Automatic reduction of periodic noise in images using adaptive Gaussian star filter

Seniha KETENCİ, Ali GANGAL
Department of Electrical and Electronics Engineering, Faculty of Engineering, Karadeniz Technical University, Trabzon, Turkey

Abstract: The reduction of noise in images is a crucial issue and an inevitable preprocessing step in image analysis. Many diverse noise sources, which disrupt source images, exist in nature and through manmade devices. Periodic noise is one such disruption that has a periodic pattern in the spatial domain, causing hills in the image spectrum. In practice, quasiperiodic noise is commonly encountered instead of periodic noise. It has a more complex frequency spectrum, such as a star shape, in place of a pure delta shape in the frequency amplitude spectrum. In this study, we consider designing a star shape Gaussian filter that is a more appropriate adaptive filter of (quasi-) periodic noise. We called this filter the adaptive Gaussian star filter (AGSF), regarding the extension of the standard Gaussian filter. The proposed method is fully automatic and consists of three steps. Firstly, the low-frequency region in the image spectrum is detected via region-growing in the frequency domain. Next, the noise coordinates are estimated, and each noise spread area is determined and labeled using region-growing in the spectrum. Finally, AGSF shape and parameters are adjusted adaptively according to estimated noise characteristics. The performance of the method is discussed in the context of different sizes and contrasts for noisy images. The results are compared with previous work in the literature and they show that the developed algorithm is quite robust in reducing both periodic and quasiperiodic noise.

Key words: Periodic noise, region growing, star filter, two-dimensional Fourier transform

1. Introduction

Noise is an undesired effect that causes disturbance in signals and images. In this context, periodic noise is an unwanted signal that interferes with the source image or signal at a random frequency, depending on its source.

Generally, this interference can be added to the image from nature, the electricity network, or electronics devices. It is known that periodic noise sources create strips in an order on an image in the spatial domain. In Figures 1A and B, original and noisy images are given as an example of periodic noise. Unfortunately, an efficient spatial filter for periodic noise reduction in an image has not been developed yet. However, recovering the image tends to become easier in the frequency domain because of evident noise peaks as shown in Figures 1C and D.

The concept of periodic noise was first described in 1963 [1]. Periodic noise has a periodic pattern in the spatial domain. In the Fourier amplitude spectrum, it is revealed as conjugate peaks having very clear spectral appearance at conjugate frequencies [2–5]. Gonzalez and Wintz stressed that filtering in the amplitude spectrum with a notch filter could be an effective removal method [5]. After that, noise reduction studies have
been focused on the survey of frequency domain, especially spectral peak detection [6,7]. A median-like filter for periodic and quasiperiodic noise was progressed by Aizenberg and Butakoff [7]. Though their method is invariant of position of the peaks in the amplitude spectrum, filter parameters are variable manually depending on the image. In their next study, spectral spikes were detected using a window predetermined size and were filtered semiautomatically with a Gaussian notch filter [8]. Ghada et al. modified the median filter in the Fourier space with nonlinear filtering [9]. RFPCM [10] clustering based on fuzzy logic was used to find location
of noise spikes by Dutta et al. [11]. They utilized a cluster filter mask to remove periodic noise regions after elimination of the central frequency region on a spectrum amplitude image. A soft morphological filter was designed in the spectrum on the basis of standard morphological operators like opening and closing by Jing et al. [12]. The Fourier amplitude spectrum of periodic and quasiperiodic noise generally has a star-shaped profile. With this knowledge, a filter with a star shape was designed by Ketenci et al. [13]. Sur and Grédici proposed an automated approach based on frequency domain statistics in their study [14] to remove quasiperiodic noise. According to their algorithm, noise spikes were detected using deviations in a model of the expected average spectrum.

Our previous study, the Gaussian star filter (GaSF) [13], and the proposed AGSF consist of basically a smoothing process, low-frequency region labeling, and noise spikes detection using a region-growing algorithm in the amplitude spectrum. Yet, linear smoothing by a $3 \times 3$ average filter is utilized in GaSF [13], while nonlinear smoothing is applied to spectrum image by a $3 \times 3$ Gaussian kernel low-pass filter in the proposed AGSF. The smoothing process in these algorithms affects low frequency region labeling, detection of noise spikes, estimation of filter parameters, and also algorithm performance. The $3 \times 3$ Gaussian kernel smoothing filter in the proposed AGSF provides more accurate low-frequency region detection using a region-growing algorithm, whereas the $R \times R$ square area is labeled as an untouched low-frequency region in the GaSF [13] method. $R$ is twice the largest Euclidean distance from the center pixel in an image spectrum to the furthest labeled pixel. GaSF’s performance [13] is restricted due to the fact that square area selection could be caused by missing noise spikes in low frequencies. Fewer missing noise spikes in low frequencies are guaranteed with the proposed AGFS. With respect to growing criteria, region-growing threshold values in GaSF [13] are set manually, which are proportional to the maximum amplitude values of untouched area and detected noise peaks. However, they are set automatically in the proposed algorithm. It means that AGSF filter parameters are determined automatically.

Differences between GaSF [13] and the proposed AGSF are specified briefly in Table 1.

<table>
<thead>
<tr>
<th>GaSF [13]</th>
<th>Proposed AGSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 3$ average filter</td>
<td>$3 \times 3$ Gaussian kernel smoothing filter</td>
</tr>
<tr>
<td>Linear smoothing</td>
<td>Nonlinear smoothing</td>
</tr>
<tr>
<td>Square low-frequency region detection</td>
<td>Irregular low-frequency region detection</td>
</tr>
<tr>
<td>Threshold values set manually</td>
<td>Threshold values set automatically</td>
</tr>
</tbody>
</table>

Most of the filters and algorithms from previous works are manual or semiautomatic methods except for studies [14] done by Sur et al.

In this paper, we propose an automatic periodic-noise-reduction algorithm with designed AGSF in Fourier space. The proposed method is partially based on a region-growing algorithm. Firstly, the low-frequency region in the Fourier amplitude spectrum of an image is determined. This step is crucial because the success of the algorithm is mainly related to it. Next, the filter parameters for each noise point are detected depending on amplitude, coordinate, and area of noise peaks. Finally, the image amplitude spectrum is filtered by the designed AGSF and a noise-reduced image is obtained in the spatial domain by an inverse Fourier transform. Compared to [14], AGSF provides the best performance for removing periodic noise in low frequencies. As can be seen from the results, the method is fairly efficient and invariant of image quality and size.
The paper is organized as follows. In Section 2, the proposed noise-reduction algorithm is presented in detail. Section 3 gives the experimental results of the proposed method and comparison with the automatic method [14]. In the last section, the proposed method is discussed in all respects.

2. Method

There are three main steps in the proposed algorithm and they are as follows: detection of the low-frequency region, detection of noise peaks and areas, and the design of an adaptive Gaussian star filter. A flowchart of the periodic noise reduction algorithm can be viewed in Figure 2.

![Flow chart of the proposed algorithm.](image)

The proposed algorithm is explained step-by-step in detail in the following subsections.

2.1. Detection of the low-frequency region

As is known, periodic noise appears as grids and lines in an image on a spatial domain. Meanwhile, it can emerge as high amplitudes at any frequency location in an image spectrum. In this step, taking a 2D-centered...
Fourier transform of the source image is the first part. Next, a low-frequency region of the centered Fourier transform of the image is determined. Finally, it is labeled as an untouched area since it contains the main image information, even if it has some noise components. The source image in the spatial domain is multiplied with $(-1)^{x+y}$, where $(x, y)$ is a set of pixel coordinates, to center the Fourier spectrum. The 2D-centered Fourier transform of the image (2D-CFTI), $F(u, v)$, is defined as

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (-1)^{x+y} f(x, y) e^{-j2\pi \left( \frac{ux}{M} + \frac{vy}{N} \right)},$$

where $f(x,y)$ is the intensity value of the image size of $M \times N$.

2D-CFTI is filtered by a $3 \times 3$ Gaussian kernel low-pass filter ($G$), whose coefficients ($h(p,q)$) are calculated using Eq. (2), in a frequency domain in order to obtain a smoother amplitude spectrum. The aim of this process is to smooth the peaks on the 2D-CFTI in order to make the region-growing algorithm more effective. A low-frequency region is determined using region-growing on the smoothed 2D-CFTI (2D-SCFTI) and it is labeled on the 2D-CFTI. $K$ is the spreading constant of the region-growing method. It is selected experimentally and falls within a range of $[0, 1]$. Growing criteria in region-growing is related to $K$. Thus, it takes an important role to detect the low-frequency region. The growing of the region continues until the amplitude spectrum value of that region is less than $K$ times the maximum amplitude value for the 2D-SCFTI.

2.2. Detection of noise coordinates and regions

Maximum noise peaks coordinates are searched for in the 2D-CFTI, except for in the untouched area. In this search, every point whose amplitude value is greater than $K$ times the maximum amplitude value of 2D-CFTI is a candidate for noise peak. Noise peaks are twins and have symmetry according to the center. This means that coordinates of other twin noise peaks can easily be determined when one of the twins is detected in the 2D-CFTI. Region parameters of twins are obtained via the growing region. $K_n$ is a constantly spreading parameter in the region-growing method for the noise area and within a range of $[0, 1]$. Growing criteria in region-growing is related to $K$. Thus, it takes an important role to detect the low-frequency region. The growing of the region continues until the amplitude spectrum value of that region is less than $K$ times the maximum amplitude value for the 2D-SCFTI.

2.3. Adaptive Gaussian star filter

The profile of a typical periodic noise in an amplitude spectrum is generally seen as a star form. Hence, the shape of a filter is designed as a star using two 2D elliptical filters positioned perpendicularly to each other.
AGSF in the frequency domain is represented by the following:

\[
AGSF (u, v) = \begin{cases} 
    S_1 (u_1, v_1) + S_2 (u_2, v_2) & \text{if } u_1 \neq u_2 \text{ and } v_1 \neq v_2 \\
    \max (S_1 (u_1, v_1), S_2 (u_2, v_2)) & \text{if } u_1 = u_2 \text{ and } v_1 = v_2 
\end{cases} 
\]

\begin{align*}
    u &= 1, 2, \ldots, M \\
    v &= 1, 2, \ldots, N \\
    u_1 &= 1, 2, \ldots, M \\
    v_1 &= 1, 2, \ldots, N \\
    u_2 &= 1, 2, \ldots, M \\
    v_2 &= 1, 2, \ldots, N
\end{align*}

\( (u_1, v_1) \) is the noise peak and the neighbor pixels of the \( n \) \( \text{th} \) noise region. Moreover, \( u_1 \) and \( v_1 \) are the coordinates of the \( n \) \( \text{th} \) twin noise peaks \( (u_2, v_2) \), found in Eq. (8) after \( u_1 \) is estimated.

\[ u_2 = M + 1 - u_1, \quad v_2 = N + 1 - v_1 \quad \text{(8)} \]

\( \alpha_{n1,2} \) and \( \beta_{n1,2} \) are the filter parameters determined via the region-growing algorithm around \( n_1^{th} \) noise twin peaks.

Distances between the \( n_1^{th} \) noise peak and the neighbor pixels of the \( n_1^{th} \) noise region are calculated by using the Euclidean distance. \( \alpha_{n1} \) is proportional to the Euclidean distance between the \( n_1^{th} \) noise peak center and the nearest edge point from the peak center of the \( n_1^{th} \) noise region. Moreover, \( \beta_{n1} \) is proportional to the Euclidean distance between the \( n_1^{th} \) noise peak and farthest edge point. \( \alpha_{n2} \) and \( \beta_{n2} \) are calculated for the \( n_2^{th} \) noise peak as well.

The parameters of AGSF are determined as

\[
\begin{align*}
    \alpha_{n1,2} &= C \cdot \min \left( \sqrt{(u_{n1,2} - u_{n2})^2 + (v_{n1,2} - v_{n2})^2} \right) \\
    \beta_{n1,2} &= C \cdot \max \left( \sqrt{(u_{n1,2} - u_{n2})^2 + (v_{n1,2} - v_{n2})^2} \right) \\
    z &= 1, 2, \ldots, Z_{n1,2}
\end{align*}
\]

where \( Z_{n1,2} \) represents the number of neighbor pixels of the \( n_1^{th} \) noise region. As given in Eq. (9), \( \alpha_{n1,2} \) and \( \beta_{n1,2} \) are \( C \) times the minimum and maximum value of distances, respectively.
3. Results and discussion

The efficiency of the proposed algorithm on a variety of images is examined for different frequency sensitivities as well as $\alpha$ and peak signal to noise ratios (PSNR), as in Eqs. (10) and (11), respectively:

$$\psi = \frac{\sqrt{(u_s - u_c)^2 + (v_s - v_c)^2}}{\sqrt{(u_c - M)^2 + (v_c - N)^2}}$$

$$PSNR = 10 \log \frac{P_{image}}{P_{noise}} = 10 \log \frac{L^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \eta(x, y)^2},$$

where $L$ is the maximum intensity value of the test image and $P_{noise}$ is the power of the two-dimensional periodical sinusoidal noise, $\eta$, added to the test image. The range of $\alpha$ is $[0, 1]$ and it is obtained over the Euclidean distance between the center point of the image $(u_c, v_c)$ and the interested noise point $(u_s, v_s)$ and divided by the Euclidean distance between the center point and corner point of the image.

$K$ and $K_n$ are proportional constants, as stated, and their convenient values are determined empirically via different sizes images disrupted by diverse periodic noise characteristics.

The parameters of the algorithm are selected experimentally as follows:

$$K = 0.02, \quad K_n = 0.15 \quad \text{and} \quad C = 3.$$  Additionally, $G = \begin{bmatrix} 0.5/8 & 1/8 & 0.5/8 \\ 1/8 & 2/8 & 1/8 \\ 0.5/8 & 1/8 & 0.5/8 \end{bmatrix}$  and $L = 255.$

To investigate the effects of $K$ and $K_n$ on the performance of the proposed algorithm, the image in Figure 1, with a size of $512 \times 512$, is covered by periodic noise. The noise is characterized by 16 noise regions and PSNR = 14.15 dB. The obtained PSNR values before and after filtering, as shown in Table 2, are given for different values of $K$ and $K_n$. According to the results, when $K$ is smaller ($K = 0.005$) than the convenient value ($K = 0.02$), only 12 of 16 noise regions could be detected and filtered, since the image region in spectrum grows over and the noise peaks located in the low-frequency region could not be detected. There are missing noise regions for $K$ and larger values ($K = 0.05$) than the identified value ($K = 0.02$). With respect to $K_n$, both smaller and larger values ($K_n = 0.05$ and $K_n = 0.5$) than the convenient value ($K_n = 0.15$), noise region are not enough to compute reasonable filter parameters.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$K_n$</th>
<th>Before filtering, PSNR (dB)</th>
<th>After filtering, PSNR (dB)</th>
<th>Number of noise region(s)</th>
<th>Number of detected noise region(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.15</td>
<td>14.15</td>
<td>21.87</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>*0.02</td>
<td>0.15</td>
<td>14.15</td>
<td>24.58</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>0.05</td>
<td>0.15</td>
<td>14.15</td>
<td>22.61</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>0.02</td>
<td>0.05</td>
<td>14.15</td>
<td>21.39</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>0.02</td>
<td>*0.15</td>
<td>14.15</td>
<td>24.58</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>0.02</td>
<td>0.5</td>
<td>14.15</td>
<td>23.45</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

As seen in Figure 3A, the sample image in Figure 3B, with a size of $512 \times 512$ pixels, is disrupted by periodic noise in low frequencies in Figure 3C with $\psi = 0.2$ and PSNR = 20 dB. The amplitude spectrum
Figure 3. a) Image corrupted by periodic noise in low frequencies. b) Original gray level image (512 × 512 pixels). c) Periodic noise component. d) Frequency amplitude spectrum of a noisy image. e) AGSF as a high-pass filter to decay the periodic noise peaks. f) Amplitude spectrum of a filtered image.
Figure 3. g) Image filtered with GaSF [13], PSNR = 34.7075 dB with \( \tau_1 = F_o(u,v,o)/25 \) [13], \( \tau_2 = 0.15*F_o(u,v,o) \) [13] and \( \tau_3 = F_n(u,v,n)/15 \) [13]. h) Image filtered with [14], PSNR = 33.0628 dB. i) Image filtered with the proposed method AGSF, PSNR = 34.1182 dB.

of the noisy image and the designed AGSF are shown in Figures 3D and 3E. The amplitude spectrum of a noise-filtered image with the proposed AGSF is shown as in Figure 3F. PSNR for the filtered image with GaSF [13] in Figure 3G, with [14] in Figure 3H, and with the proposed AGSF in Figure 3I are computed to be about 34.7075 dB, 33.0628 dB, and 34.1182 dB, respectively. As understood, the best improvement in filtered images is obtained by GaSF [13] with manual setting parameters. In [14], an average power spectrum line was obtained using robust linear regression-fitting in a power law distribution of the spectrum. According to the average line, the upper boundary is described under a standard deviation rule and outliers are found up to it. This means that noisy spikes in low frequencies are missed due to the fact that they are located inside the drawn upper boundary. Thus, concerning automated methods, the proposed AGSF performs better than [14] for noise reduction in low frequencies.
Figure 4. a) Image corrupted by periodic noise in high frequencies. b) Original gray level image (512 × 512 pixels). c) Periodic noise component noise. d) Image filtered with GaSF [13], PSNR = 61.5554 dB with $\tau_1 = F_\omega(u_\omega,v_\omega)/60$ [13], $\tau_2 = 0.25 * F_\omega(u_\omega,v_\omega)$ [13], and $\tau_3 = F_n(u_n,v_n)/30$ [13]. e) Image filtered with [14], PSNR = 46.6584 dB. f) Image filtered with the proposed AGSF, PSNR = 44.0864 dB.
As seen in Figure 4A, the original image in Figure 4B, with a size of 512 × 512 pixels, is covered by periodic noise in high frequencies in Figure 4C with $\psi = 0.6$ and PSNR = 20 dB. The images denoised with GaSF [13,14], and the proposed AGSF are shown in Figures 4D–4F. PSNR for the filtered image with GaSF [13,14], and the proposed AGSF are about 61.5554 dB, 46.6584 dB, and 44.0864 dB, respectively. GaSF [13]

Figure 5. a) Real periodic noisy image, a school (1024 × 768 pixels). b) Image filtered with [14]. c) Noise-reduced image with proposed AGSF. d) Real periodic noisy image, a man (500 × 352 pixels). e) Image filtered with [14]. f) Noise-reduced image with proposed AGSF.
Figure 5. g) Real periodic noisy image, a clown (294 × 294 pixels). h) Image filtered with [14]. i) Noise-reduced image with the proposed AGSF.

With manual setting parameters performed the best among these methods. Considering automated methods, [14] performed better than the proposed AGSF for noise reduction in high frequencies.

\[ \varepsilon = \frac{\text{Number of noise-reduced image with AGSF}}{\text{Number of disrupted image by periodic noise}} \]  

The test images are in a JPEG format. They have different sizes and different spatial textures. \( \varepsilon \) is about 99\% for \( \psi = 0.2, 0.3, 0.4, \) and 0.6, and PSNR = 15 dB, 18 dB, and 20 dB, as shown in Table 3. It is provided that \( \varepsilon \) of the algorithm has approximately the same values for specified \( \alpha \) and PSNR, even if the tested images are resized, as shown in Table 3.
Table 3. Algorithm efficiencies for different noise characteristics.

<table>
<thead>
<tr>
<th>Efficiency ($\psi$)</th>
<th>PSNR = 15 dB</th>
<th>PSNR = 18 dB</th>
<th>PSNR = 20 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 0.2$</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>$\psi = 0.3$</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\psi = 0.4$</td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>$\psi = 0.6$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In Figure 5, the school, the man, and the clown are naturally noisy images corrupted by periodic noise, especially in low frequencies. They are filtered by [14] and by the proposed method as well. It is observed that the images filtered with AGSF have the best visibility when the proposed method is compared with [14].

4. Conclusion
We have proposed an automatic periodic noise reduction algorithm based on low-frequency region detection, estimation of noise coordinates and areas in frequency domain, and adjustment of AGSF shape and parameters. The performance of the method is discussed in the context of texture and size for noisy images. The results show that the proposed method reduces periodic noise while preserving the useful details of the image. The proposed algorithm is compared to an automatic method [14]. As can be inferred from the results, the developed method, which determines parameters automatically, has better performance in reducing (quasi-) periodic noise in low frequencies than [14].

References