A modeling and simulation study about CO$_2$ amount with web-based indoor air quality monitoring

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Abstract: Human breath causes indoor air quality (IAQ) to get worse in overcrowded places such as schools, hospitals, and offices where people spend most of their time, and it is rarely felt. For a healthy and comfortable living environment, pollutant gases must also be monitored in addition to temperature and humidity control. In this respect, carbon dioxide (CO$_2$) is considered as one of the main indoor air pollutants to determine ventilation requirements. In this study, a mathematical model and simulator software have been developed to predict CO$_2$ concentrations under different indoor conditions and to visualize the predicted results that come from the model. During the study some experiments are conducted to validate the model at a faculty building of Sakarya University. In these experiments, first CO$_2$ concentrations are predicted using the model and then they are measured to validate the model for target classrooms under different indoor conditions depending on the number of students, their physical characteristics, and activities. The study shows that increasing the number of students has considerable impacts on the amount of CO$_2$ produced and it can be used as the minimum value of the outdoor ventilation rate. The modeling and simulator software can be used for analyzing alternative building designs such as different ventilation types, occupant profiles, and numbers and dimensions of windows, doors and rooms in terms of IAQ.

Key words: Indoor air quality, multicompartment IAQ model, simulator, software monitoring, CO$_2$ estimation

1. Introduction
Carbon dioxide (CO$_2$) is a colorless and odorless gas that is emitted through human activities. CO$_2$ is an important indoor air pollutant, and its concentration is usually considered as an indicator for indoor air quality (IAQ) and adequate ventilation. High concentrations of CO$_2$ can cause tiredness, headache, difficulty breathing, wooziness, unconsciousness, an increase in heart rate, and even death [1]. Aside from toxic gases and particles, there are some IAQ measurement and analysis studies that focused on CO$_2$ concentrations [2]. Climate control by just measuring the room temperature is not sufficient because people who stay there for a long time do not realize that the air quality gets worse. Likewise, unnecessary ventilation may increase living expenses. The problem of bad air quality and more energy expenses is even more severe in public places like schools, hospitals, and government buildings.

Modeling and simulation studies may help in analyzing IAQ status in terms of energy efficiency and comfort. These studies may be more helpful once they are used for examining air conditioning systems before
they are installed or before the buildings are built. Thermal discomfort and bad IAQ cause performance and productivity decline for both staff and students in schools. Several studies have analyzed IAQ in school buildings and presented that they have insufficient ventilation and unacceptable thermal comfort and IAQ. Chaloulakou and Mavroidis [3] employed the IAQ model developed by Hayes [4,5] to investigate the indoor/outdoor carbon monoxide (CO) ratios of a school in Greece. Model predictions were satisfactory when outdoor concentration changes were not sharp. The highest CO concentrations were measured during the early hours of weekdays. Santamouris et al. [6] monitored airflows and distribution of indoor CO$_2$ concentrations in naturally ventilated schools before, during, and after courses. They found that the indoor CO$_2$ concentration was higher than the limit threshold value for about half of the classrooms. The threshold for CO$_2$ concentration is 1000 ppm or 1800 mg/m$^3$ as set by the US Environmental Protection Agency [7] and the American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE) [6,8]. In another study, Clements-Croome et al. [9] measured IAQ effects on student performance at 20 schools in the United Kingdom. During the study, minimum suitable ventilation rate, thermal comfort, and acceptable air quality were investigated to help ensure healthy working conditions for education. Bako-Biro et al. [10] analyzed CO$_2$ concentration and other parameters during 3 weeks at 8 schools in England. During the experiments, it was observed that the CO$_2$ concentration was over 5000 ppm in classrooms, which is much higher than the average recommended level of 1500 ppm and the preferred level of 1000 ppm.

Occupants and their behaviors such as working, writing, and typing in an office or eating, cooking, and sleeping at a home environment may also be important for IAQ and energy efficiency. For example, these two metrics are observed in an IAQ simulation of buildings to improve energy efficiency and to provide better IAQ control. Jian et al. [11] made IAQ measurements according to window and door open/closed and presented CO$_2$ concentration as the best predictor of occupant behavior.

The main goal of this study is to assess IAQ in buildings while preserving the energy efficiency. To achieve the goal, a software modeling/simulation tool has been developed to analyze and improve IAQ in the buildings. Using the tool, a building structure and alternative scenarios can be tested easily, and an optimal solution can be sought using just a computer. In the study, a mathematical model has been developed to estimate CO$_2$ pollutant distribution for the Department of Computer Engineering of Sakarya University. The classrooms are used as a test bed to validate the model and the simulation tool. The CO$_2$ concentrations and the air flow rates are first computed and then measured for each closed area, and the findings are also compared with ASHRAE standard values.

2. Theoretical background

2.1. Multicompartment IAQ model

IAQ is used to denote whether or not indoor air is clean. The modeling of an indoor environment and pollutant distribution in the environment is one of the most important steps to calculate the indoor pollutant concentration and to determine the IAQ of the environment [12]. Multicompartment IAQ models are based on the mass balance equation and used to predict pollutant concentrations depending on the source (infiltration, source production, etc.) and sink parameters (exfiltration, deposition, removal by ventilation, etc.) of indoor environments. If these parameters are detailed enough, then the mass balance equation, which is given in Eq. (1) [13–15], can be used to see the change rate of pollutant concentration:
\[ V_i \frac{dC_i}{dt} = \sum_{k=0}^{S} G_{i,k} + pC_i Q_{0,i} - C_i Q_{i,0} + \sum_{j=1}^{N} C_j Q_{j,i} - \sum_{j=0}^{N} C_i Q_{i,j}, \]  

where: 
\( V_i \) is the volume of the \( i \)th compartment (m\(^3\)) (classroom or corridor for this study), 
\( C_i \) is the pollutant concentration in the \( i \)th compartment (g/m\(^3\)), 
\( t \) is time (min), 
\( p \) is the penetration factor of outdoor air to the \( i \)th compartment (dimensionless), 
\( G_{i,k} \) is the pollutant generation rate of sources in the \( i \)th compartment (g/min), \( k = 0, 1, 2, \ldots, S \), and \( S \) is the total number of sources (or occupancies for this study), 
\( C_0 \) is the outdoor pollutant concentration (g/m\(^3\)), 
\( Q_{0,i} \) is the infiltration rate (m\(^3\)/min) (air flow rate from outdoor to the \( i \)th compartment), 
\( Q_{i,0} \) is the exfiltration rate (m\(^3\)/min) (air flow rate from the \( i \)th compartment to outdoor), 
\( C_j \) is the pollutant concentration in the \( j \)th compartment (mg/m\(^3\)), 
\( Q_{j,i} \) is the air flow rate from the \( j \)th compartment to the \( i \)th compartment (\( i \neq j \)) (m\(^3\)/min), and 
\( Q_{i,j} \) is the air flow rate from the \( i \)th compartment to the \( j \)th compartment (m\(^3\)/min), 
where \( j = 0 \) for outdoors, \( j = 1, 2, \ldots, N \), and \( N \) is the total number of compartments [13,15].

In this study, the mass-balance model has been improved by adding gases in closed environments. The earlier model was developed considering the particles for IAQ monitoring by Nazaroff and Cass [14,15] and was also used for mechanically ventilated environments [16,17]. An analytical solution of the model given in [13,18] is detailed in the Appendix of this paper. During the research, it has been observed that CO\(_2\) is the most important factor for IAQ, especially in crowded places like schools or hospitals.

2.2. CO\(_2\) generation rate

Human metabolic production of CO\(_2\) is usually the most important source in densely occupied spaces. Other indoor CO\(_2\) sources are plant respiration, direct fire, and outdoor CO\(_2\) concentration. CO\(_2\) gas produced by occupants is counted as an adequate indicator of IAQ [19–23] and it is used to demonstrate the acceptability of an indoor environment in terms of human body odor and population density [2]. Most IAQ studies in schools show that the CO\(_2\) concentration reaches very high values, which requires improvements in the ventilation control systems to reduce the amount of pollutants in indoor environments [6,24,25].

Indoor CO\(_2\) concentration relies on the characteristics of the occupants (the number of people and their body sizes, ages, and behaviors). As a function of physical activity level \( M \) (MET) and DuBois surface area \( A_D \) (m\(^2\)), oxygen (O\(_2\)) consumption rate \( V_{O_2} \) (L/s) and CO\(_2\) generation rate \( V_{CO_2} \) (L/s) of a human are calculated by Eqs. (2) and (3), respectively:

\[ V_{O_2}(A_D M) = \frac{0.00276 \times A_D M}{(0.23 RQ + 0.77)}, \]  
\[ V_{CO_2}(V_{O_2}) = RQ \times V_{O_2}. \]  

Respiratory quotient \( RQ \) is the ratio of generated CO\(_2\) and consumed O\(_2\) and its default value is equal to 0.83 for an average adult size (dimensionless) [2]. \( A_D \) is based on body height \( H \) (m) and body weight \( W \) (kg) as
taken from Eq. (4) [26].

\[ A_D(H, W) = 0.20247H^{0.725}W^{-0.425} \]  (4)

Metabolic equivalent (MET) levels for typical physical activities are given in Table 1 [2,27]. The CO\(_2\) generation rate increases depending on MET level [2,28].

<table>
<thead>
<tr>
<th>Activity</th>
<th>Metabolic equivalent (MET)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seated, quiet</td>
<td>1.0</td>
</tr>
<tr>
<td>Seated, reading</td>
<td>1.0</td>
</tr>
<tr>
<td>Writing</td>
<td>1.0</td>
</tr>
<tr>
<td>Typing</td>
<td>1.1</td>
</tr>
<tr>
<td>Standing, relaxed</td>
<td>1.2</td>
</tr>
<tr>
<td>Standing, filing</td>
<td>1.2</td>
</tr>
<tr>
<td>Walking (0.9 m/s)</td>
<td>2.0</td>
</tr>
<tr>
<td>Exercise</td>
<td>3.0–4.0</td>
</tr>
</tbody>
</table>

Both CO\(_2\) generation and ventilation requirements change linearly with the real number of occupancies, \(S\) (dimensionless) [20]. The amount of CO\(_2\) produced \(G\) (L/s) and minimum outdoor air ventilation requirement \(V_b\) in the breathing zone can be determined using Eqs. (5) and (6), respectively:

\[ G = \sum_{k=0}^{S} G_{i,k} = V_{CO_2} \times S \]  (5)

\[ V_b = R_p \times S + R_a \times A \]  (6)

where \(R_p\) is the outdoor rate per person (L/s person), \(R_a\) is the outdoor rate per area (L/s m\(^2\)), and \(A\) is the floor area of the zone (m\(^2\)) [29]. These parameters are determined by ASHRAE Standard 62.1 for various indoor environments [30].

2.3. Simulation

Analytical modeling, simulation, and measurement are methods for system evaluation. In recent years, modeling of indoor pollutant sources has shifted from experimental and simple models to more complex mass transfer models. In these situations, analytical modeling becomes difficult and inefficient to obtain high-resolution results. A simulator hides the complexity of the model from the user, so he/she can focus on actual analysis. In many IAQ studies, simulation has been used widely because it generates results close to real values and allows the demonstration of various scenarios in a short time. For example, numerical simulation methods have been preferred for the detection of pungent gases in the environment [31–35]. Tsujita et al. [36] created a physical simulation environment by placing sensors at different points and measured the amount of gas in their study. In order to obtain realistic test results, models can be set for different time intervals through the simulation.

Simulation programs allow users to assess the performance of large multizone buildings when buildings are designed and IAQ is concerned. For example, Chen et al. [37] presented a cosimulation method using EnergyPlus and Champs-Multizone for whole building simulation. Integrated simulation environments can
analyze both energy efficiency and IAQ. Integrated environments were found to be more useful for analyzing the interaction between IAQ and energy efficiency measures.

Simulation is also important for developing and evaluating new ideas. For example, new scenarios can be tested using a simulation tool and resulting feedbacks can be used for control. The biggest difference of this study from existing simulators is that it has a dynamic and versatile structure. In certain environments, realistic scenarios can be used to make predictions about IAQ as well as instantaneous changes in scenarios to examine the effects on the results. In addition, building characteristics of different architectures can be introduced to the system and information about IAQ can be obtained with the possible scenarios even in the design phase of the building. User-friendly interfaces simplify system control with notifications, not just graphical representations. Even the necessary sensors and communication tools can be placed in a building and the simulator can be easily converted to an IAQ monitoring system.

In this study, first an analytical model has been developed for IAQ predictions. Then the model is configured to produce predictions for the target places and these predictions are analyzed by the simulator software. Finally, real measurements are conducted with electronic devices instrumented with CO\textsubscript{2} sensor systems to validate the model.

3. Experimental studies
3.1. Test-bed description

In this study, the first floor of the Faculty of Computer and Information Sciences at Sakarya University is selected as the test area/location, and the classrooms are modeled and simulated for IAQ analysis (Figure 1). The first floor contains 9 classrooms (C1101, . . . , C1109) and one large common corridor interconnecting each classroom with the air flow rates. The classrooms are naturally ventilated and have three windows (the dimensions of a window are 1.1 m \times 1.3 m) and one interior door (2.05 m \times 0.9 m). The corridor has a total of 7 m\textsuperscript{2} of large openable windows.

![Figure 1. Schematic representation of the classrooms on the first floor in the faculty building.](image-url)
The geometric properties of the classrooms and corridor are presented in Table 2. It is assumed that the classrooms and corridor provide comfort conditions in terms of temperature and humidity during modeling.

<table>
<thead>
<tr>
<th>Compartment</th>
<th>Volume (m$^3$)</th>
<th>Surface area (m$^2$)</th>
<th>Floor area (m$^2$)</th>
<th>Total open window areas (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1101, C1102</td>
<td>298</td>
<td>325</td>
<td>102</td>
<td>2.16</td>
</tr>
<tr>
<td>C1103–C1108</td>
<td>190</td>
<td>228</td>
<td>65</td>
<td>2.16</td>
</tr>
<tr>
<td>C1109</td>
<td>222</td>
<td>257</td>
<td>76</td>
<td>2.16</td>
</tr>
<tr>
<td>Corridor</td>
<td>640</td>
<td>475</td>
<td>218</td>
<td>7</td>
</tr>
</tbody>
</table>

The classroom/corridor characteristics, the measurement air flow rates, temperature and relative humidity, initial pollutant concentrations obtained from Eq. (1), and predicted CO$_2$ values and measured values are stored in database tables for later analysis, and they are organized in a related fashion to reduce the replications. The classrooms’ schedules and student populations are stored in another table. The relationships between the database tables are presented in Figure 2. The total number of students in the classrooms and corridor (varying time) according to the course schedule of the computer engineering department is used as scenario input for predicting the CO$_2$ distribution.

In the model, physical activity level for an student of average size (height: 1.70 m, weight: 66 kg) is set as 1.0 MET from Table 1. The average body volume of a student is used as 0.06 m$^3$ to compute the effective volume of a classroom. $V_{O2}$ and $V_{CO2}$ values per student are computed as 0.4347 g/min (0.0051 L/s) and
0.4635 g/min (0.0042 L/s) for 1.0 MET, respectively. People outdoor rate \( (R_p) \) and area outdoor rate \( (R_a) \) are used to determine minimum ventilation rates, and they are obtained from ASHRAE Standard 62.1, \( R_p = 3.8 \) L/s person \( (0.228 \text{ m}^3/\text{min person}) \) and \( R_a = 0.3 \text{ L/s m}^2 \) \( (0.018 \text{ m}^3/\text{min m}^2) \) for a lecture classroom [30].

The sample period of measurement is suggested as 8–15 min for continuous observation [38,39], so in this study the prediction time interval \( \tau \) is set as 10 min.

### 3.2. Simulator description

The database server and the simulator software both run at different locations at Sakarya University. The modeling software is developed by using MATLAB, and the simulator software is implemented by using ASP.NET. The simulator provides multiple web-based interfaces for entering model parameters and scenarios. These parameters are stored in the database, and the modeling software uses them to create the virtual environment. After the virtual environment is modeled, scenario files should be introduced to the system for simulating the IAQ. The analytic results produced by the modeling software are stored periodically in the database, and the simulator reads them to create graphical outputs.

The simulator displays several graphical reports about the IAQ of the building model based on various scenarios. The simulator is used with a one-semester schedule as the default scenario, and the start/finish time of the courses and total number of students registered in the courses can be defined in the schedule. The simulation can be started by selecting an arbitrary day from the semester. Users can open and close the windows/doors and change the number of students in each classroom by the provided interface. The IAQ impacts of each intervention can be monitored with the graphical interface of the simulator.

The simulator processes the data concurrently with threads coming from different objects. For example, a thread method is used for making observations about the status of each classroom defined in the general floor plan as shown in Figure 3. In the graphical interface, if the estimated CO\(_2\) concentration is less than 1000 ppm, then the background color of the classroom becomes green, at 1000–1400 ppm the color becomes yellow, and if it is more than 1400 ppm the color becomes red to reflect IAQ status. The IAQ status of a classroom can easily be inferred from the graphical output.

### 3.3. Measurements for validation

At the last phase of the study, the data obtained from the mathematical model and simulations are validated by the measurements conducted in target classrooms and the corridor. The device used for measurements is an AZ Instrument 77535 CO2/Temp./RH Meter, and it is capable of reading CO\(_2\) concentration, air temperature, and relative humidity with a measuring range of 0–9999 ppm CO\(_2\) and accuracy of 30 ppm + 5% of the reading (0–5000 ppm). It has a self-calibration property and calibrates itself before any measurement process begins. The device shows the measurement data on its small LCD screen or outputs data via an RS-232 port. For continuous measurements, a piece of software is developed to be run on a computer for logging the data from the RS-232 and converting data to Ethernet packets. During the experiments, one device is installed at each target place, and they are configured for continuous measurements. The validation data are collected from multiple devices as planned and then analyzed. CO\(_2\) prediction and measurement values have been taken between 0900 and 1200 hours on different days as shown in Figures 4a–4c. Doors and windows are mostly closed during the measurements. The air temperature and relative humidity measurements in the classrooms were 21.4–27.5 °C and 33.6%–54.9%, respectively.
Figure 3. General view of the simulator output for the classrooms.

Figure 4. (a) Measured and predicted CO$_2$ concentrations (ppm), (b) STUDENT count, (c) volumetric air flow rate in classroom C1103.

4. Results

The system developed in this study consists of two parts: analytic modeling software and online simulator software. These two software programs communicate with each other through the database server.

At the beginning, a user can specify the building’s properties and scenarios that include daily activities in that building by means of the simulator interfaces. The analytic model software then accesses the database server and computes the estimated IAQ values according to the scenarios. Users can get detailed information about IAQ without dealing with complex environment modeling calculations through the graphical and textual representations offered by simulator web interfaces. This system enables users to determine the minimum and maximum conditions for ensuring the best IAQ while a building under examination is in the design stage.
With the simulator, a user can have detailed information by clicking links for the rooms of the graphical output. Some important information about a classroom is presented at the graphical interface, such as open/closed windows, door status, instant temperature, humidity, current student number, predicted CO$_2$ and O$_2$ ppm values, minimum outdoor air ventilation requirement, and calendar information.

To improve the prediction capability, besides their quantity the characteristics of the occupants are also added to the model, such as their physical activity, body weight and height, and time spent in a classroom. For example, CO$_2$ estimations calculated by the model and the number of students in the classroom/corridor that comes from the real course schedule of the department can be seen as a function of time. The statuses of the door/windows (open/closed) are also considered in the model.

Figures 5 and 6 show that CO$_2$ concentration amounts increase linearly depending on the total number of students in the environment and mostly exceed the threshold limit. According to weekly prediction and measurement results in nearly half of the environments the CO$_2$ amount is well over the recommended limit.

**Figure 5.** (a) Estimated CO$_2$ amounts (ppm) in classrooms C1103 and C1104, (b) student counts based on the course schedule.

In Figure 5a the CO$_2$ concentration value in room 1104 appears to be almost 2 times larger than in room 1103. This is because on Monday both rooms start from the same point; however, a larger population stays for a longer time in room 1104, causing more CO$_2$ production. This offset remains there until the end of the week due to insufficient ventilation. The pattern for CO$_2$ production shows similarities depending on the number of students; however, prior accumulations would not drop down quickly unless a long period of ventilation takes place. This shows us the importance of the infiltration factors of Eq. (1). It can be inferred from Figure 5b that the stay time and the number of students are both important for CO$_2$ production.

Figure 7 demonstrates indoor data produced by the model graphically for classroom C1103 in a shorter period of time (0900–2100 hours on Monday). We have observed that not only the student counts but also opened/closed doors/windows have an impact on the CO$_2$ amounts. Reduction in CO$_2$ concentration shows inertia due to the large volume of the classroom. A sudden drop was not expected in this situation; instead, the slope declines gradually as in Figure 7a. The CO$_2$ amount reaches over the threshold in 2 h even though the door and the windows are open between 0900 and 1100 hours in Figure 7. Similarly, Figure 7 plots how the minimum outdoor ventilation rate changes, computed by Eq. (6) with respect to the number of students shown.
Figure 6. (a) Estimated CO\textsubscript{2} amounts (ppm) in classroom C1109 and the corridor, (b) student count in the corridor based on general course schedule of the department.

in Figure 7.

Figure 7. Hourly estimation study for classroom C1103 based on course schedule on a Monday: (a) CO\textsubscript{2} distribution, (b) student count, (c) door opened/closed, (d) number of opened windows, (e) ventilation rate calculated by Eq. (6).

Many measurement studies have been conducted to validate the model; however, only the results for class C1103 are presented here due to space limits (see Figure 4).
5. Discussions

Observed conditions (such as number of students, doors and windows opened/closed) are entered into the model and the results are compared with the real measurements. As can be seen from Figure 4a, CO$_2$ readings and predictions follow each other and deviations seem to be reasonable amounts. The maximum difference occurred at a reading value of 5000 ppm and the drift was about 750 ppm; the average and the standard deviation of differences are calculated as 80.43 and 457.78, respectively. The differences are due to factors such as the meter position, measurement errors, and average student profile used in calculations. These results validate the mathematical model, and the model can be used to predict conditions where measurements are not possible. The mathematical model was also tested and validated in our previous study [40]. Figure 4c shows the volumetric air flow rate (m$^3$) for room 1103. It is calculated after observing the door and window opened/closed states.

Another very important outcome of the study is that CO$_2$ amounts far exceed the recommended indoor levels that should be reconsidered by the university administrators to create a better environment for learning. Alternative solutions are reported to the university administrators, such as assigning smaller number of students to the classrooms or installing ventilation systems in the buildings. IAQ directly affects students’ health, academic performance, and productivity. High indoor CO$_2$ levels (over the standard level, 1000 ppm) cause complaints such as headache, drowsiness, loss of attention, and nausea [41]. Thus, predicting indoor CO$_2$ concentration, especially for schools, and determining minimum ventilation requirements are needed to prevent and/or reduce high amounts of CO$_2$.

The simulator developed in this study informs users about the IAQ status of a classroom on a specified date in addition to the current situation by warning messages, and it provides the prediction of CO$_2$ amounts according to different student profiles, various schedules, and classrooms with different sizes and features.

6. Conclusions

In this paper, a modeling and simulation software tool developed for predicting CO$_2$ concentrations has been presented. The model is based on mass-balance equations, and it takes into account ventilation flows and metabolic CO$_2$ generation by occupants and predicts the time-varying CO$_2$ concentrations in a multizone environment. A portion of a university building is used as a test bed to demonstrate the capabilities of the model. Many real measurements with instrumentation have been conducted on different days during the semester in various classes to validate the model. The experiments show that the predicted CO$_2$ levels are very close to the real measurements, and this verified model can be used as a tool to investigate alternatives in the design phase of the buildings to improve the IAQ status of the rooms. A web-based approach is preferred for the simulator tool, which allows analyzing the modeling data from any place. The simulator presents colorful graphical outputs for the IAQ status of rooms of interest in a building.

During the experiments, excessive CO$_2$ amounts were measured in the classrooms of the computer engineering department, and the situation was reported to the school administration. In a future study, the web-based simulator will be improved to show online data collected from real sensors installed in the classrooms.

References


Appendix

\[ V_{i}^{\frac{dC_{i}}{dt}} = \sum_{k=0}^{S} G_{i,k} + pc_{i}Q_{i,0} - c_{i}Q_{i,0} + \sum_{j=1}^{N} c_{j}Q_{j,1} - \sum_{j=0}^{N} c_{j}Q_{j,1} = S_{i} - L_{i}C_{i} \]

Source parameter: \[ S_{i} = \frac{1}{v_{i}} \left( \sum_{k=0}^{S} G_{i,k} + pc_{i}Q_{i,0} + \sum_{j=1}^{N} c_{j}Q_{j,1} \right) \]

Sink parameter: \[ L_{i} = \frac{1}{v_{i}} \left( Q_{i,0,0} + \sum_{j=1}^{N} c_{j}Q_{j,1} \right) \]

\[ \frac{dC_{i}}{dt} + \frac{L_{i}C_{i}}{v_{i}} = \frac{S_{i}}{v_{i}} \]

\[ \frac{dC_{i}}{dt} + \frac{L_{i}C_{i}}{v_{i}} = \frac{S_{i}}{v_{i}} \]

\[ C_{i} = \begin{cases} v, & C_{i}' = v' \\ \frac{C_{i}P(t) + u(t)}{C_{i}u'} = \frac{Q(t)u(t)}{(uC_{i})'} \\ C_{i} = v, C_{i}' = v' \end{cases} \]

\[ C_{i}u' = C_{i}P(t)u(t) - \frac{u'}{u} = P(t) \]

\[ \ln(u') = P(t) - e^{P(t)dt} \]

\[ u = e^{\frac{L_{i}t}{v_{i}}} = e^{\frac{L_{i}}{v_{i}}h_{i}dt} \]

\[ (uv)' = \frac{S_{i}}{v_{i}}e^{\frac{L_{i}}{v_{i}}t} \]

\[ (e^{r_{i}}C_{i})' = \frac{S_{i}}{v_{i}}e^{\frac{L_{i}}{v_{i}}t} \]

\[ f(e^{r_{i}}C_{i})' = \int \frac{S_{i}}{v_{i}}e^{\frac{L_{i}}{v_{i}}t} \rightarrow e^{r_{i}}C_{i} = \frac{S_{i}}{v_{i}}e^{\frac{L_{i}}{v_{i}}t} + K \]

\[ C_{i} = \left( \frac{S_{i}}{L_{i}}e^{\frac{L_{i}}{v_{i}}t} + K \right) \frac{1}{e^{\frac{L_{i}}{v_{i}}t}} = \frac{S_{i}}{L_{i}} + Ke^{\frac{L_{i}}{v_{i}}t} \]

\[ t = 0, C_{i} = \frac{S_{i}}{L_{i}} + K = C_{i0}, \text{ initial concentration} \]

\[ K = C_{0} - \frac{S_{i}}{L_{i}} \]

\[ C_{i} = \frac{S_{i}}{L_{i}} + (C_{0} - \frac{S_{i}}{L_{i}})e^{-\frac{L_{i}}{v_{i}}t} = \frac{S_{i}}{L_{i}} \left( 1 - e^{-\frac{L_{i}}{v_{i}}t} \right) + C_{0}e^{-\frac{L_{i}}{v_{i}}t} \]

\[ t \rightarrow \infty, C_{i} = \frac{S_{i}}{L_{i}} \left( 1 - e^{-\frac{L_{i}}{v_{i}}t} \right) + C_{0}e^{-\frac{L_{i}}{v_{i}}t} = \frac{S_{i}}{L_{i}} = C_{i}, \text{ steady state concentration} \]

\[ C_{i} = \frac{S_{i}}{L_{i}} \left( 1 - e^{-\frac{L_{i}}{v_{i}}t} \right) + C_{0}e^{-\frac{L_{i}}{v_{i}}t} = C_{f} \left( 1 - e^{-\frac{L_{i}}{v_{i}}t} \right) + C_{0}e^{-\frac{L_{i}}{v_{i}}t} = C_{f} + (C_{0} - C_{f})e^{-\frac{L_{i}}{v_{i}}t} \]