Comparing of phase shifting method and one-dimensional continuous wavelet transform method for reconstruction using phase-only information

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Abstract: The one-dimensional continuous wavelet transform method has some advantages in hologram reconstruction, when used in the only phase, compared with the phase shifting method. This paper aims to discuss these advantages. One advantage is related to image quality. Another advantage is less power spent and saving time, because the one-dimensional continuous wavelet transform method uses only one hologram and the phase shifting method uses four holograms for recording and reconstruction processes. One final advantage is that the one-dimensional continuous wavelet transform method can also be used in real-time applications. Within the context of the ongoing optimization studies, this study will make a significant contribution to the literature because of these mentioned advantages.

Key words: Hologram, continuous wavelet transform, phase shifting method, phase, image quality, optimization

1. Introduction
The amplitude and phase information of three-dimensional (3D) object fields can be recorded and reconstructed via holography [1]. Holographic interferometry, computer-generated holography, and digital holography are major applications of holography [2,3]. The digital holography technique used in this study was first introduced into the literature by Schnars and Jüptner [4]. Holography enables full digital recording and reconstruction. In the recording process, the digital holographic image is recorded with a camera. The reconstruction process is performed digitally by the Fresnel–Kirchhoff integral (FKI) on a computer without an optical arrangement [5–7]. Most reconstruction methods are realized by pure amplitude or pure phase information [8]. In the reconstruction process, using the phase information of a hologram contributes to obtaining the highest image quality with minimum power loss. In order to find the phase information of the hologram, there are various methods. These methods are the Fourier transform method (FTM) [9–11], phase shifting method (PSM) [12–14], and one-dimensional continuous wavelet transform method (1D-CWTM) [15,16]. The phase shifting method was introduced for the first time by Yamaguchi and Zhang [13]. To create a phase hologram and reconstruct a 3D image, more than one hologram is required for this method. Unlike the PSM, the 1D-CWTM uses only one hologram for both processes. In addition, the PSM needs more power than the 1D-CWTM for the reconstruction of the hologram on the camera.

Moreover, the image quality is an importance parameter for the reconstructed images of holographic methods. In this study, the PSM and 1D-CWTM are examined by using numerical and experimental results for the first time. Both methods are compared with respect to three main points: the image quality of the...
reconstructed image, the total time for the recording and reconstruction process, and the total power requirement in the recording and reconstruction process. How many holograms are needed for the reconstruction process in both methods? As the number of holograms increases the recording and reconstruction process times increase.

This paper is presented in four sections. In Section 2, the hologram simulation is achieved digitally. After that the phase is calculated by using 1D-CWTM and PSM and the phase holograms are created. Then reconstructed 3D images are obtained by FKI method, using these phase holograms. As a result, the reconstructed 3D images are presented. Additionally, in this section the normalized root means square (NRMS) values are given to compare the image quality of the two methods. In Section 3, the digital experimental results, which are obtained by using the methods explained in Section 2, are presented. In the last section, the simulation and experimental results are discussed.

2. Digital hologram recording and 3D image reconstruction

In an interferometric setup, a hologram is created from the interference of the two waves coming from the object and reference arms. When the beam coming from the object arm generates the object wave, the beam reflected from the reference arm constitutes the reference wave. The mathematical expression of this hologram is defined by Eq. (1) [9].

\[
I(x, y) = |A_O(x, y)|^2 + |A_R(x, y)|^2 + 2 |A_O(x, y)||A_R(x, y)| \cos(kz + \phi(x, y))
\]  

(1)

Here \((x, y)\) are the coordinates on the hologram plane. The first term on the right side of Eq. (1) gives the intensity information of the object wave \((A_O(x, y))\), while the second term is about the intensity information of the reference wave \((A_R(x, y))\). The third term on the right side of the equation defines a function of phase difference \((\phi(x, y))\) between the object and reference wave.

The digital hologram of the object, which is given in Figure 1a, is created digitally. Each point of the diffracted field amplitude of the letter S (used as the object in this study) is accepted as 1. The hologram pattern obtained by the mathematical expression of the hologram given in Eq. (1) is presented in Figure 1b.

Using the phase information of the hologram during the 3D object reconstruction process improves the image quality and decreases the power loss. To obtain the phase information in this study, the 1D-CWTM and PSM were used, respectively [14,16].

If the phase information is found through the 1D-CWTM, the continuous wavelet transform is performed for each line of the hologram; this is given in Eq. (1) (see Eq. (2)) [17].

\[
W(s, b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} I(x) \psi_{s, b}^* \left(\frac{x - b}{s}\right) dx
\]

(2)

Here \(s\) defines the scaling parameter associated with the frequency and \(b\) defines the shifting parameter, which changes according to the \(x\) position. The definition of \(*\) gives the complex conjugate. This conjugate expression is obtained by shifting and scaling the mother Morlet wavelet \((\psi(x))\) [18]. In this study, all processes for the algorithm of the 1D-CWTM are performed with MATLAB toolbox.

The Morlet wavelet function, which has the plane wave modulated by Gaussian function, is expressed in Eq. (3) [17].

\[
\psi(x) = \pi^{-1/4} \exp(i\omega_0 x) \exp\left(-\frac{x^2}{2}\right)
\]

(3)
Eq. (3) can be rearranged by scaling and shifting. The scaled and shifted Morlet wavelet function is given in Eq. (4).

\[ \psi_{s,b} \left( \frac{x - b}{s} \right) = \pi^{-1/4} \exp \left( i w_0 \left( \frac{x - b}{s} \right) \right) \exp \left( -\frac{1}{2} \left( \frac{x - b}{s} \right)^2 \right), \]  

(4)

where \( w_0 \) is spatial frequency and equals 4. The complex amplitude of the wavelet function is described with the imaginary part \( \text{Im} (W(s,b)) \) and real part \( \text{Re} (W(s,b)) \) of the \( W(s,b) \) and this function is given in Eq. (5).

\[ |W(s,b)| = \sqrt{(\text{Re} (W(s,b)))^2 + (\text{Im} (W(s,b)))^2} \]  

(5)

The phase information obtained from the 1D-CWTM, which is given in Eq. (2), is found with Eq. (6). Moreover, the calculated phase value is between \([ -\pi, +\pi ]\).

\[ \phi(x) = \tan^{-1} \left( \frac{\text{Im} (W(s,b))}{\text{Re} (W(s,b))} \right) \]  

(6)

By using the phase information obtained by the 1D-CWTM, the phase hologram, whose amplitude is 1, can be created. In Eq. (7), the mathematical expression of the created phase hologram is given.

\[ p(x,y) = \exp (i\phi(x,y)) \]  

(7)

To reconstruct the 3D image from the phase hologram created by the 1D-CWTM (see Eq. (7)), the FKI is used (see Eq. (8)) [19].

\[ P_f (\xi, \eta) = -\frac{1}{i\lambda} \iint p(x,y) \frac{1}{z} \exp (-ikz) \exp \left\{ -\frac{i}{2z} \left[ (\xi - x)^2 + (\eta - y)^2 \right] \right\} dxdy \]  

(8)
Figure 2a shows the phase pattern of the letter S created by the 1D-CWTM digitally and accepted as the sample object. The reconstructed image from phase hologram created by the 1D-CWTM, in which the scaling interval is 2−9, is presented in Figure 2b. The 3D appearance of this image is shown in Figure 2c.

The single image is obtained without the filtering process from the reconstructed image obtained by using the 1D-CWTM. In this procedure, the zero diffraction order and twin image problem are eliminated directly.

The phase shifting digital holography technique is the second method of obtaining phase information from the digital hologram. This method, which was introduced for the first time by Yamaguchi and Zhang, requires more than one hologram to create a phase hologram as opposed to the other methods. Different holograms are generated at the desired value by shifting the phase of reference wave from the basic hologram definition given in Eq. (1) [17]. While the initial reference wave is accepted as 0 and this phase value is shifted with the degree
of $\phi = 0, \pi/2, 3\pi/2$ respectively, 4 different holograms can be achieved. The mathematical definition of the 4 holograms, which are obtained by shifting the reference wave, is expressed in Eq. (9) [9,20,21].

$$I_0 = A_R^2 + A_O^2 + 2A_RA_O \cos \phi$$

$$I_{\pi/2} = A_R^2 + A_O^2 + 2A_RA_O \cos (\phi + (\pi/2))$$

$$I_\pi = A_R^2 + A_O^2 + 2A_RA_O \cos (\phi + \pi)$$

$$I_{3\pi/2} = A_R^2 + A_O^2 + 2A_RA_O \cos (\phi + (3\pi/2))$$

It is possible that the phase information of the hologram is obtained directly depending on the recorded intensity information of each generated hologram (see Eq. (10)) [13].

$$\phi (x,y) = \arctan \frac{I_{3\pi/2} - I_{\pi/2}}{I_0 - I_\pi}$$

The phase hologram, whose amplitude is 1, can be created by substituting the phase information, which is obtained by using PSM given in Eq. (10), in Eq. (7). To reconstruct the 3D image of the object from the created phase hologram, the FKI expressed in Eq. (8) is used.

The phase pattern of the letter S obtained by the PSM is given in Figure 3a. The reconstructed image from the phase hologram created by the PSM is presented in Figure 3b. The 3D appearance of this image is shown in Figure 3c. In addition, the single image is obtained without the filtering process from the reconstructed image obtained using the PSM. In this procedure, the zero diffracted order and twin image problems are eliminated directly.

To compare the image quality of the reconstructed image from the phase hologram, which is obtained by using the 1D-CWTM and PSM, respectively, the normalized root mean square (NRMS) values are calculated (see Eq. (11)). To calculate the NRMS value, the amplitude and phase information of the holograms are utilized [19].

$$NRMS = \sqrt{\frac{\iint (|\Gamma_A (\xi, \eta)|^2 - |\Gamma_p (\xi, \eta)|^2)^2 \, d\xi \, d\eta}{\iint (|\Gamma_A (\xi, \eta)|^2)^2 \, d\xi \, d\eta}}$$

Here $\Gamma_A$ defines the complex field information of the hologram and $\Gamma_p$ ($\Gamma_p = \exp (i \phi)$) expresses the phase hologram created by using phase information.

In Table 1, the NRMS values of the image reconstructed from the numerical holograms are calculated with Eq. (11) by using the 1D-CWTM and PSM. According to these values, the image quality of the 3D image obtained by using the 1D-CWTM seems to be high. These results show that less power is consumed in the step of the hologram recording and reconstruction processes in the 1D-CWTM. According to the digital recording and reconstruction processes digitally, less time is needed for the 1D-CWTM than for the PSM, because it requires four holograms for recording and reconstruction processes in the PSM and each individual hologram’s recording and reconstruction processes take more time.
Figure 3. The appearance of digitally recorded and reconstructed images obtained by using the PSM: (a) The phase information pattern created digitally by using the PSM; (b) The 2D appearance of the reconstructed image by the PSM; (c) The 3D appearance of the reconstructed image.

Table 1. The NRMS values calculated digitally using the 1D-CWTM and PSM.

<table>
<thead>
<tr>
<th>Methods to Obtain Phase</th>
<th>Digital Results of NRMS Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D-CWTM</td>
<td>0.9995</td>
</tr>
<tr>
<td>PSM</td>
<td>1.0498</td>
</tr>
</tbody>
</table>

3. The experimental hologram recording and 3D image reconstruction

In the experimental part of this study, the lensless Fourier digital holography (LFDH) setup is used for hologram recording. It is given in Figure 4. The beam in the system, which is used for hologram recording, is obtained by He–Ne laser. The object and reference wave, which are obtained with the laser wavelength of 633 nm, create the
interference pattern on a charge-coupled device (CCD) camera in this system. The CCD camera used during the recording process has a resolution of 640 by 480 pixels.

![Diagram of LFDH recording setup](image)

**Figure 4.** The LFDH recording setup. In this setup, the size of the used object is 0.7 cm, the distance of the object from the CCD is 95 cm, and the angle between reference and object waves is 1.54°.

The beam coming from the red He–Ne laser has an output power of 10 mW and is divided into two arms via a beam splitter (BS). The beam coming from the first arm is reflected on the sample object using a mirror 1 (M1). The waves reflected from the object are called object waves. After the beam from the second arm passes through the neutral density filter (NDF), mirror 2 (M2), and three-axis spatial filter (SF) consecutively, it is recorded on the camera directly as the reference wave. While the reference wave comes onto the recorded plane of the CCD camera at a right angle, the reflected wave from the object occurs at a very small angle. The object and the reference wave constitute the interference pattern (hologram) on the CCD camera. This hologram

![Sample object and hologram](image)

**Figure 5.** The images of the sample object used in the experimental setup: (a) Sample object, star; (b) The hologram pattern of star.
is transferred onto the personal computer (PC) using an interface. The 3D image reconstruction and signal processing parts are done on the computer.

A star is used as an object in the hologram recording process and it is shown in Figure 5a. The hologram pattern of the star created experimentally is presented in Figure 5b.

The hologram pattern given in Figure 5b is processed digitally after being transferred to the computer via an interface. In this process, the phase information of the hologram is obtained by using the 1D-CWTM and PSM described in Section 2 (see Eqs. (6) and (10)). The FKI given in Eq. (8) is used to reconstruct the hologram pattern by using this phase information. In Figure 6a, the phase pattern of the star, found by using the 1D-CWTM, is given. The reconstructed image from the phase hologram created by the 1D-CWTM, is given in Figure 6b and is scaled using a 2–3 interval. The 3D appearance of this image is presented in Figure 6c.

The second method explained in detail for finding the phase information in Section 2 is the PSM. In the PSM, the phase information is found by recording the numbers of holograms in this study. To create 4

Figure 6. The appearance of the experimental recorded and digital reconstructed images obtained by using the 1D-CWTM: (a) The phase pattern created experimentally by using the 1D-CWTM; (b) The 2D appearance of reconstructed image by the 1D-CWTM; (c) The 3D appearance of the reconstructed 3D image.
holograms, the phase of the reference wave must be changed. This phase is shifted by $\phi = 0, \pi/2, \pi, 3\pi/2$ respectively. To realize this process experimentally, the Piezo stage shown in Figure 4 is used. The NPXY200 Piezo stage, which is connected to the reference arm, is computer controlled and has been a working closed loop. The reconstruction process is performed using the FKI (see Eq. (10)) after finding the phase information of the hologram created by the PSM in the computer.

In Figure 7a, the phase pattern of the star, found with the PSM, is given. In addition to this, the reconstructed image from phase hologram created by the PSM is presented in Figure 7b. The 3D appearance of this reconstructed image is shown in Figure 7c.

By using Eq. (11), given in Section 2, the NRMS values are calculated to compare the image quality of the reconstructed image. The calculated NRMS values are given in Table 2.
**Table 2.** The NRMS values calculated experimentally using the 1D-CWTM and PSM.

<table>
<thead>
<tr>
<th>Methods to Obtain Phase</th>
<th>Optical Results of NRMS Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D-CWTM</td>
<td>0.9989</td>
</tr>
<tr>
<td>PSM</td>
<td>1.0689</td>
</tr>
</tbody>
</table>

In Table 2, the NRMS values of the image are calculated with Eq. (11) by using the 1D-CWTM and PSM. These values are reconstructed from the experimental hologram. According to these values, the image quality of the 3D image obtained using the 1D-CWTM seems to be better. These results show that the minimum power is consumed in the step of reconstruction and the hologram recorded in the 1D-CWTM. Since the PS method uses 4 holograms, the hologram recording process in the laboratory takes quite a long time and consumes more power, whereas time and power are notably diminished by using the 1D-CWTM.

4. Discussion

In this study, the 1D-CWTM is compared to the PSM within the context of ongoing optimization studies, which is about the image quality, saving time, and use in real-time applications. While reconstructing the 3D images from the digital hologram, the phase information obtained digitally and experimentally is used. The 1D-CWTM and PSM are used to obtain this phase information. According to the calculated NRMS values, higher image quality is calculated using the 1D-CWTM. In addition, the recording and reconstruction processes of the hologram by the 1D-CWTM are carried out in minimum time because of its required hologram number, because the PSM uses 4 holograms for both processes and the 1D-CWTM uses only one hologram. This situation shows that the 1D-CWTM can be used in real-time applications and the PSM cannot.

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References


