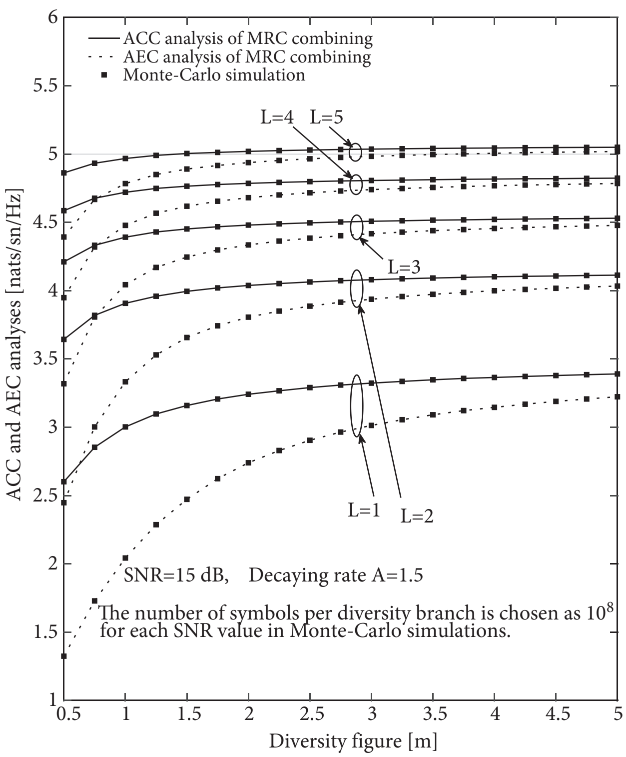

Figure 1. The ACC and AEC performances of the L-branch MRC combining over mutually interdependent Nakagami-m-fading channels.

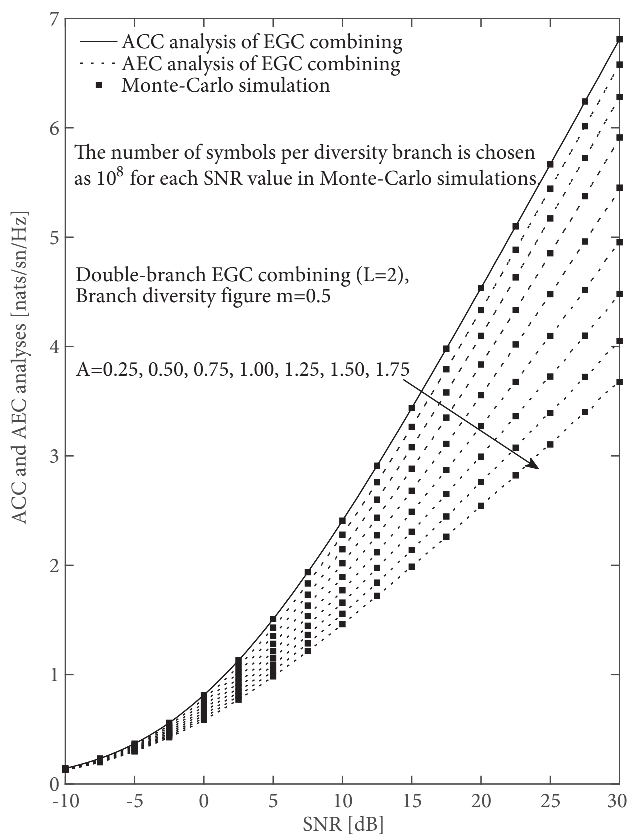

Figure 2. The ACC and AEC performances of the double-branch ($L=2$) EGC combining over mutually interdependent Nakagami-m-fading channels.

As observed in both in Figure 1 and Figure 2, the ACC increases as the number of the diversity branches, i.e. $L\in\mathbb{N}$ increases, which means that the highest rate rises with a negligible probability of error during transmission. It is also observed that as either the fading block length or the asymptotic decay rate of the buffer occupancy increases, the AEC increases, resulting in more reliability against the fading block length and the asymptotic decay rate of the buffer occupancy. As the fading conditions get better (i.e. as the quality of the transmission increases), both the fading block length and thus the asymptotic decay rate of the buffer occupancy approach zero, which yields that the ACC and AEC performances certainly overlap, as expected.

In Figure 3, the ACC and AEC of the AF multihop transmission are illustrated, showing that both monotonically decrease with the number of hops. As the fading conditions get better, the ACC and AEC performances certainly overlap, similar to the fact mentioned above for those of MRC and EGC combining schemes.

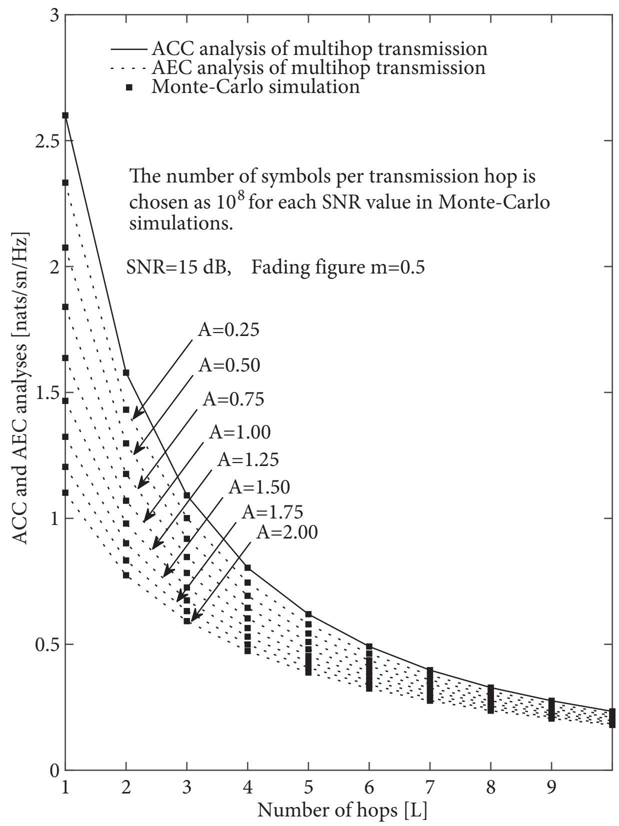

Figure 3. The ACC and AEC performances of the AF multihop transmission over mutu3 ally interdependent Nakagami-m-fading channels.

Finally, the concepts of ACC and AEC performances are well utilized in all figures as two crucial QoS metrics providing the averaged maximum achievable rate in all diversity combining and transmission schemes given in the Table. Note that our two MGF-based approaches presented above are more general as compared to the ones proposed in [9–12,20,22] since both can also be used for the ACC and AEC analyses of RMSC and SC diversity combining schemes and cascaded fading channels. Furthermore, both MGF-based approaches provide numerical techniques to find the performance lower-bounds. However, these special cases are beyond the scope of this paper due to space limitation.

5. Conclusion

In this paper, we proposed two MGF-based approaches for both ACC and AEC analyses of diversity combining and transmission schemes over generalized fading channels. Specifically, in contrast not only to [9,10], which are basically unified with respect to generalized fading channels, but also to [11], which is unified for both EGC and MRC combining schemes with respect to fading channels, our MGF-based ACC analysis is explicitly generic enough to unify the ACC analysis for popular diversity combining and transmission schemes over generalized fading channels. In addition, our MGF-based AEC analysis is more generic than those proposed in [20–22], such that it remarkably unifies not only for a variety of diversity combining and transmission schemes but also for correlated and uncorrelated fading distributions. Finally, we show that analysis-based and simulation-based results are in perfect agreement.

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