A new technique for optimum excitation of switched reluctance motor drives over a wide speed range

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Abstract: Optimum performance of switched reluctance motors (SRMs) over a wide range of speed control is an essential approach for many industrial applications. However, the doubly salient structure and deep magnetic saturation make magnetization characteristics of SRMs a highly nonlinear function of rotor position and current magnitude. This, in turn, makes the control of SRM drives a challenging task. As the control of SRMs depends on the inductance profile, it requires an adaptive control technique for optimum operation over a wide range of operating speeds. This paper presents an adaptive control technique for optimum excitation of SRM drives. The proposed control technique accurately considers the effect of back-emf voltage for high- and even low-speed operation. It determines the most efficient switch-on angle as a function of motor speed and current magnitude. Moreover, the optimum switch-off angle is defined to enhance motor output torque/power without negative torque production. The proposed technique simplifies the SRM control in order to cut down the complexity and cost; it offers easy implementation and can be used for sensor and sensorless operation of SRM drives. The proposed control technique uses an analytical solution. For adequate modeling, the nonlinear magnetization characteristics of the SRM are obtained using finite element analysis. The SRM, converter, and control algorithm are modeled using the MATLAB/Simulink environment. The simulation results are compared with a closed-loop switch-on angle controller in order to show the feasibility of the proposed control technique. In addition, experimental results are obtained to prove the promising performance and simplicity of the proposed controller.

Key words: Switched reluctance motor, adaptive control, excitation angles, switch-on angle, switch-off angle, finite element analysis, MATLAB/Simulink

1. Introduction

The switched reluctance motor (SRM) is the simplest of all electric machines. It promises a reliable and low-cost variable-speed drive for several industrial applications [1–3]. However, the doubly salient structure causes highly nonlinear magnetic characteristics, which complicate the control of SRM drives [4, 5]. The SRM control strategy depends on inductance profile and requires an online adaptive control strategy to achieve high performance over a wide range of operating speeds [6–8].

In many applications, it is desirable to achieve the highest torque/ampere ratio over the widest range of possible speeds [9–11]. The developed torque of SRMs can be optimized by applying appropriate switch-
on (\(\theta_{on}\)) and switch-off (\(\theta_{off}\)) angles, but this may require sophisticated and complex digital controllers for real-time variation of switching angles as a function of motor current and speed [9, 12].

For decades, many types of research have been directed towards optimizing SRM performance based on the optimum solution for \(\theta_{on}\) and \(\theta_{off}\) angles. Using the ideal inductance profile, the conventional approach for \(\theta_{on}\) was developed [12]. This solution can provide acceptable performance over a limited speed range [13]. In [5], the conventional approach was used to obtain an initial value of \(\theta_{on}\), and then within a certain range around this initial value an experiment was conducted to find the most efficient set of angles. In [6], a continuous tuning of \(\theta_{on}\) under steady state was used to minimize total power consumption, but the control strategy depended on energizing switching angles and required a complicated process to shorten its searching time. In [4], a closed-loop \(\theta_{on}\) control was designed to force the first peak of phase current to occur at the angle where rotor poles begin to overlap with stator poles (\(\theta_m\)). This method requires two subalgorithms to monitor the first peak of phase current and its position. In [7], the excitation angles were optimized based on a fuzzy adaptive controller, which required good experimental knowledge of machine performance. In [13], an analytical solution was developed for excitation angle optimization. However, this method neglects stator resistance, which has a great effect on current profile. In [14], the field reconstruction method was used for switching angle optimization. It may deliver a fast but complicated solution. In [15], automatic control of switch-off angle in the face of switch-on angle automatic control was introduced to maximize drive efficiency. The formula of optimized switch-off angle as a function of speed and reference current magnitude was obtained from the experimental results.

In this paper, an adaptive control technique for optimum \(\theta_{on}\) and \(\theta_{off}\) angles is developed. The proposed technique uses an analytical solution that can provide an eligible candidate for industrial applications. It cuts down the complexity and provides a fast and accurate solution that offers simple structure, easy and low cost of implementation, and high dynamics. It can provide optimum motor performance over the possible range of operating speeds. It does not basically require experimental measurements; the inductance data can be obtained easily using finite element analysis (FEA).

The rest of this paper is organized as follows: Section 2 presents the problem description. Section 3 illustrates the proposed controller for optimum \(\theta_{on}\) and \(\theta_{off}\) angles. Section 4 and Section 5 contain the simulation and experimental results, respectively. Finally, Section 6 gives the conclusions drawn from this research.

2. Problem description
The SRM cannot run directly from a DC or AC line; it requires discrete commutation from one phase to another [16]. The control strategy of SRM drives mainly depends on its inherited inductance profile. For the sake of simplicity, a linear inductance profile is considered for explanation as shown in Figures 1a and 1b. The motor coils must be excited in the increasing inductance zone (\(dL/d\theta > 0\)), and they should be deenergized before the negative inductance zone (\(dL/d\theta < 0\)) in order to avoid negative torque production. The amount of allowed time for phase current to rise/fall is a function of motor speed, current magnitude, and \(\theta_{on}\) and \(\theta_{off}\) angles.

The motor current profile differs greatly for low- and high-speed operation. At low speeds, the motor current can rise and decay quickly to reach its commanded reference value. Therefore, the switch-on instant can be delayed (close to \(\theta_m\)) as illustrated in Figure 1a. On the contrary, at high speeds, the phase current can neither rise nor decay quickly enough to reach its preferred level. For that reason, the motor phase winding is switched on early to allow the phase current to reach its desired level, as shown in Figure 1b [3–5]. Therefore, the optimum \(\theta_{on}\) should satisfy two conditions [4–6].

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1. It should make the phase current reach its reference value.

2. It should force the first peak of phase current to occur at angle \( \theta_m \).

On the other hand, the optimum \( \theta_{off} \) should make phase current decay to zero at \( \theta_z \). If the phase current decayed to zero after \( \theta_z \), a considerable amount of negative torque will be produced. In addition, if the current falls to zero before \( \theta_z \), the amount of generated positive torque will be decreased.

![Figure 1. Ideal inductance and optimum current waveforms at a) low speed, b) high speed.](image)

3. The proposed control technique

The motor for which the analysis is applied is a 6/4 three-phase SRM. Its magnetic circuit structure and main geometry configurations are shown in Figure 2. The motor dimensional and control parameters are given in the Table. As FEA is widely accepted for SRM characteristics calculation and performance analysis [17, 18], it is used to obtain the magnetization characteristics of the 6/4 SRM. Figure 3 shows the FEA-calculated inductance \( L(i, \theta) \) and torque \( T(i, \theta) \) characteristics. The unaligned and aligned positions are defined by angle \( \theta = 0^\circ \) and angle \( \theta = 45^\circ \), respectively.

![Table 1. The design data of 6/4 SRM prototype in mm.](table)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Output power</td>
<td>2.5 hp</td>
<td>Stator pole arc angle (( \beta_s ))</td>
<td>29°</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>60 V</td>
<td>Rotor pole arc angle (( \beta_r ))</td>
<td>31°</td>
</tr>
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<td>Rated speed</td>
<td>1500 r/min</td>
<td>Air gap length (( l_g ))</td>
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<tr>
<td>Yoke outside diameter (( D_{oy} ))</td>
<td>135</td>
<td>Stack length</td>
<td>90</td>
</tr>
<tr>
<td>Shaft diameter (( D_{sh} ))</td>
<td>27</td>
<td>Unaligned inductance (( L_u ))</td>
<td>0.8 mH</td>
</tr>
<tr>
<td>Bore diameter (( D_{bore} ))</td>
<td>74.2</td>
<td>Aligned inductance (( L_a ))</td>
<td>5 mH</td>
</tr>
<tr>
<td>Height of stator pole (( h_{ts} ))</td>
<td>18</td>
<td>( \theta_m )</td>
<td>12.5°</td>
</tr>
<tr>
<td>Height of rotor pole (( h_{tr} ))</td>
<td>13</td>
<td>( \theta_u = \theta_z )</td>
<td>45°</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>64</td>
<td>Core material</td>
<td>M-27 steel</td>
</tr>
</tbody>
</table>
3.1. Determination of optimum $\theta_{on}$

The conventional approach for optimum $\theta_{on}$ is given as [5, 12]:

$$\theta_{on} = \theta_m - \frac{L_u I_{ref} \omega}{V_{dc}},$$

(1)

Figure 2. The basic geometrical dimensions of the SRM.

Figure 3. The FEA-calculated inductance $L(i, \theta)$ and static torque $T(i, \theta)$ characteristics.
where $L_u$ is the minimum inductance, $V_{dc}$ is the DC bus voltage, $I_{ref}$ is the reference current, and $\omega$ is the rotor speed. Eq. (1) assumes that the phase-inductance is constant over region $[\theta_g, \theta_m]$. This approach can give reasonable performance under low speeds (up to base speed) unless $\theta_{on}$ becomes less than $\theta_g$. For speed higher than rated speed, Eq. (1) starts to break down as back-emf voltage becomes more prominent.

The determination of optimum $\theta_{on}$ over a wide range of speed control depends on the accurate estimation of back-emf voltage. This can be achieved if the real inductance profile of the SRM is considered.

We start from the voltage equation, as follows [19, 20]:

$$V = Ri + L(i, \theta) \frac{di}{dt} + \frac{dL(i, \theta)}{d\theta} i \omega,$$

where $V$ is the phase voltage and $R$ is the phase resistance. Solving the voltage equation for phase current ($i$) during its energizing process gives:

$$i(t) = \frac{V_{dc}}{R + K_b\omega} [1 - e^{-\frac{R + K_b\omega}{L_u} t_r}],$$

where $k_b = \frac{dL(i, \theta)}{d\theta}$. $t_r$ is the required rise time for phase current to reach $I_{ref}$:

$$t_r = \frac{-L(i, \theta)}{R + K_b\omega} \ln[1 - I_{ref}(\frac{R + K_b\omega}{V_{dc}})].$$

In order to get the first peak of phase current at $\theta_m$, $\theta_{on}$ should be advanced by $\omega t_r$ and $\theta_{on}$ becomes:

$$\theta_{on} = \theta_m - \omega t_r = \theta_m - \omega \frac{-L(i, \theta)}{R + K_b\omega} \ln[1 - I_{ref}(\frac{R + K_b\omega}{V_{dc}})].$$

Despite Eq. (5) giving the optimum solution of $\theta_{on}$, it represents $\theta_{on}$ as a function of multiple variables ($\theta_m$, $\omega$, $R$, $L(i, \theta)$, $k_b$, $I_{ref}$, and $V_{dc}$). $R$ and $V_{dc}$ have almost constant values. $\theta_m$ is the angle at which rotor poles begin to overlap with stator poles. It depends on motor geometrical structure and is found to be $12.5^\circ$. $\omega$ and $I_{ref}$ are control variables. Therefore, the determination of optimum $\theta_{on}$ depends mainly on $L(i, \theta)$ and its slope $k_b(i, \theta)$.

3.1.1. Accurate calculation of $L(i, \theta)$ and $k_b(i, \theta)$

The analysis of the inductance profile reveals the fact that in the minimum inductance zone $L(i, \theta)$ is only a function of rotor position $L(i, \theta) = L(\theta)$ (see Figure 3). Based on that fact, $L(i, \theta)$ can be fitted against $\theta$ using an exponential function as given by Eq. (6). After that, $k_b(i, \theta)$ can be calculated as the derivative of inductance as illustrated by Eq. (7). It should be noted that $k_b(i, \theta)$ is negative over $[\theta_g, 0)$ and positive over $[0, \theta_m]$.

$$L = ae^{b|\theta|} + c, \quad \theta_g \leq \theta \leq \theta_m$$

$$k_b(i, \theta) = \begin{cases} -\frac{dL(i, \theta)}{d\theta} = -abe^{b|\theta|}, & (\theta_g \leq \theta \leq 0) \\ +\frac{dL(i, \theta)}{d\theta} = +abe^{b|\theta|}, & (0 \leq \theta \leq \theta_m) \end{cases}$$
Here, \( a = 0.01161, b = 0.261, \) and \( c = 0.8173 \) are fitting coefficients. The coefficients with 95\% confidence bounds are calculated using the MATLAB fitting toolbox. Figure 4a shows a comparison between FEA-calculated inductance and its fitting curve. The variation of \( k_b(i, \theta) \) against rotor position is given in Figure 4b.

The accurate fitting of \( L(i, \theta) \) and \( k_b(i, \theta) \) is not enough for accurate determination of optimum \( \theta_{on} \). According to Eq. (5), \( \theta_{on} \) is calculated backward from \( \theta_m \). As given by Figure 1 but under consideration of the actual inductance profile, the difference between \( \theta_m \) and \( \theta_{on} \) is directly proportional to motor speed. As the motor speed increases, the difference \( (\Delta \theta = \theta_m - \theta_{on}) \) increases. Hence, for accurate determination of \( \theta_{on} \), the effective amount of inductance \( L(i, \theta) \) and \( k_b(i, \theta) \) should be calculated over interval \( \Delta \theta \). In order to achieve that, an initial value for \( \theta_{on} \) is required.

The calculation procedure for accurate \( \theta_{on} \) is shown in Figure 5. First, an initial value for the switch-on angle is calculated using the conventional approach that is given by Eq. (1). Then the effective values of \( L(i, \theta) \) and \( k_b(i, \theta) \) are calculated as the average values over the period \([\theta_{on}, \theta_m]\) based on Eqs. (6) and (7). Finally, these values are used to determine the optimum \( \theta_{on} \) accurately using equation (5).

It should be noted that the initial calculation of \( \theta_{on} \) using the conventional approach may have a little impact on the effective values of \( L(i, \theta) \) and \( k_b(i, \theta) \). This impact can be eliminated by adding an additional stage that calculates effective values of \( L(i, \theta) \) and \( k_b(i, \theta) \) as a function of accurate \( \theta_{on} \) if required. The effective values of \( L(i, \theta) \) and \( k_b(i, \theta) \) are plotted and fitted against initial \( \theta_{on} \) as shown in Figures 6a and 6b respectively. For easy implementation, these effective values are fitted against initial \( \theta_{on} \) as third-degree polynomials as follows:

\[
L(i, \theta)_{eff} = a_1 \theta_{on}^3 + a_2 \theta_{on}^2 + a_3 \theta_{on} + a_4, \tag{8}
\]

\[
k_b(i, \theta)_{eff} = b_1 \theta_{on}^3 + b_2 \theta_{on}^2 + b_3 \theta_{on} + b_4, \tag{9}
\]

where \( a_1 = 1.718554e^{-08}, a_2 = 6.45122e^{-07}, a_3 = 5.725676e^{-06}, a_4 = 9.0429e^{-04} \) and \( b_1 = 8.8571e^{-09}, b_2 = 1.0006e^{-07}, b_3 = 1.7266e^{-06}, b_4 = 2.2657e^{-05} \) are fitting coefficients that are calculated using the MATLAB fitting toolbox.

![Figure 4. Fitting curve for \( L(i, \theta) \) and \( k_b(i, \theta) \) in minimum inductance zone.](image-url)
Figure 5. The procedure for accurate calculation of optimum \( \theta_{on} \).

Figure 6. Fitting for the effective values of \( L(i, \theta) \) and \( k_b(i, \theta) \).

3.2. Determination of optimum \( \theta_{off} \)

As mentioned earlier, the optimum \( \theta_{off} \) should make motor phase current decay to zero at \( \theta_z \). The analytical solution for \( \theta_{off} \) is very complicated because inductance at the instant of switch-off is a function of current magnitude and rotor position. A simplified solution can be introduced based on the fact that the flux rise time is equal to its falling time under single pulse control with neglected stator resistance [13]. Figure 7 shows the phase flux linkage and phase current in an electric period. After the phase winding is switched on, the phase flux-linkage rises up from zero, reaching its peak at \( \theta_{on} \). Meanwhile, after the phase winding is switched off, the flux-linkage falls down, reaching zero at \( \theta_z \). With the flux-linkage’s rising time \( t_{rise} \) and falling time \( t_{fall} \):

\[
    t_{rise} = \frac{\theta_{off} - \theta_{on}}{\omega} = t_{fall} = \frac{\theta_z - \theta_{off}}{\omega} \quad \Rightarrow \quad \theta_{off} = \frac{1}{2}(\theta_{on} + \theta_z). \tag{10}
\]

Eq. (10) is derived based on single pulse control under high-speed operation. This equation starts to introduce a noticeable error while the speed is decreasing. The error becomes unacceptable under current chopping mode. This error can be handled with a compensation factor \( k \), and Eq. (10) becomes:

\[
    \theta_{off} = \frac{1}{2}(\theta_{on} + \theta_z) + k(1 + w \frac{I_{max}}{I_{ref}}), \tag{11}
\]

where \( k \) is a curve-fitting parameter that is adjusted with motor speed as a third-degree polynomial as given in Figure 8. \( I_{max} \) is the maximum permissible current that can follow in motor winding. Mostly, \( I_{ref} = I_{max} \) during the chopping process. \( w = 0.02 \) is a weight factor to suitably adjust the increase of \( \theta_{off} \) compensation in the case of \( I_{ref} < I_{max} \). In general, in order to avoid production of negative torque, angle \( \theta_z \) should be equal to \( \theta_a \). This may differ slightly according to the motor’s inherited inductance profile. In this paper, \( \theta_z \) is set to \( 45^\circ \). The structure of the \( \theta_{off} \) controller is illustrated in Figure 9.
4. Simulation results and discussion

For clear verification of the proposed control technique, a closed-loop switch-on angle (CL $\theta_{on}$) is used for the purpose of comparison [4]. Despite the other types of $\theta_{on}$ control techniques, CL $\theta_{on}$ is the best to guarantee an optimum solution for $\theta_{on}$ as it is a closed loop control. This controller forces the first peak of phase current ($\theta_{peak}$) to occur at $\theta_m$ and also allows the peak phase current to reach its reference level. The structure of the CL $\theta_{on}$ controller is shown in Figure 10. It compares the peak value of phase current ($I_{peak}$) to its reference level ($I_{ref}$), and the angle of the first peak position ($\theta_{peak}$) to $\theta_m$. The error signal is processed through a PID controller whose output compensates the conventional approach. The controller parameters are $KP = 0.5$ and $KPI = 0.2^o/A$, and PID gains are $KP = 0.5$, $KI = 12$, and $KD = -0.004$.

A complete simulation model of the 6/4 SRM, its drive system, and the control algorithms is achieved using the MATLAB/Simulink environment. The FEA-calculated magnetization characteristics are used for SRM simulation in the form of lookup tables [21, 22].

The motor performance under a sudden change in speed and load torque is shown in Figures 11a–11d. The motor starts with a constant load torque of 4.5 Nm and then load torque is decreased from 4.5 Nm to 3 Nm at 0.7 s. The speed is suddenly changed from 1500 r/min to 2500 r/min at 0.9 s as shown in Figure 11a. The online automatic control of $\theta_{on}$ is shown in Figure 11b. $\theta_{on}$ of the proposed controller has a clear difference from the CL $\theta_{on}$ controller because of the required settling time for the CL $\theta_{on}$ controller to reach steady state. The proposed controller provides the optimum solution of $\theta_{on}$ instantaneously according to motor speed and current magnitude. Its performance depends on the effective values of $L(i, \theta)$ and $k_b(i, \theta)$. The proposed control
technique provides an adaptive solution for $\theta_{on}$ by accurate determination of the effective values of $L(i, \theta)$ and $k_b(i, \theta)$ as illustrated in Figure 11c. It is clear that $L(i, \theta)$ and $k_b(i, \theta)$ are fully adaptive with motor speed and load torque. The online control of $\theta_{off}$ is given in Figure 11d.

A clear verification of optimum solution for $\theta_{on}$ and $\theta_{off}$ angles is given in Figures 11e–11h. The proposed controller provides optimum $\theta_{on}$, which forces the first peak of phase current to always occur at angle $\theta_m = 12.5^\circ$ (see Figure 11e). It also allows the peak value of phase current ($I_{peak}$) to reach its recommended reference level as shown in Figures 11f and 11g. For the CL $\theta_{on}$ controller, despite it reaching a steady-state value of 12.5°, it needs settling time and provides lower dynamic performance compared to the proposed controller.

The production of no negative torque can be guaranteed as illustrated in Figure 11h. After the instant of switching off, the phase current decays to zero at angle $\theta_z \cong 45^\circ$, which ensures no production of negative torques. Thus, the switch-off angle is optimum over the entire speed range. The current, inductance, and torque waveforms at low and high speeds are shown in Figures 12a and 12b, respectively. It is clear that the position of the first peak of phase current occurs at the end of the minimum inductance zone ($\theta_m = 12.5^\circ$), the current decays to zero at $\theta_z \cong 45^\circ$, and the torque profile has no negative torque at all. This ensures the optimum solution of $\theta_{on}$ and $\theta_{off}$.

![Figure 10. The structure of closed-loop switch-on angle controller (CL $\theta_{on}$ controller).](image)

5. Experimental verification

The controller performance is experimentally verified with a 6/4 three-phase SRM. The SRM specifications are given in the Table. The control algorithm is implemented using a Texas Instruments TMS320F28379D digital signal processor (DSP). The SRM is coupled to an electromagnetic brake (MAGTORL model 4605c), which acts as a mechanical load. An absolute shaft encoder (RM44SI0010B10F2E10) is used to provide the absolute rotor position through a synchronous serial interface (SSI) that is connected to SPI terminals of the DSP through an interface circuit. The estimated variables ($\theta_{on}$, $\theta_{off}$, and phase rotor position) are output using the digital analog converters (DACs) of the DSP board. High-accuracy and linear LAH50-P current transducers (CTs) are used for phase current measurements. The phase voltages and supply voltage are measured using a high-speed and linearity-based op-amp circuit. A three-phase transformer, three-phase diode rectifier, and capacitor are used to provide DC power. The data are collected and plotted using a data acquisition board (DAQ NI USB-6009) and LabView software. The DSP is programmed using MATLAB/Simulink with the C2000 microcontroller support package in addition to Code Composer Studio (CCS) software. Figure 13 illustrates
the schematic diagram of the measurement platform while Figure 14 shows the practical implementation of this platform.

Figures 15a–15f show the controller behavior under transient conditions. They illustrate the actual motor speed and $\theta_{on}$ and $\theta_{off}$ variation under a sudden step change in speed from 200 r/min to 1600 r/min. Both the original and filtered signals are included. A clear adjustment of $\theta_{on}$ and $\theta_{off}$ is seen to track changes
in motor speed and current magnitude. As the motor speeds up, $\theta_{on}$ decreases automatically to allow motor current to reach its reference level. $\theta_{off}$ is also adjusted to force current decay to zero at $\theta_z$.

**Figure 12.** The current, inductance, and torque wave-forms.

**Figure 13.** Schematic diagram of the measurement platform.
A detailed verification of the performance and effectiveness of the proposed control technique is given in Figures 16a–16f. This figure gives the experimental phase current against rotor position at a variety of operating speeds (704 r/min, 1071 r/min, 1317 r/min, 1514 r/min, 1846 r/min, 1966 r/min). At all the operating speeds,
the controller provides a very good adjustment of $\theta_{on}$ to force the first peak of phase current to always occur at $\theta_m = 12.5^\circ$. It also controls $\theta_{off}$ to make phase current decay to zero at $\theta_z \cong 45^\circ$.

Figure 16. Experimental results at variety of operating speeds.

6. Conclusions
This paper presented an adaptive analytical solution for optimum $\theta_{on}$ and $\theta_{off}$ angles of SRM drives over a wide range of speed control. The proposed control technique determines the optimum $\theta_{on}$ that provides the most efficient operation. It accurately considers the effect of back-emf voltage by adaptive calculation of the effective amount of phase inductance $L(i, \theta)$ and its derivative $k_b(i, \theta)$. In addition, it calculates the optimum $\theta_{off}$ in order to avoid production of negative torque even at high speeds. The proposed control technique uses analytical solutions that offer a simple structure, low cost of implementation, and high dynamics. The motor,
its converter, and control algorithms are modeled using MATLAB/Simulink. The experimental and simulation results show that the proposed technique provides optimum operation over an extended speed range.

References


