IONOLAB-MAP: An automatic spatial interpolation algorithm for total electron content

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Received: 23.11.2016 • Accepted/Published Online: 13.12.2017 • Final Version: 27.07.2018

Abstract: Investigation of the variability of total electron content (TEC) is one of the most important parameters of the observation and monitoring of space weather, which is the main cause of signal disturbance in space-based communication, positioning, and navigation systems. TEC is defined as the total number of electrons on a ray path. The Global Positioning System (GPS) provides a cost-effective solution for the estimation of TEC. Due to various physical and operational disturbances, TEC may have temporal and spatial domain gaps. Global ionospheric maps (GIMs) provide worldwide TEC with 1- to 2-h temporal resolution and 2.5° × 5° spatial resolution in latitude and longitude, respectively. The GIM-TEC with the highest possible accuracy can be obtained 10 days after the recording of the signals. Therefore, a high-resolution and accurate interpolation of TEC is necessary to image and monitor the regional distribution of TEC in near-real time. In this study, a novel spatiotemporal interpolation algorithm with automatic gridding is developed for 2-D TEC imaging by data fusion of GPS-TEC and GIM-TEC. The algorithm automatically implements optimum spatial resolution and desired temporal resolution with universal kriging with linear trend for midlatitude regions and ordinary kriging for other regions. The theoretical semivariogram function is estimated from GPS network data using a Matern family, whose parameters are determined with a particle swarm optimization algorithm. The developed algorithm is applied to the Turkish National Permanent GPS Network (TNPGN-Active), a dense midlatitude GPS network. For the first time in the literature, high spatial resolution TEC maps are obtained between May 2009 and May 2012 with a 2.5-min temporal update period. These TEC maps will be used to investigate the spatiotemporal variability of the ionosphere over the diurnal and annual trend structure, including seasonal anomalies and geomagnetic and seismic disturbances over ionosphere.

Key words: Ionosphere, total electron content, kriging, global positioning system, 2D interpolation, optimization

1. Introduction

The ionosphere is an important layer for short-wave and satellite communication, navigation, positioning, and guidance systems. In order to compensate for phase and amplitude errors in the signals, the structure of the ionosphere must be understood and its variability must be continuously monitored [1].

One of the most important parameters for monitoring the variability of the ionosphere is total electron content (TEC). TEC is defined as the total number of electrons in a cylinder with a cross-sectional area of 1 m² along a ray path. The unit of TEC is TECU, and 1 TECU equals 10¹⁶ electrons/m².

The Global Positioning System (GPS) provides cost-effective solutions for estimating TEC [2,3]. The

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space and time variability of the ionosphere can be investigated by observing the variability of GPS-TEC. Unfortunately, due to various physical or operational disturbances, GPS network stations are sparse in space and the recorded data may have temporal gaps.

In order to obtain regular and dense TEC values in a given region, TEC can be interpolated spatially, which is called a TEC map.

A widely used source of TEC maps are the global ionosphere maps (GIMs) on the International GNSS Service (IGS) website (ftp://igscb.jpl.nasa.gov). The resolution of a GIM is $2.5^\circ \times 5^\circ$ in latitude and longitude, respectively, with 1 to 2 h of temporal resolution [4]. Although GIMs provide a general and global distribution of TEC, spatiotemporal resolution is not sufficient to capture local and regional variability. Therefore, a reliable, automatic, robust, and near-real time mapping tool is necessary for monitoring the regional ionosphere.

In the literature, several methods are used for TEC mapping. Although inverse distance weighting (IDW), multiquadratic function fitting, spline methods, and spherical cap harmonic analysis do not include any statistical information on the nature of the ionosphere, these methods have low computational complexity and are easy to implement [5–11]. A neural network is a powerful method when extrapolation is considered. However, this method has high computational complexity [5,12]. The TEC mapping technique, on the basis of a Kalman filter data assimilation scheme, suffers from similar drawbacks, and small-scale variability of the ionosphere can be missed due to its low temporal resolution [13]. Kriging is proven to be a strong candidate for regional mapping of TEC [9,14–19]. One of the most important advantages of kriging over the other listed mapping techniques is its inherent inclusion of the spatial correlated structure of TEC, whereas its main drawback is its dependence on uniform distributions of samples and its choice of semivariogram function. In [19,20], it was shown that kriging gives the lowest error when the ionospheric trend is known. In [20], it was shown that the trend of TEC for a midlatitude region can be modeled by a linear function. Therefore, in [14–16], universal kriging with linear trend (UK1) was applied to a midlatitude region. In order to capture the spatiotemporal variability fully, the Matern family is used for the estimation of the theoretical semivariogram function. The major problem with this application is the possible sparsity or nonuniform distribution of GPS stations, which causes depleted data sets. In such cases, the number of samples in the experimental semivariogram may decrease significantly and the estimation of the theoretical semivariogram function may not be reliable. Additionally, clustered or nonuniform distribution GPS stations may cause errors in kriging interpolation [19,20].

In this study, an automatic regional 2-D imaging algorithm for TEC, namely IONOLAB-MAP, is introduced. IONOLAB-MAP is developed to produce robust TEC interpolation by fusing the data from a GPS network and a GIM so that the missing or depleted data sets can be interpolated, resulting in a robust and reliable estimation of semivariogram functions. In this way, TEC maps reconstructed using kriging algorithms, with semivariograms estimated from the Matern family, can have spatial resolutions that are defined according to the number of spatial samples in a denser data set. The error variance bounds in the new algorithm indicate that the TEC maps can achieve the highest possible accuracy and spatial resolution. The algorithm can be used as a module with a user-friendly function structure, where the user needs to define only the date, time, and region borders. The predetermined GPS station coordinates and GPS recording directories can be provided within the inputs of the function. IONOLAB-MAP and error variance maps are generated automatically, and the outputs are saved in a predetermined format in a user-defined directory.

In Section 2, the interpolation of the missing GPS-TEC values is detailed. The developed algorithm is provided in Section 3 along with details on universal and ordinary kriging. The implementation of the algorithm over a TNPGN-active regional GPS network is presented in Section 4.
2. Spatiotemporal interpolation of missing TEC

GPS-TEC can be disrupted for a certain period due to various physical and operational disturbances. Therefore, interrupted TEC estimates must be corrected to monitor the ionosphere for 24 h. In this study, TEC values are corrected by using the GIM-TEC estimates as completely as possible, as detailed in [14–16].

The spatial interpolation technique aims to interpolate the missing TEC value of any station from its neighbors within a given radius. The full derivation of the spatial interpolation technique is provided in [15,16].

TEC values of a given station \( u \) for a day \( d \) and time \( t \) can be defined as \( z_{u; d; t} \). The estimate of the missing TEC value, \( \hat{z}_{u; d; t} \), can be obtained by using TEC values of the neighbors of the station \( u \) within \( R_r \) km radius on day \( d \) and time \( t \) as follows:

\[
\hat{z}_{u; d; t} = \sum_{v=1}^{N_{u; R_r}} \alpha_{u; d; t; R_r} (v) z_{v; d; t; R_r}
\]

where \( \alpha_{u; d; t; R_r} (v) \) is the interpolation coefficient of the \( v \)th neighbor of the station \( u \); \( N_{u; R_r} \) is defined as the number of neighbors used for the interpolation of TEC within \( R_r \) km radius; and \( z_{v; d; t; R_r} \) denotes the TEC value of the \( v \)th neighbor.

The interpolation coefficient of the \( v \)th neighbor, \( \alpha_{u; d; t; R_r} (v) \), can be obtained by solving the following minimization problem:

\[
\sum_{d_n=d_i}^{d_s} \left| \sum_{v=1}^{N_{u; R_r}} \alpha_{u; d; t; R_r} (v) z_{v; d; t; R_r} - z_{u; d; t} \right|_2^2
\]

from day \( d_i \) to day \( d_s \) prior to day \( d \). The minimization in Eq. (2) can be obtained in closed form, and the interpolation coefficients can be obtained as:

\[
\alpha = \left( \sum_{d_n=d_i}^{d_s} z_{v; d; t}^T z_{v; d; t} \right)^{-1} \left( \sum_{d_n=d_i}^{d_s} z_{v; d; t} z_{v; d; t} \right)
\]

where \( \alpha \) denotes the optimized interpolation coefficient vector and is given as:

\[
\alpha = \left[ \begin{array}{cccc} \alpha_{u; d; t; R_r} (1) & \cdots & \alpha_{u; d; t; R_r} (v) & \cdots & \alpha_{u; d; t; R_r} (N_{u; R_r}) \end{array} \right]^T
\]

The TEC vector of the neighbors on day \( d_n \) can be expressed as:

\[
z_{v; d; t} = \left[ z_{1; d; t; R_r} \cdots z_{v; d; t; R_r} \cdots z_{N_{u; R_r}; d; t; R_r} \right]^T
\]

Using Eq. (2), any missing TEC value can be calculated from its neighbors. The interpolation coefficients are obtained from Eq. (3), as given in detail in [15,16].

3. Automatic regional mapping

TEC maps require a higher and denser distribution in order to capture spatial ionospheric variability. Higher resolution TEC distributions can be obtained by regional GPS networks, where more advanced mapping methods can be implemented for any desired time interval. Kriging, which is known as the best linear unbiased estimator (BLUE), is one of the advanced mapping techniques widely used in the literature for the interpolation of
geophysical signals [17,19,20]. In order to reduce kriging errors, samples must be distributed uniformly [19,21]. Thus, TEC values are redistributed over a new grid structure. The arrangement of grid points is detailed in the next section.

3.1. Underlying grid structure

Any grid structure with the desired density can be placed over the region of interest, and the TEC estimates of each grid point can be calculated with interpolation methods.

To obtain uniformly distributed sample points, the region of interest can be divided into \( N_\theta \) points in the direction of latitude and \( N_\phi \) points in the direction of longitude, as shown in Figure 1. A major challenge is to decide the number of sample points for interpolation without loss of representation of underlying ionospheric variability.

\[
\begin{align*}
N_\theta &= \frac{\theta_f - \theta_i}{\Delta \theta} + 1, \\
N_\phi &= \frac{\phi_f - \phi_i}{\Delta \phi} + 1,
\end{align*}
\]

where \( \Delta \theta \) and \( \Delta \phi \) are intervals of latitude and longitude, respectively. It is advisable to generate uniform square spatial sampling in order to minimize the kriging error [19,22]. Therefore, \( \Delta \theta \) is taken as equal to \( \Delta \phi \), and both are called \( \Delta \). Then the number of grid points, \( N_s = N_\theta N_\phi \), can be calculated as:

\[
N_s = \left( \frac{\theta_d}{\Delta} + 1 \right) \left( \frac{\phi_d}{\Delta} + 1 \right)
\]

where \( \theta_d = \theta_f - \theta_i \) and \( \phi_d = \phi_f - \phi_i \). \( N_s \) must be determined with respect to the semivariogram [23]. \( \Delta \) can be computed by solving the second-order equation given in Eq. (8). Since \( \Delta \) defines the interval of the grid points, it must be real and positive. The equation has two distinct real roots, but only one is positive. Therefore, \( \Delta \) can be calculated as:
\[ \Delta = \frac{(\theta_d + \phi_d) + \sqrt{(\theta_d + \phi_d)^2 + 4 \theta_d \phi_d (N_s - 1)}}{2 (N_s - 1)}. \]  

TEC estimates can be obtained using the interpolation algorithm on a redistributed uniform grid. Universal kriging with linear trend (UK1) and ordinary kriging algorithms are provided in the next section.

### 3.2. Automatic regional mapping using UK1 algorithm

The UK1 algorithm estimates TEC, \( \hat{z}_K \), using the linear combination TEC samples, \( z_s \), at any position in space, \( x = [\theta \phi]^T \), where \( \theta \) and \( \phi \) denote latitude and longitude, respectively. \( \hat{z}_K \) can be calculated as:

\[ \hat{z}_K (x) = \sum_{n_u=1}^{N_u} \lambda_{n_u} z_s (x_{n_u}) \]  

where \( x_{n_u} = [\theta_{n_u} \phi_{n_u}]^T \) defines the measurement points. Weight \( \lambda_{n_u} \) at \( n_u \) sample point is obtained by solving the constrained optimization [14–17,19,20].

The strength of the statistical correlation as a function of distance in kriging can be calculated by using a semivariogram function. Since the experimental semivariogram may be computed only for certain pairs at irregular distances, \( h \), a theoretical model is usually fitted to obtain a continuous function for all possible \( h \) values. In the literature, typical theoretical semivariogram functions are chosen as exponential, Gaussian, and spherical [14,15].

In the case of the ionosphere, TEC values vary significantly with respect to time and one semivariogram function may not represent this complicated variability. Therefore, in this study, the theoretical semivariogram function is chosen to be the Matern family, which includes all the above-listed functions and is given as:

\[ \gamma_M (h) = \begin{cases} c_0, & h = 0 \\ c_0 + \sigma_0^2 \left[ \frac{2 h^{\nu}}{3 d^{\nu + 1}} K_{\nu}(\frac{h}{d}) \right], & h > 0 \end{cases} \]  

where \( c_0 \) is the nugget effect; \( \sigma_0^2 \) is the partial sill; \( K_\nu (\cdot) \) is the modified Bessel function of the second kind and order \( \nu \); and, finally, \( d \) is defined as the range [14–16]. Different correlation models can be obtained by changing the smoothing parameter, \( \nu \).

Parameters \( c_0, \sigma_0^2, \nu, \) and \( d \) can be obtained by using the particle swarm optimization (PSO) method due to its heuristic nature. The advantages of PSO are its relative ease in implementation and convergence speed for manipulating optimum points with few parameters [14–16,23].

The UK1 algorithm can be repeated for any desired temporal interval. Since the parameters of the Matern semivariogram function are obtained using PSO, the UK1 maps are obtained automatically with high spatial resolution for any desired time interval.

### 3.3. Automatic regional mapping using ordinary kriging algorithm

The ordinary kriging (OK) algorithm estimates TEC using the linear combination TEC samples at any position in space \( x \), as shown in Eq. (10). The main difference between UK1 and OK comes from the underlying trend estimation. OK assumes that TEC over a region has a fixed yet unknown trend. Unlike the UK1 algorithm,
the experimental semivariogram is calculated directly over the TEC values for the OK algorithm. The Matern semivariogram is chosen as the theoretical semivariogram function. By running the PSO algorithm several times, the parameters of the Matern function can be obtained with minimum cost with respect to the experimental semivariogram. Then the weights of the sample points given in Eq. (10) are obtained with respect to the OK algorithm by solving the constrained optimization.

The OK algorithm can be repeated for any desired temporal interval. Since the parameters of the Matern semivariogram function are obtained using PSO, OK maps are obtained automatically with high spatial resolution for any desired time interval. In the next section, the results of the developed algorithms are explained for automatic mapping of TEC.

4. Results

In this study, a user-friendly, robust, reliable, and automatic regional spatial interpolation algorithm, called IONOLAB-MAP, is developed. The outlined algorithm of IONOLAB-MAP, given in Figure 2, can be applied to any GPS network. This algorithm operates with two different input options. GPS-TEC can be obtained from a directory or given as an input. Before the augmentation of the GPS network with GIM-TEC, the values are checked and possible erroneous values are corrected using the methods given in [2,12,24]. For the purpose of reducing instabilities in kriging that occur in the inverse operation [19,20], GPS-TEC is augmented with corrected GIM-TEC. After augmentation of GPS-TEC with GIM-TEC, the missing GPS-TECs are recovered with the spatiotemporal interpolation method given in Section 2. In order to reduce kriging errors, a uniform grid structure is constituted over the given region. TEC estimates of the uniformly distributed grid points are calculated using a first-order interpolation algorithm. With respect to the region, the algorithm chooses the kriging method automatically and high-resolution TEC maps are generated. Finally, the outputs are saved in a predetermined format in a user-defined directory.

In this study, IONOLAB-MAP is applied to the Turkish National Permanent GPS Network (TNPGN-Active). The TNPGN-Active network is composed of 144 GNSS stations, which have continuously operated since May 2009. TEC is estimated as IONOLAB-TEC, whose temporal resolution is 2.5 min [2,12,24]. However, the typical temporal resolution of GIM is 2 h. In order to equalize the temporal resolution of GIM-TEC with IONOLAB-TEC, GIM-TECs are interpolated by C-spline [15,16]. Then IONOLAB-TEC is augmented with corrected GIM-TEC, and the missing IONOLAB-TEC values are recovered based on the spatiotemporal interpolation method provided in [15,16]. The disrupted IONOLAB-TEC values are interpolated with one neighborhood \((N_w; R_r = 1)\) and one day \((d_i = d_s = d - 1)\) at a given time. Any TNPGN-Active station is surrounded by four GIM points. The disrupted IONOLAB-TEC of any TNPGN station is estimated from four separate GIM points. The new value of the missing IONOLAB-TEC is calculated by taking the median of these four different GIM-TEC values, temporally interpolated for the desired hour.

Before the kriging algorithm is implemented, the uniformly distributed grid structure given in Section 3.1 is placed over Turkey. A common yet unsubstantiated practice is to limit estimation to lags with a minimum of 30 pairs in order to calculate the experimental semivariogram [17]. The number of grid points \((N_s)\) is determined to provide minimum pairs. Then the TEC estimates of each grid point are calculated using the IDW algorithm. Hence, the TEC of each grid point is calculated from the weighted average of the sparser data.

As indicated previously, the underlying trend function must be known to process the UK1 algorithm. Previous studies show that the trend of TEC for a midlatitude region can be modeled by a linear function [25]. Hence, in this study, the trend of the TNPGN-Active is chosen as linear [14–16].
The spatial correlation structure of TEC can be represented by the experimental semivariogram. In order to obtain a continuous semivariogram function with respect to distance, the experimental semivariogram must be fitted to a theoretical semivariogram. In this study, the Matern family is chosen as the theoretical semivariogram function, as discussed in the previous section. The parameters of the Matern function are estimated by using PSO, and the Matern function is fitted on the experimental isotropic semivariogram [14]. In order to shorten the time, the boundaries of the parameters are restricted. The lower bound of each parameter \((c_0, \sigma^2, \nu,\text{ and } d)\) is defined as zero. The upper bounds of the parameters are defined as the minimum value of the experimental isotropic semivariogram, the maximum value of the experimental isotropic semivariogram, and the maximum value of distance, respectively.

Previous studies showed that uniform square sampling can be increased six times by using the kriging
algorithm [19,20]. Thus, in this study, in order to obtain denser TEC maps, TEC estimates are redistributed on a high spatial resolution (six times larger) grid structure. The number of grid points, $N'_s$, can be found as $N'_s = 6N_s$ after the kriging algorithm. TEC estimation of each new grid point is calculated using UK1, given in Section 3.2, and the IONOLAB-MAP is generated. Lastly, IONOLAB-MAP and data can be saved in a predetermined format.

In Figure 3, isotropic UK1 TEC maps and corresponding variance maps over Turkey are provided for 25 March 2011 at 09:00 UT, which was a quiet day for the ionosphere. Figure 3a illustrates the UK1 map obtained by using reduced GPS data. The kriging algorithm is run over 50 TNPGN-Active stations that are chosen randomly. Figure 3b illustrates the UK1 map obtained by using full TNPGN data. Figure 3c illustrates the UK1 map obtained by using augmented TNPGN-Active stations with GIM-TEC. In Figure 4 corresponding variance maps are illustrated. As shown here, the recovery of the missing data becomes important, because it both improves the spatial resolution and lowers the error in variance. Additionally, it can easily be seen from the same figure that augmentation with GIM-TEC reduces extrapolation errors.

![Figure 3](image-url)

**Figure 3.** Isotropic UK1 maps on 25 March 2011, a nondisturbed day of the ionosphere, at 09:00 UT: a) UK1 maps using reduced TNPGN data, b) UK1 maps using full TNPGN data, c) UK1 maps using augmented data.

In Figure 5, isotropic UK1 TEC maps and corresponding variance maps are provided for 25 October 2011 at 09:00 UT, a disturbed day in the ionosphere during which a severe geomagnetic storm took place [26]. Figure 5a illustrates the UK1 map obtained by using reduced GPS data. The kriging algorithm is run over 50 TNPGN-Active stations chosen randomly. Figure 5b illustrates the UK1 map obtained by using full TNPGN
Figure 4. Isotropic UK1 variance maps on 25 March 2011, a nondisturbed day of the ionosphere, at 09:00 UT: a) variance maps using reduced TNPGN data, b) variance maps using full TNPGN data, c) variance maps using augmented data.
Using the developed algorithm, UK1 TEC maps or OK TEC maps can be obtained automatically at any desired time interval with high spatial and temporal resolution.

5. Conclusion
The ionosphere must be continuously monitored so as to remove its effect on the signals of space-based communication, positioning, and navigation systems. The space and time variability of the ionosphere can be investigated by observing the variability of TEC, which can be estimated using ground-based or space-based measurement systems. Unfortunately, measurement systems can be sparse in space and recorded data may have temporal gaps. Therefore, in order to obtain TEC values for a desired point, TEC can be interpolated temporally and/or spatially. On the other hand, TEC maps must be prepared to obtain regular and dense TEC values in a given region.

In this study, a user-friendly, robust, reliable, and automatic TEC mapping algorithm is developed with two different kriging algorithms for a GPS network augmented with GIM-TEC. Because it uses the inherent statistical knowledge of TEC, kriging has a great advantage over other mapping techniques. The developed automatic mapping technique with UK1 algorithm is applied to a midlatitude regional GPS network. It is important to use the kriging algorithm over nearly uniformly distributed sample points. However, the kriging method shows interpolation error in addition to extrapolation error. First, in order to reduce kriging error,
**Figure 6.** Isotropic UK1 variance maps on 25 October 2011, a day with severe geomagnetic disturbance, at 09:00 UT: a) variance maps using reduced TNPGN data, b) variance maps using full TNPGN data, c) variance maps using augmented data.

**Figure 7.** Normalized root mean square error (NRMSE) with respect to the number of training stations on 8 September 2011, one day before the geomagnetic disturbance: a) 00:00 UT, b) 10:00 UT.

missing GPS-TECs are estimated from the GIM and the borders of the region are framed with it. Hence, two different networks are gathered, and the missing data are estimated by using a reliable and robust spatiotemporal...
interpolation technique. Therefore, the novel mapping algorithm, which differs from other mapping techniques by using the estimation algorithm, operates with minimum error in all circumstances. In order to calculate the experimental semivariogram, the first-order interpolation method is used for grid points, which are arranged automatically. Then the Matern function is fitted on the experimental isotropic semivariogram using a nonlinear optimization routine, PSO. Additionally, grid size is arranged to optimize the number of sample points, and the algorithm automatically switches between universal and ordinary kriging according to the region. Hence, high space-time resolution automatic kriging maps are obtained and the outputs can be saved in the desired format in the user-defined directory.

Acknowledgment

This study was supported by the grants of TÜBİTAK 115E915 and TÜBİTAK 114E541 and the joint TÜBİTAK 112E568 and RFBR 13-02-91370-CT-a projects.

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