Design optimization of distribution transformers with nature-inspired metaheuristics: a comparative analysis

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Abstract: Many economies in the world have adopted energy-efficiency requirements or incentive programs mandating or promoting the use of energy-efficient transformers. On the other hand, increases in transformer efficiency are subject to increases in transformer weight and size, sometimes as much as 50% or more. The transformer manufacturing industry is therefore faced with the challenge to develop truly optimum designs. Transformer design optimization (TDO) is a mixed-integer nonlinear programming problem having a complex and discontinuous objective function and constraints, with the objective of detailed calculation of the characteristics of a transformer based on national and/or international standards and transformer user requirements, using available materials and manufacturing processes, to minimize manufacturing cost or total owning cost while maximizing operating performance. This paper gives a detailed comparative analysis of the application of five modern nature-inspired metaheuristic optimization algorithms for the solution of the TDO problem, demonstrated on three test cases, and proposes two algorithms, for which it has been verified that they possess guaranteed global convergence properties in spite of their inherent stochastic nature. A pragmatic benchmarking scheme is used for comparison of the algorithms. It is expected that the use of these two algorithms would have a significant contribution to the reduction of the design and manufacturing costs of transformers.

Key words: Distribution transformer, transformer design optimization, high efficiency, metaheuristics, swarm intelligence, differential evolution

1. Introduction

Electrical energy undergoes on average four voltage transformations between being generated and being consumed, and as a result a large number of transformers of different classes and sizes with a wide range of operating voltages are employed in the transmission and distribution network. Traditionally, transformers at the end of this chain providing power to end users at domestic consumer voltage levels (usually 400 V or less) are called distribution transformers [1,2].

The estimated global stock of distribution transformers in 2014 was 118 million units. Total installed power capacity was estimated to be 13,848 GVA, with an average unit rating of 117 kVA. Installed capacity is forecast to reach 22,400 GVA in 2030 [2]. The global market for distribution transformers is forecast to exceed USD 20 billion by 2018 [3].

Distribution transformers are very efficient compared with other electrical equipment, having losses in the order of 2%–3% or better. However, energy losses in distribution transformers are highly significant due to the huge number installed, as given above.

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Due to the substantial savings potential, as of 2014 some 15 economies (including the European Union), representing approximately 54% of the installed stock of distribution transformers by capacity, have adopted energy-efficiency requirements or incentive programs promoting energy-efficient designs.

It is estimated by the European Commission that roughly 2.5% of energy in the European Union is consumed as a result of transformer losses. The European Commission defined Ecodesign standards for transformers and introduced them in the new European Standard EN 50588-1, which was put in force on 01.07.2015. The standard will be implemented in two steps; in the first step, already effective from July 2015, the maximum allowable no-load losses will be reduced by 30% as compared to the $C_0C_k$ (AC') loss combination alternative of the European Standard EN 50464-1 that it replaced. In the second step, effective from July 2021, maximum allowable no-load losses will be reduced by another 10%, and the load losses by $\sim 30\%$. Furthermore, the 10%–15% tolerances on losses allowed in the previous standard have been zeroed in the new one.

It is important to note that increases in transformer efficiency are normally also subject to increases in transformer weight and size. This occurs due to increases in the quantity of material used in the design, to reduce either the no-load or load losses, or both.

Although not prepared for the loss values stated in the European Standards, the illustration of size and weight differences between standard and efficient 100 kVA and 400 kVA distribution transformers given in [2] reveals that the weight of a 100 kVA transformer would increase by 37% and that of the 400 kVA would increase by 48%.

2. Transformer design optimization

After the recent introduction of new regulations mandating the use of high efficiency distribution transformers, the transformer manufacturing industry is faced with the challenge to develop indisputably the best (optimum) designs since, in today’s highly competitive market environment, it will be too difficult to expect customers to fully compensate for the inevitable increase in material costs.

Transformer design optimization (TDO) is a mixed-integer nonlinear programming problem having a complex and discontinuous objective function and constraints, with the objective of detailed calculation of the characteristics of a transformer based on national and/or international standards and transformer user requirements, using available materials and manufacturing processes, to minimize the manufacturing cost or total owning cost (TOC) while maximizing operating performance [1,4].

There are several different types of objective functions defined in the literature for TDO, but the most commonly used ones are minimization of transformer manufacturing cost and minimization of TOC, which can be defined as the life cycle costs associated with purchasing and operating a transformer. A detailed description of the objective function types can be found in Chapter 2 of [1].

Minimization of main material cost, manufacturing cost, and TOC are the three objective function options available in the software prepared for this study.

Even though governing bodies impose regulations mandating the use of high-efficiency transformers, transformer customers are advised to continue using TOC since the regulations are only for setting a minimum level for the transformer efficiency [5].

Transformers and transformer design optimization are two areas that have been extensively studied in the literature. For instance, a literature survey conducted in 2009 [6] revealed general backgrounds of research and developments in the field of transformer design and optimization for the past 35 years, based on more than 420 published articles, 50 transformer books, and 65 standards.
Other surveys in the areas of transformers and transformer design optimization can be found in review papers [7–11].

It is clearly stated in the literature that global transformer design optimization is still an appealing research area since several approaches for its accomplishment have yet to be explored [4].

3. Methodology

3.1. Purpose and scope of the study

The purpose of this study is to compare the application of several modern nature-inspired metaheuristic optimization algorithms to global transformer design optimization and suggest suitable algorithms dedicated to the TDO problem as well as to meet the challenging requirements of the transformer industry. Only those algorithms that have not been previously used for the TDO problem are considered in this study.

Twenty algorithms were investigated within the scope of this study, and results of the best five of them are presented in this paper.

The scope of the study is the design optimization of distribution transformers with the following technical characteristics, where the objective is to minimize the main material cost:

- Three-phase, oil-immersed distribution transformers
- Wound core construction
- Copper foil for low voltage (LV) and enameled copper round wire for high voltage (HV) conductors

3.2. Objective function, design variables and constraints

The objective function used in this study is to minimize the transformer main material cost, as defined with the following formula:

\[
\text{min } Z(\vec{x}) = \min \sum_{j=1}^{8} c_j f_j(\vec{x}),
\]

where \(c_j\) and \(f_j\) are the unit cost (USD/kg) and the weight (kg) of each component \(j\) of the eight main materials, and \(\vec{x}\) is the vector of the design variables [4]. The eight main materials and their unit costs are LV winding material (12.01 USD/kg), HV winding material (12.01 USD/kg), core material (6.01 USD/kg), insulating paper (7.72 USD/kg), duct strips (8.58 USD/kg), insulating liquid (1.72 USD/kg), tank sheet steel (1.03 USD/kg), and corrugated panel material (1.20 USD/kg), as given in Chapter 2 of [1].

Design variables are similar to those given in [12]; however, based on our experience in the transformer industry, two more were added to have the result in a single run instead of having several runs for different combinations of these two parameters. The eight design variables used in this study, together with their type and unit of measure, are given below:

1. Number of turns of the LV winding (integer)
2. Magnetic induction (real, Gauss)
3. Width of the core leg (real, mm) – dimension D in Figure 1
4. Height of the core window (real, mm) – dimension G in Figure 1
5. Current density of the LV conductor (real, A/mm$^2$)
6. Current density of the HV conductor (real, A/mm$^2$)
7. Number of end cooling ducts in the LV winding (integer)
8. Number of end cooling ducts in the HV winding (integer)

The TDO problem must satisfy the following constraints:

- The designed no-load losses must be smaller than guaranteed no-load losses plus tolerance
- The designed load losses must be smaller than guaranteed load losses plus tolerance
- The designed total losses must be smaller than guaranteed total losses plus tolerance
- The transformer impedance voltage must be between a minimum and a maximum impedance voltage plus tolerance
- The total heat produced by the total losses of the transformer must be smaller than the total heat that can be dissipated by the combined effects of conduction, convection, and radiation
- The transformer temperature rise must be smaller than maximum temperature rise
- $l_b j \leq x_j \leq u_b j$, $j = 1, 2, \ldots, 8$, where $l_b$ and $u_b$ are lower and upper boundaries on the design variables, respectively
- $x_j \geq 0$, $j = 1, 2, \ldots, 8$

Figure 1. Active part (core and windings) of a wound core type distribution transformer.

Weights of the 8 main materials in the objective function are calculated by using the conventional design method given in Chapter 2 of [1], which, in our opinion based on our experience in the transformer industry, is the most realistic and complete method available in the literature to the best of our knowledge. The method was coded in MATLAB, extended where necessary to cover other power/voltage ratings based on past experience. Mathematical functions of core loss and heat transfer curves were obtained by curve-fitting. A routine was added to calculate the minimum width of the corrugated panel with which the heat produced in the transformer can be dissipated.
3.3. Optimization algorithms

Metaheuristic algorithms included in this study are very briefly described below, where basically relevant references to the algorithms are given and key parameters used in this study are specified. According to surveys [6–11], these algorithms, all of which are population-based, have not been previously used for the TDO problem.

3.3.1. Artificial bee colony (ABC) algorithm

The ABC algorithm, proposed by Karaboğa [13], is a swarm intelligence-based algorithm that simulates the simplified food searching behavior of honey bees.

ABC has three key parameters: limit, modification rate MR, and scout production period SPP. The key parameter values used in this study are limit = 10 × np × D, SPP = np × D, and MR = 0.9, as used in [14], where np = number of population and D = dimension of the problem, which is 8 for this study.

The constrained optimization version of ABC is used in this study [13]. The Delphi code, available from [14], has been converted to MATLAB.

3.3.2. Backtracking search optimization algorithm (BSA)

The BSA, developed by Civicioglu [15], is a population-based evolutionary algorithm.

The BSA has two key parameters: mix rate mixrate and scale factor F. In this study, for the first key parameter mixrate = 1 is used as suggested in [15], and among the five alternatives available for F, the Levy-like pseudo-stable walk option that simulates inverse gamma distribution is chosen, which yielded slightly better results as compared to the other alternatives.

The MATLAB code used in this study for the BSA is given in [16].

3.3.3. Competitive-Adaptive Differential Evolution Algorithm (b6e6rl)

b6e6rl is a population-based differential evolution algorithm developed by Tvrđik [17].

There are two key parameters in b6e6rl: competition control parameters n0 and δ, which are taken as n0 = 2 and δ = 1 / (5 × 12), as suggested in [18]. The MATLAB code for b6e6rl used in this study is also given in [18].

3.3.4. Cuckoo search (CS) algorithm

CS, developed by Yang and Deb [19], is a swarm intelligence-based optimization algorithm based on brood parasitism of some cuckoo species.

There is basically a single key parameter for CS, discovery rate pa, which is taken as pa = 0.25 in this study as suggested in [19].

The MATLAB code for the constrained optimization version is used in this study [20].

3.3.5. Flower pollination algorithm (FPA)

The FPA was developed by Yang [19] and is inspired by the pollination process of flowering plants.

There is basically a single key parameter for FPA, switching probability p, which is taken as p = 0.8 in this study as suggested in [19].
The MATLAB code for the constrained optimization version is used in this study [21].

3.4. Parameter tuning of algorithms

Impacts of key parameters on the performance of optimization algorithms were analyzed for all of the five algorithms by conducting parameter tuning studies, and it was observed that these algorithms are in general insensitive to their key parameters for the TDO problem, with the exception of the BSA, for which the Levy-like pseudo-stable walk option of its key parameter $F$ yielded slightly better results as compared to the other alternatives, as pointed out in Section 3.3.2.

3.5. Penalty calculation method

In this study, the static penalty method is used for no-load loss and load loss-related constraints, and the death penalty is used for impedance voltage and temperature rise constraints for all algorithms.

3.6. Benchmarking methodology

Based on input from senior transformer design engineers, the following “pragmatic” method was devised to compare stochastic algorithms, as seen from the point of view of a transformer design engineer:

- Robustness
  - Accuracy - An algorithm is rated as “fair” if mean accuracy of multiple consecutive test runs (minimum 10) as compared to the global optimum is 99.5% (in other words, mean error $\varepsilon_{\text{mean}} = 0.5\%$)
  - Precision - An algorithm is rated as “fair” if minimum accuracy in any of the multiple consecutive test runs (minimum 10) as compared to the global optimum is 99% (in other words, maximum error $\varepsilon_{\text{max}} = 1\%$)

- Speed - An algorithm is rated as “fair” if the CPU time of a single test run is 15 s (CPU time, measured with the cputime function of MATLAB, is roughly equal to clock time on a mainstream notebook when MATLAB is the only program running)

Based on the above, the scheme in Table 1 was used to score the tests.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Score</th>
<th>$\varepsilon_{\text{mean}}$ %</th>
<th>$\varepsilon_{\text{max}}$ %</th>
<th>CPU s</th>
</tr>
</thead>
<tbody>
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<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>Excellent</td>
<td>90</td>
<td>0.1</td>
<td>0.2</td>
<td>3</td>
</tr>
<tr>
<td>Very good</td>
<td>80</td>
<td>0.2</td>
<td>0.4</td>
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<td>Good</td>
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<td>9</td>
</tr>
<tr>
<td>Fairly good</td>
<td>60</td>
<td>0.4</td>
<td>0.8</td>
<td>12</td>
</tr>
<tr>
<td>Fair</td>
<td>50</td>
<td>0.5</td>
<td>1.0</td>
<td>15</td>
</tr>
<tr>
<td>Poor</td>
<td>40</td>
<td>0.6</td>
<td>1.2</td>
<td>18</td>
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<td>1.4</td>
<td>21</td>
</tr>
<tr>
<td>Very poor</td>
<td>20</td>
<td>0.8</td>
<td>1.6</td>
<td>24</td>
</tr>
<tr>
<td>Extremely poor</td>
<td>10</td>
<td>0.9</td>
<td>1.8</td>
<td>27</td>
</tr>
<tr>
<td>Unacceptable</td>
<td>0</td>
<td>1.0</td>
<td>2.0</td>
<td>30</td>
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</table>
4. Computational results and discussion

4.1. Technical characteristics and constraints of the transformers studied

Design optimizations were performed for three transformers with the following technical characteristics, in addition to those given in Section 3.1:

- Three-phase, oil-immersed distribution transformers
- 160, 400, and 630 kVA power ratings
- Primary/secondary voltages 20/0.4 kV
- Wound core construction
- Copper foil for low voltage (LV) and enameled copper round wire for high voltage (HV) conductors
- Loss and short-circuit impedance values in accordance with designs in [1] given below:
  - 160 kVA $P_0 = 425$ W, $P_k = 2350$ W, $U_k = 4\%$
  - 400 kVA $P_0 = 750$ W, $P_k = 4600$ W, $U_k = 4\%$
  - 630 kVA $P_0 = 1100$ W, $P_k = 8900$ W, $U_k = 6\%$
  where $P_0$ = no-load losses, $P_k$ = load losses, and $U_k$ = short-circuit impedance
- Other characteristics in accordance with the standard IEC 60076-1

Active part (core and windings) of a wound core type distribution transformer is shown in Figure 1.

4.2. Performance tests

The software prepared for this study using MATLAB 2014a consists of a main program, one subroutine for each of the five optimization algorithms, and one subroutine for design calculation, which is shared by all of the optimization algorithm subroutines. The user interface of the TDO software is shown in Figure 2.

Performance tests have been conducted on a mainstream notebook with 2.60 GHz Intel Core i5-3320M CPU and 4 GB RAM:

- For the three types of transformers with specifications as given in Section 4.1
- By using the five algorithms (ABC, b6e6rl, BSA, CS, FPA)
- With the following generation and population values
  - Number of generations: 1000, 1500, 2000, 2500
  - Population size: 20, 40, 60, 80, 100
- Key parameter values used for each algorithm are as given in Section 3.3
- No stopping criterion was used
- Tests were repeated 20 times for each algorithm

Ranges of values were used for number of generations and population size parameters for all algorithms instead of fixed values since the best combinations of these two would be different for each algorithm.
4.3. Benchmark of algorithms

Based on the definitions given in Section 3.6, a robustness score where accuracy and precision factors are taken into account with equal weights, and a total score where weight of robustness is 80% and speed is 20%, are calculated for each test run. A list of the top 20 robustness scores is given in Table 2; lists of top robustness scores and top total scores for each algorithm are given in Tables 3 and 4, respectively, and the scores in these tables are overall for the three types of transformers. Figure 3 shows a comparison of the best robustness scores of algorithms for each transformer type and overall.

It should be noted that we first considered using the 4 test cases (160, 400, 630, and 1000 kVA 20/0.4 kV transformers) given in [4] for comparison with the results of our proposed method. However, some essential
Table 2. Benchmark of algorithms – top 20 robustness scores.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Generation</th>
<th>Population</th>
<th>$\varepsilon_{\text{mean}} %$</th>
<th>$\varepsilon_{\text{max}} %$</th>
<th>CPU time</th>
<th>Robust score</th>
<th>Speed score</th>
<th>Total score</th>
</tr>
</thead>
<tbody>
<tr>
<td>b6e6rl</td>
<td>1000</td>
<td>100</td>
<td>0.01</td>
<td>0.02</td>
<td>15.8</td>
<td>99.1</td>
<td>47.4</td>
<td>88.8</td>
</tr>
<tr>
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<td>100</td>
<td>0.01</td>
<td>0.04</td>
<td>31.3</td>
<td>98.6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
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<td>100</td>
<td>0.01</td>
<td>0.04</td>
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<td>98.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>b6e6rl</td>
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<td>80</td>
<td>0.01</td>
<td>0.05</td>
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<td>98.4</td>
<td>57.2</td>
<td>90.2</td>
</tr>
<tr>
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<td>0.01</td>
<td>0.05</td>
<td>18.9</td>
<td>98.4</td>
<td>37.0</td>
<td>86.1</td>
</tr>
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<td>0.01</td>
<td>0.13</td>
<td>23.9</td>
<td>96.2</td>
<td>20.4</td>
<td>81.1</td>
</tr>
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</tr>
<tr>
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<td>67.6</td>
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<td>0.27</td>
<td>6.1</td>
<td>90.8</td>
<td>79.5</td>
<td>88.6</td>
</tr>
</tbody>
</table>

Table 3. List of top robustness scores for each algorithm.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Generation</th>
<th>Population</th>
<th>$\varepsilon_{\text{mean}} %$</th>
<th>$\varepsilon_{\text{max}} %$</th>
<th>CPU time</th>
<th>Robust score</th>
<th>Speed score</th>
<th>Total score</th>
</tr>
</thead>
<tbody>
<tr>
<td>b6e6rl</td>
<td>1000</td>
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<td>0.01</td>
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<td>CS</td>
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<tr>
<td>FPA</td>
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<td>0.17</td>
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<td>63.6</td>
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<td>ABC</td>
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<td>0.55</td>
<td>8.5</td>
<td>75.2</td>
<td>71.7</td>
<td>74.5</td>
</tr>
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</table>

Table 4. List of top total scores for each algorithm.

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<th>Algorithm</th>
<th>Generation</th>
<th>Population</th>
<th>$\varepsilon_{\text{mean}} %$</th>
<th>$\varepsilon_{\text{max}} %$</th>
<th>CPU time</th>
<th>Robust score</th>
<th>Speed score</th>
<th>Total score</th>
</tr>
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<tr>
<td>b6e6rl</td>
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<td>80</td>
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<td>0.05</td>
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<td>0.13</td>
<td>9.7</td>
<td>94.4</td>
<td>67.6</td>
<td>89.1</td>
</tr>
<tr>
<td>BSA</td>
<td>2000</td>
<td>60</td>
<td>0.08</td>
<td>0.27</td>
<td>5.7</td>
<td>89.0</td>
<td>80.9</td>
<td>87.4</td>
</tr>
<tr>
<td>FPA</td>
<td>2500</td>
<td>20</td>
<td>0.16</td>
<td>0.54</td>
<td>3.5</td>
<td>78.5</td>
<td>88.2</td>
<td>80.5</td>
</tr>
<tr>
<td>ABC</td>
<td>2000</td>
<td>60</td>
<td>0.24</td>
<td>0.56</td>
<td>6.9</td>
<td>74.1</td>
<td>77.0</td>
<td>74.6</td>
</tr>
</tbody>
</table>

information needed to be able to make one-to-one comparisons, such as no-load loss and gradient curves, was not provided in [4]. We therefore made the comparison of algorithms among themselves instead of comparing our results with the results of previous studies. Nevertheless, a comparison of the proposed method with the 630 kVA design example given in Chapter 2 of [1] is presented in Section 4.5.
4.4. Evaluation of results
The performance tests clearly showed that the competitive-adaptive differential evolution algorithm (b6e6rl) outperforms all the others from the robustness point of view; it is, on the other hand, the slowest of all the five algorithms studied. The robustness of CS is close to that of b6e6rl, and it is slightly faster that b6e6rl even though CS needs to calculate the objective function twice for each iteration.

The BSA, FPA, and ABC algorithms are faster than b6e6rl and CS, and the speeds of these three algorithms are comparable. BSA is the most robust one among them, its score being close to the robustness of CS. However, the robustness score reduces rapidly between BSA and FPA, and furthermore between FPA and ABC as well.

4.5. Comparison of proposed method with previous studies
In this section, the 630 kVA design example in Chapter 2 of [1] is used as a reference design and optimized with the proposed method in this study.

Two alternatives were prepared for comparison; in the first one, the same magnetic induction value of 1.7 T was used as in the reference design and kept fixed. In the second alternative, magnetic induction was allowed to vary.

As some of the important variables of the reference design, such as diameter of HV winding conductor, height of core window, etc., have discrete-like or integer values, the discrete version of the TDO software prepared for this study was used to have a comparison on the same basis. The results obtained are given in Table 5, where the first alternative is 8.4% and the second alternative is 10.2% more economical compared to the reference design.

**Table 5.** Comparison of 630 kVA optimization results with reference design in [1].

<table>
<thead>
<tr>
<th>Design variables/ constraints /weights</th>
<th>Unit</th>
<th>Lower/upper bounds</th>
<th>Example in [1]</th>
<th>Alt-I</th>
<th>Alt-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>LV number of turns</td>
<td>-</td>
<td>10–50</td>
<td>15</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Magnetic induction</td>
<td>Tesla</td>
<td>1.40–1.75</td>
<td>1.70</td>
<td>1.70</td>
<td>1.75</td>
</tr>
<tr>
<td>Width of core leg</td>
<td>mm</td>
<td>100–500</td>
<td>220</td>
<td>261</td>
<td>257</td>
</tr>
<tr>
<td>Core window height</td>
<td>mm</td>
<td>100–500</td>
<td>261</td>
<td>281</td>
<td>281</td>
</tr>
<tr>
<td>LV current density</td>
<td>A/mm²</td>
<td>1.00–5.00</td>
<td>4.76</td>
<td>4.96</td>
<td>4.96</td>
</tr>
<tr>
<td>HV current density</td>
<td>A/mm²</td>
<td>1.00–5.00</td>
<td>4.13</td>
<td>4.13</td>
<td>4.13</td>
</tr>
<tr>
<td>LV number of ducts</td>
<td>-</td>
<td>0–12</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>HV number of ducts</td>
<td>-</td>
<td>0–12</td>
<td>12</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Thickness of LV cond</td>
<td>mm</td>
<td>-</td>
<td>0.79</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Diameter of HV cond</td>
<td>mm</td>
<td>-</td>
<td>1.80</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>No-load losses</td>
<td>W</td>
<td>1100 + 15%</td>
<td>1202</td>
<td>1097</td>
<td>1210</td>
</tr>
<tr>
<td>Load losses</td>
<td>W</td>
<td>8900 + 15%</td>
<td>9587</td>
<td>9903</td>
<td>9789</td>
</tr>
<tr>
<td>Total losses</td>
<td>W</td>
<td>10,000 + 10%</td>
<td>10,789</td>
<td>11,000</td>
<td>11,000</td>
</tr>
<tr>
<td>Impedance</td>
<td>%</td>
<td>6 ± 10%</td>
<td>5.76</td>
<td>5.54</td>
<td>5.48</td>
</tr>
<tr>
<td>Main material cost</td>
<td>USD</td>
<td></td>
<td>8423</td>
<td>7712</td>
<td>7560</td>
</tr>
<tr>
<td>Weight of active part</td>
<td>kg</td>
<td>885</td>
<td>824</td>
<td>803</td>
<td></td>
</tr>
<tr>
<td>Oil weight</td>
<td>kg</td>
<td>563</td>
<td>511</td>
<td>506</td>
<td></td>
</tr>
<tr>
<td>Total weight</td>
<td>kg</td>
<td>1899</td>
<td>1840</td>
<td>1811</td>
<td></td>
</tr>
</tbody>
</table>
In both alternatives, calculated total losses are equal to the maximum value of the relevant constraint plus the tolerance; this can be considered as a justification of the proper functioning of the optimization algorithm used.

It should be noted that the reference design has dual primary voltages rated at 20 kV and 6.6 kV; however, the losses and weights given in Table 5 for this design have been calculated as if the transformer has a single primary voltage rated at 20 kV.

5. Conclusions
In this study, 20 modern nature-inspired metaheuristic optimization algorithms were investigated by conducting performance tests for suitability for the TDO problem, and the results of the best five algorithms are presented in this paper. These five algorithms are the ABC, BSA, b6e6rl, CS, and FPA, none of which have previously been used for the full TDO problem.

A pragmatic benchmarking scheme was developed for comparison of the algorithms. It is quite straightforward and can easily be adapted to specific needs.

The comparison of the results of performance tests conducted by using three different distribution transformer ratings proved that the competitive-adaptive differential evolution (b6e6rl) algorithm yields results with better than 99.9% accuracy and precision in a single run, in less than 30 s. Furthermore, it was mathematically proved in [19] and [22] that the CS algorithm, the performance of which is very close to that of b6e6rl, can satisfy the global convergence requirements and thus has guaranteed global convergence properties. The results of the performance tests conducted in this study verify the validity of this property, and hence both b6e6rl and CS can be considered as a viable alternatives to deterministic methods for the solution of the TDO problem.

It should be noted that, when some or all of the design variables used for the TDO problem are continuous, as generally is the case in the available literature, including this study, the resulting optimum solution would be a theoretical one due to nonstandard dimensions; hence, the design engineer needs to convert the theoretical solution to a feasible one. This problem is addressed in another study by the same authors of this paper [23], where a discrete transformer design optimization method is proposed that yields solutions with commercially available or productionally feasible dimensions, thus eliminating the need for the additional efforts of the design engineer.

It should also be noted that both theoretical and practical optimization methods should be used together; a design engineer should first determine where the theoretical optimum solution lies, and then use practical optimization to approach the theoretical optimum as much as possible. The reason for this is that practical optimization can sometimes get trapped at a local optimum far away from the theoretical optimum, and without using theoretical optimization the design engineer may not be aware of such a situation. In short, theoretical and practical optimization methods should complement each other in a transformer design optimization process.

References


