

## Lambda optimization of constraint violating units in short-term thermal unit commitment using modified dynamic programming

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**Abstract:** This paper presents a new approach with a three-stage optimization algorithm for the least-cost optimal solution of the unit commitment problem. In the proposed work, the optimal schedule is obtained by optimizing the lambda operator for the states that violate the inequality constraints. The objective of the work is to minimize the fuel cost when subjected to various constraints such as load balance, minimum up/down time, ramp limit, and spinning reserve. This method of committing the units yields the least-cost solution when applied to the IEEE 10-unit systems and 7-unit Indian utility practical systems scheduled for 24 h and the results obtained are compared with the existing methods.

**Key words:** Dynamic programming, economic dispatch, lambda optimization, unit commitment

### 1. Introduction

Electric power is the basic necessity for any economic activity of a country. As a chief fuel resource, coal provides about 30% of the global primary energy needs, generates 41% of the world's electricity, and is used in the production of 70% of the world's steel. With the available global life span of another 190 years approximately, the world's primary fuel resource must be conserved to meet the growing demand. Moreover, the demand for electricity is not the same at all times of the day and this also necessitates the conservation of the resource by operating the units economically, which is realized through optimal unit commitment. Due to the ever increasing need for power in all sectors, more optimization is required at the generation level in order to conserve the fuel and minimize the production cost.

Unit commitment in power systems refers to the optimization problem of determining the schedule of generating units that minimize the operating cost subject to a variety of constraints [1]. Obviously it is not necessary to commit a unit when it is not required and this determination of ON/OFF must be done according to some predetermined criteria that satisfy the constraint as well as minimize the cost. The research for the unit commitment problem (UCP) is aimed at fast computing techniques and low production costs. In solving the UCP [2,3], certain optimization methods are involved, which ensures bridging the gap between demand and supply. The UCP is solved using both mathematical and nonmathematical approaches. The solving methodologies have been advanced from simple rule-of-thumb methods to highly complex methods. Many mathematical approaches exist in the literature, like priority lists [4,5], Lagrangian relaxation [6], mixed integer programming (MIP) [7], branch and bound (B&B) [8], Bender's decomposition [9], and dynamic programming (DP) [10–12]. Priority list methods are fast but highly heuristic and give schedules with relatively high operating costs. Moreover, the

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quality of the final solution is not guaranteed to be good. Lagrangian relaxation uses Lagrange multipliers and solves time-dependent constraints, but due to the duality gap, the solutions obtained are not optimal. MIP methods have only been applied to small UCPs and have required major assumptions, which limit the solution space, although they provide exact solutions theoretically. The B&B algorithm uses an intelligent structure to search for the solution. The solution obtained from the B&B method is a local optimum and the computation time is large for long-term commitment.

Many nonmathematical approaches like evolutionary programming [13], fuzzy logic [14], artificial neural networks [15–17], simulated annealing (SA) [18], tabu search [19,20], and genetic algorithms (GAs) [21] provide solutions to UCPs. The GA is a stochastic and parallel search method based on the mechanics of natural selection and natural genetics. SA is a stochastic optimization technique that theoretically converges and produces a global optimal solution. However, the GA and SA demand high computational time, many times larger than the mathematical approaches. The metaheuristic approaches alone cannot guarantee the optimality of the solution and it is difficult to estimate the quality of the solution accuracy due to parameter adjustments. Hence, the mathematical method chosen must be efficient in terms of generating the optimal initial solutions. Among the various methods available [22–24], one extensively employed method is DP, which provides the exact solution for the UCP. DP is based on Bellman’s principle of optimality, and in the deterministic process of finding the functional objective, DP is continuously applied to the subproblems, thereby producing the optimal decision as the outcome. Practically it is very difficult to formulate the problem as it calls for a number of states, but the problem could be framed with straightforward constraints to obtain the optimal schedule. DP is a powerful mathematical model that overrides the priority list method in committing the units optimally. In this paper the deterministic approach is modified in three stages, which promises an optimal solution with less computation time.

## 2. Unit commitment problem

UC is a complex multistage decision-making process in electric power systems that involves determining the ON/OFF schedules of the units to meet the forecasted load. This process helps the dispatcher manage the uncertainties and various constraints. The period of 24 h (1 day) is considered to solve the UCP. The objective is to minimize the total fuel costs while satisfying the system constraints and other local constraints.

### 2.1. Objective function

$$\text{Min } TFC = \left[ \sum_{t=1}^T \sum_{i=1}^n (a_i + b_i P_{g_{i,t}} + c_i P_{g_{i,t}}^2) \right] + SUC \quad (1)$$

TFC - Total fuel cost

t - Time interval (1 to 24 h)

n - Number of generating units

$P_{g_{i,t}}$  - Power generation of  $i$ th unit at time period  $t$

$a_i, b_i, c_i$  - Cost coefficients

SUC - Startup cost

**2.2. Constraints**

- Subject to system constraints

*Unit initial conditions*

First hour schedule is based on the unit initial status

*Unit status restrictions*

Certain units under all load conditions are assigned with the must-run status

*Power balance constraint*

$$\sum_{i=1}^n Pg_{i,t} = P_L \tag{2}$$

Where

$P_L$  – Load at time interval  $t$

*Reserve constraint*

$$\sum_{i=1}^n Pg_{i,max} \geq P_L + R_t \tag{3}$$

Where

$R_t$  – Reserve at time interval  $t$

$Pg_{i,max}$  – Maximum generation limit in MW of  $i$ th unit

- Subject to local constraints

*Inequality constraints*

$$Pg_{i,min} \leq Pg_{i,t} \leq Pg_{i,max} \tag{4}$$

Where

$Pg_{i,min}$  – Minimum generation limit in MW of  $i$ th unit

*Minimum up and minimum down time constraints*

- Indicate that a unit must be ON/OFF for a certain number of hours before it can be committed or decommitted

$$T_{i,t}^{ON} \geq T_i^{UP} \tag{5}$$

$$T_{i,t}^{OFF} \geq T_i^{DOWN} \tag{6}$$

Where

$T_{i,t}^{ON}$  – ON time of  $i$ th unit in interval  $t$ ,  $t = 1$  to  $T$

$T_{i,t}^{OFF}$  – OFF time of  $i$ th unit in interval  $t$ ,  $t = 1$  to  $T$

$T_i^{UP}$  – Minimum ON time of  $i$ th unit

$T_i^{DOWN}$  – Minimum OFF time of  $i$ th unit

*Startup cost (SUC)*

$$SUC_{i,t} = \begin{cases} \text{hot startup cost, if downtime} \leq \text{coldstart hours} \\ \text{cold startup cost, otherwise} \end{cases} \tag{7}$$

$$SUC_{i,t} = S_{oi}[1 - D_i e^{-\left(\frac{T_{off,i}}{T_{down,i}}\right)}] + E_i \tag{8}$$

$S_{oi}$ - Cold startup cost

$D_i$  and  $E_i$ - Startup cost coefficients

*Ramp rate limits*

The operating ranges of all online units are restricted by their ramp rate limits.

$$\begin{aligned} P_i - P_i^0 &\leq UR_i \\ P_i^0 - P_i &\leq DR_i \end{aligned} \quad (9)$$

$P_i$ - Power generation of unit  $i$

$P_i^0$ - Power generation of unit  $i$  at previous hour

$UR_i$ - Ramp-up rate limit for unit  $i$  at hour  $t$

$DR_i$ - Ramp-down rate limit for unit  $i$  at previous hour

*Initial status*

Unit status is taken into consideration at the beginning of the schedule.

Based on the load data, Eqs. (7) and (8) are used to calculate the startup cost. A simple UC problem for a 10-unit system with 24 h of load can be realized with number of states resulting as  $(2^n)^T = (2^{10})^{24} =$  too many states. These huge states have to be checked to evaluate the best least-cost state for each load variation over the specified time horizon.

Unit initial status, must-run constraints, and power balance constraints are met while solving the DP subproblem itself. Must-run constraints decrease the startup cost during the later demand stages in solving the UCP.

Reserve constraint is considered in solving the UCP and is assumed as a proportional percentage from the maximum limit of the generating unit during each stage of the DP solution. Ramp rate constraints are not violated and it is checked during every stage in the forward DP. Although penalty factors are normally introduced whenever ramp constraints violate the limits, in this work no such factors are introduced as the limits are stable during every transition.

Startup cost and shut-down cost constraints represented in Eq. (7) are applied for solving the IEEE 10-unit system and Eq. (8) is applied for solving the NTPS 7-unit Indian utility system. The DP starts initially after checking whether the initial status conditions are met. This status restriction is for 24 h, represented for 10 unit and 7 unit systems, respectively.

Inequality constraints (min and max generation limit) are refined by lambda optimization for constraint violating units and it is validated throughout every stage of the demand. Many constraints are satisfied throughout the unit commitment problem solving and the inequality constraints are optimized through three stages.

### 3. Modified dynamic programming for unit commitment

Although unit commitment and economic dispatch decisions are interdependent, for certain load changes the satisfaction of one constraint would result in the violation of the other constraint. A method could be proposed that can modify and update the commitment order [25] so as to produce a schedule that satisfies the constraints and yields the optimal solution, thereby minimizing the production cost. This paper focuses on the modified dynamic programming (MDP) method for solving the UCP as it demands superiority for suboptimal solutions in the case of any divergence in terms of accuracy. This feature of MDP is used to find the optimal state

in the second and third stages of the proposed algorithm. In the proposed MDP three-stage technique, the number of states during each hour are considerably reduced and therefore the decision-making process is fast. The decomposition does not always guarantee the convergence [26], yet the assumptions made would result in convergence of the final schedule. In the proposed work, the states diverge for higher loads of the 10-unit system and the convergence occurs by customizing the economic dispatch algorithm, thereby yielding a suboptimal solution with minimum processing time. DP has been applied to the power system UC problem as a powerful method in finding out the schedule of the generating units as observed from various works in the literature [27,28]. In this paper, MDP is applied to the UCP. MDP identifies the best solution over fuzzy dynamic programming (FDP) when applied to the 7-unit thermal power station (TPS) in India in terms of cost and processing time. Since fuel cost is a major cost component in the total cost calculation, reducing the fuel cost by 0.5% can result in saving millions of dollars per year for very large power system utilities. The MDP method is efficient compared to ordinary DP in terms of the search for states performed using trial and error logic in the already stored feasible states' list. This is explained in the flowchart shown in Figure 1. It is observed that for the 7-unit system the final commitment converged through all stages, but in the case of the 10-unit system for higher demands the states diverge and the final optimal state was identified by refining the algorithm.

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### Algorithm

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Step 1: Perform ED using lambda projection method

Step 2: Check if the inequality constraints are violated

Fix the generation to min or max

Step 3: Check and validate the fixed generation schedule using lambda calculation

```
{
  for maximum violation and fixation
  validate  $\lambda_{new} \leq \lambda$ , if true the schedule is optimal
  else go to step 4
  for minimum violation and fixation
  validate  $\lambda_{new} \geq \lambda$ , if true the schedule is optimal
  else go to step 4
}
```

Step 4: Modify the dispatch calculation and go to step 2

Step 5: Apply the constraints

Step 6: Select the state that promises low startup cost during each hour

Step 7: Modify the schedule to satisfy the up-time and down-time constraints by trial and error method

Step 8: Finalize the UC schedule

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### 4. Lambda optimization

The first stage is the optimization of incremental fuel cost parameter  $\lambda$ . In the actual dispatch calculation, the generation scheduling is tested for the inequality constraint violation, and if the constraints are violated, the maximum/minimum values are fixed. The fundamental rule that tells when the optimum has been reached is checked through Kuhn–Tucker conditions. Accordingly, this fixed generation of the particular units is checked

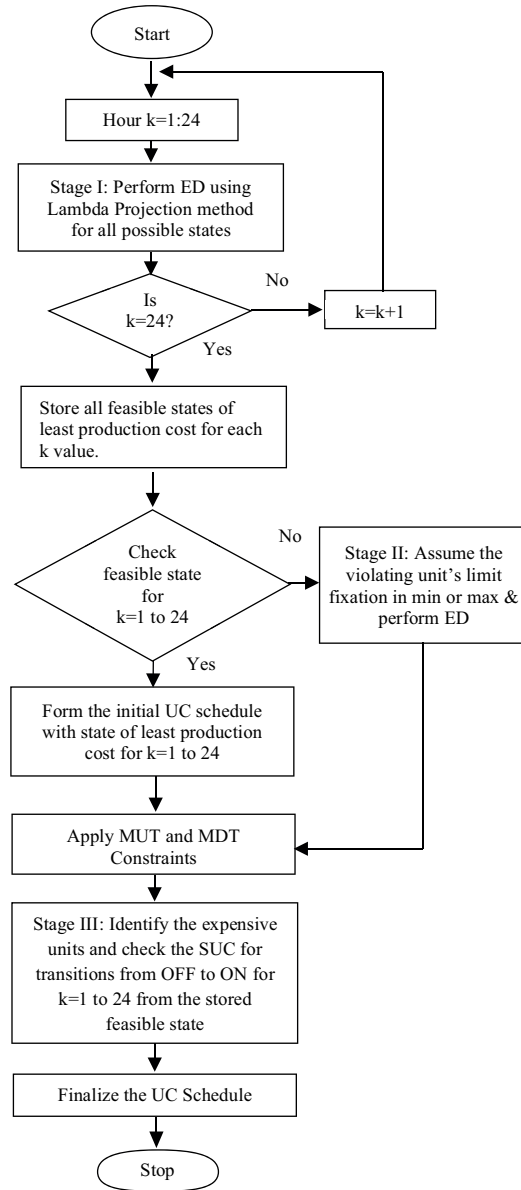


Figure 1. Various stages of optimization algorithm.

for their validity through  $\lambda$  checking. Obviously, in the checking process, in certain cases the fixed unit may not be eligible to form a state to meet the load if it does not satisfy the following Kuhn–Tucker conditions.

$$\begin{aligned} \frac{dF_i}{dPg_i} &= \lambda, \quad Pg_{i,min} < Pg_i < Pg_{i,max} \\ \frac{dF_i}{dPg_i} &\leq \lambda, \quad Pg_i = Pg_{i,max} \\ \frac{dF_i}{dPg_i} &\geq \lambda, \quad Pg_i = Pg_{i,min} \end{aligned} \tag{10}$$

This checking helps to perform the optimal scheduling. Using DP the least-cost state for all loads ranging from first hour to last hour is determined. The second stage of the algorithm checks for the nonavailability of states

for any load. If no state exists for a particular load, the economic dispatch is modified and the least-cost state is identified. In the third stage, while applying the constraints the least-cost state is obtained in two steps: in step I by considering the startup cost coefficients, and in step II by calculating the startup cost for every possible state through algorithmic search using a trial-and-error method. This method of searching the state ends up satisfying both the UC and economic dispatch decisions. After applying MUT and MDT constraints and fixing the expensive units in ON status, the startup cost is calculated for consecutive hours. The required data for the NTPS 7-unit system [14] are given in Table 1. The load data for this system are given in Table 2.

**Table 1.** Unit data of 7-unit system for 24 h [14].

Unit	P <sub>min</sub> MW	P <sub>max</sub> MW	Running cost		
			c <sub>i</sub>	b <sub>i</sub>	a <sub>i</sub>
1	15	60	750	70	0.255
2	20	80	1250	75	0.198
3	30	100	2000	70	0.198
4	25	120	1600	70	0.191
5	50	150	1450	75	0.106
6	50	150	4950	65	0.0675
7	75	200	4100	60	0.074
Unit	Startup cost			Minimum Up time, h	Minimum Down time, h
	Soi	D <sub>i</sub>	E <sub>i</sub>		
1	4250	29.5	10	3	3
2	5050	29.5	10	3	3
3	5700	28.5	10	3	3
4	4700	32.5	9.0	3	3
5	5650	32	9.0	5	5
6	14,100	37.5	4.5	5	5
7	11,350	32	5.5	6	6

**Table 2.** Load data of 7-unit system for 24 h [14].

Hour [h]	Load [MW]	Hour [h]	Load [MW]
1	840	13	545
2	757	14	538
3	775	15	535
4	773	16	466
5	770	17	449
6	778	18	439
7	757	19	466
8	778	20	463
9	770	21	460
10	764	22	434
11	598	23	530
12	595	24	840

The schedule of a 7-unit Indian utility system is obtained, which yields a total cost of Rs 1,58,0269.417. Startup cost is calculated from Eq. (2) based on the unit data available. Table 3 is obtained based on stage I. Since the SUC is very high this schedule is considered the worst case. However, in the proposed method, the total cost is reduced by identifying the high startup cost coefficients and the least possible state using a trial-and-error method.

**Table 3.** Schedule of 7-unit Indian utility system: worst case.

Hour	Schedule	GC (Rs)	SUC (Rs)	TFC (Rs)
1	1111111	86,237.69		86,237.69
2	1111111	77,574.65		77,574.65
3	1111111	79,404.67		79,404.67
4	1011111	79,160.62		79,160.62
5	1011111	78,822.85		78,822.85
6	1011111	79,731.23		79,731.23
7	1011111	77,394.7		77,394.7
8	1111111	79,712.22	34209	113,921.22
9	1111111	78,893.71		78,893.71
10	1111111	78,283.22		78,283.22
11	1001111	60,347.18		60,347.18
12	1001111	60,050.01		60,050.01
13	1000111	54,653.6		54,653.6
14	1000111	53,933.45		53,933.45
15	1000111	53,628		53,628
16	1000111	46,998.65		46,998.65
17	1000111	45,472.35		45,472.35
18	1000111	44,586.24		44,586.24
19	1000111	46,998.65		46,998.65
20	1000111	46,726.62		46,726.62
21	1000111	46,455.93		46,455.93
22	1000111	44,145.99		44,145.99
23	1000111	53,123.13		53,123.13
24	1111111	86,237.69	7487.42	93,725.11
Total cost (Rs) = 1,580,269.47				

The startup cost of the 2nd unit at the 8th hour is high and the 3rd stage of the algorithm searches for the optimal state. The state that obeys the MUT and MDT constraints is checked and the schedule is given in Table 4. If the 2nd unit is kept committed during the 4th, 5th, 6th, and 7th hours instead of decommitting as in the worst case, the startup cost of the 2nd unit at the 8th hour, Rs 34,209, is saved. If the 2nd unit is committed during the 4th, 5th, 6th, and 7th hours, the ON status cost is only Rs 310. Since the total cost is high, the schedule obtained at the end of the first stage is considered as the worst case. The total fuel cost obtained for the schedule at the end of the third stage for hours 4 to 7 is Rs 1,546,331.707/-, realizing a net savings of Rs 33,937.71/-.

**Table 4.** Best state of the schedule for hours 4 to 7 of 7-unit Indian utility system.

Hour	SCHEDULE	SUC (Rs)	TFC (Rs)
4	1111111	-	79,200.05
5	1111111	-	78,893.71
6	1111111	-	79,712.22
7	1111111	-	77,574.66
8	1111111	-	79,712.22
Total fuel cost for 24 h (Rs) = 1,546,331.707/- SUC- Startup cost, TFC- total fuel cost.			



This is considered as the schedule of the best case. The schedule obtained at the end of the third stage is more economical than the schedule obtained at the end of the first stage. The algorithm is checked for the IEEE 10-unit system. The required unit data for the 10-unit generating system are given in Table 5. The load data for this system are given in Table 6.

**Table 5.** Unit data of IEEE 10-unit system for 24 h.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Pmax (MW)	455	455	130	130	162
Pmin (MW)	150	150	20	20	25
a (\$/h)	1000	970	700	680	450
b (\$/MWh)	16.19	17.26	16.60	16.50	19.70
c (\$/MWh <sup>2</sup> )	0.00048	0.00031	0.002	0.00211	0.00398
MUT <sub>i</sub> (h)	8	8	5	5	6
MDT <sub>i</sub> (h)	8	8	5	5	6
Hcost <sub>i</sub> (\$)	4500	5000	550	560	900
Ccost <sub>i</sub> (\$)	9000	10,000	1100	1120	1800
Chour <sub>i</sub> (h)	5	5	4	4	4
Ini State (h)	8	8	-5	-5	-6
	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
Pmax(MW)	80	85	55	55	55
Pmin (MW)	20	25	10	10	10
a (\$/h)	370	480	660	665	670
b (\$/MWh)	22.26	27.74	25.92	27.27	27.79
c (\$/MWh <sup>2</sup> )	0.00712	0.00079	0.00413	0.00222	0.00173
MUT <sub>i</sub> (h)	3	3	1	1	1
MDT <sub>i</sub> (h)	3	3	1	1	1
Hcost <sub>i</sub> (\$)	170	260	30	30	30
Ccost <sub>i</sub> (\$)	340	520	60	60	60
Chour <sub>i</sub> (h)	2	2	0	0	0
Ini State (h)	-3	-3	-1	-1	-1

**Table 6.** Load data of IEEE 10-unit system for 24 h.

Hour [h]	Load [MW]	Hour [h]	Load [MW]
1	700	13	1400
2	750	14	1300
3	850	15	1200
4	950	16	1050
5	1000	17	1000
6	1100	18	1100
7	1150	19	1200
8	1200	20	1400
9	1300	21	1300
10	1400	22	1100
11	1450	23	900
12	1500	24	800

Considering the best state, all units are committed during the 10th hour and the total cost of the schedule increases due to rearranging the commitment order previously set for MUT and MDT constraint satisfaction.

No states exists for the demand of 1450 MW and 1500 MW since the value of the new lambda in the lambda projection method approaches infinity.

The problem persists for higher demands. For example, when demand is 1450 MW, all units are violating the limits according to Kuhn–Tucker conditions.

The calculation of the new lambda is given in the following equation.

$$\lambda_{new} = \frac{[New\ Load + (Sum\ of\ all\ nonviolating\ (b/c)coefficients)]}{[Sum\ of\ nonviolating\ unit's\ (1/c)coefficients]} \tag{11}$$

The value becomes infinite since all units violate the limits for the loads of 1450 MW and 1500 MW. For example, the generation fixed to maximum level and minimum level respectively during any hour is treated as optimal only if it satisfies the following respective conditions.

$$\lambda_{new} \leq \lambda \tag{12}$$

$$\lambda_{new} \geq \lambda \tag{13}$$

This constraint checking helps to produce an optimal generation scheduling. The economic dispatch is performed until the above constraints are met.

Consequently, for the higher loads, if the schedule of the respective unit is fixed for its limit violation, the power balance constraint is not met. This is shown in Table 7. In order to commit the units and satisfy the equality constraints, the algorithm is refined. The economic dispatch is optimized for the state that does not meet the power balance constraint and the units involved in the state are fixed in either of the limits.

**Table 7.** Schedule of infeasible state for loads of 1450 MW and 1500 MW ( $P_{gi} < P_L$ ).

Load (MW)	Generation schedule (MW)										$P_{gi}$ [MW]	Total cost (\$)
	1	2	3	4	5	6	7	8	9	10		
1450	455	455	130	0	25	20	25	10	10	0	1130	25,039.66
	455	455	130	130	162	20	25	10	10	10	1407	31,649.26
1500	455	455	130	130	25	20	25	10	0	0	1250	26,962.4
	455	455	130	130	162	0	25	10	10	10	1387	30,831.21

In this case, since the maximum fixation does not promise the equality constraint satisfaction, the corresponding units that are violating the maximum limits are fixed to minimum generation and the remaining units' schedule is adjusted to obtain the near-optimal solution, which is shown in Table 8.

**Table 8.** Schedule of new feasible states for loads of 1450 MW and 1500 MW ( $P_{gi} = P_L$ ).

Load [MW]	Generation schedule (MW)										$P_{gi}$ [MW]	Total cost (\$)
	1	2	3	4	5	6	7	8	9	10		
1450	455	455	130	130	162	0	53	0	55	10	1450	31,923.69
1500	455	455	130	130	162	0	85	0	55	28	1500	33,316.26

In this subproblem, it is observed that the ramp rate limit is not violated. This is done only for the diverged state. The best optimal state is attached to the schedule that arrived in the first stage and the constraints are applied at the end of the second stage. The algorithm searches for the least startup cost state by discarding the expensive units and the final optimal schedule is obtained.

## 5. Simulation results

MDP using Kuhn–Tucker conditions was implemented in MATLAB and the problem was tested for a 7-unit NTPS system and the IEEE 10-unit system. The test runs are performed with a 1.8 GHz (4GB RAM) Intel Core i3 processor under the Windows 8 operating system. The shut-down cost has been considered as zero for every unit. Based on the schedule obtained at the end of the second stage, the inclusion of the ramp rate is achieved with the modification of states by a backward sequence for satisfying the MUT and MDT constraints. The final UC schedules of the 7-unit Indian utility system and IEEE 10-unit system have the least cost of operation. Total costs of UC schedules for 7- and 10-unit systems are compared with other methods stated in the literature and are given in Tables 9 and 10, respectively.

**Table 9.** Total cost of UC schedule for 7-unit Indian utility system.

Method	TOTAL COST (Rs)
DP approach [14]	1,552,926
FDP approach [14]	1,547,340
MDP – worst	1,580,269.417
MDP – best	1,546,331.707

**Table 10.** Total cost of UC schedule for IEEE 10-unit system.

Method	TOTAL COST (\$)
Ref [15] – HNN	588,750
Ref [21] – GA	610,500
Ref [29] – GA	609,023.69
Ref [30] – GA	591,715
Ref [31] – GA	623,441
MDP – best	581,541.9892

The UC solution for the 7-unit system is obtained by MDP and is given in Table 11. Various stages of the MDP are shown in the flowchart in Figure 1

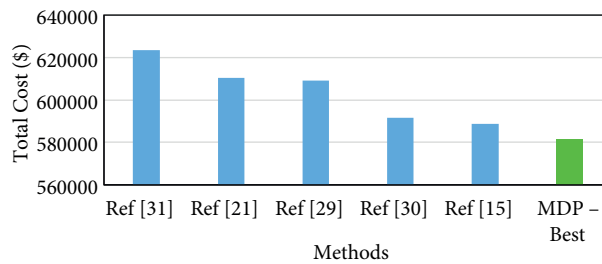
The commitment order converged through the first and third stages without passing through the second stage in the case of the 7-unit system, whereas for the 10-unit system economic dispatch was refined in the second stage in order to achieve the optimal schedule. In both the cases the ramp rate constraint is considered. The total fuel cost of MDP compared with other methods for the 10-unit system and 7-unit Indian utility system is shown in Figures 2 and 3, respectively.

The processing time of MDP is optimal for both the 7-unit and 10-unit system. The processing times of various methods are compared with MDP as shown in Table 12.

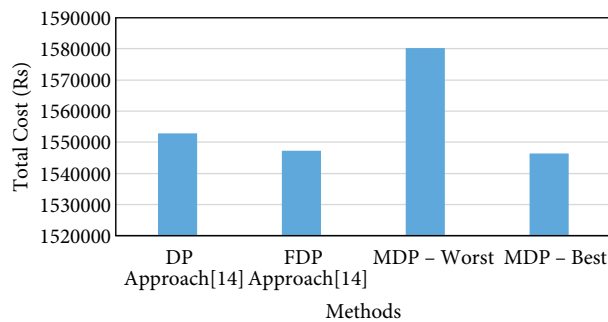
Although some divergence occurs at higher loads for the IEEE 10-unit system, the MDP method provides a more economical solution (using constraint relaxation) than the other methods stated in literature. As shown in Table 11, the MDP produces the least cost of operation with less processing time. The final UC schedule for the IEEE 10-unit system that yields the least cost of operation is shown in Table 13.

**Table 11.** Final best schedule of 7-unit Indian utility system – best case.

Hour	Schedule	GC (Rs)	SUC (Rs)	TFC (Rs)
1	1111111	86,237.69	-	86,237.69
2	1111111	77,574.65	-	77,574.65
3	1111111	79,404.67	-	79,404.67
4	1111111	79,160.62	-	79,200.05
5	1111111	78,822.85	-	78,893.71
6	1111111	79,731.23	-	79,712.22
7	1111111	77,394.7	-	77,574.66
8	1111111	79,712.22	-	79,712.22
9	1111111	78,893.71	-	78,893.71
10	1111111	78,283.22	-	78,283.22
11	1001111	60,347.18	-	60,347.18
12	1001111	60,050.01	-	60,050.01
13	1000111	54,653.6	-	54,653.6
14	1000111	53,933.45	-	53,933.45
15	1000111	53,628	-	53,628
16	1000111	46,998.65	-	46,998.65
17	1000111	45,472.35	-	45,472.35
18	1000111	44,586.24	-	44,586.24
19	1000111	46,998.65	-	46,998.65
20	1000111	46,726.62	-	46,726.62
21	1000111	46,455.93	-	46,455.93
22	1000111	44,145.99	-	44,145.99
23	1000111	53,123.13	-	53,123.13
24	1111111	86,237.69	7487.42	93,725.11
Total cost (Rs) = 1,546,331.707				



**Figure 2.** Total cost comparison chart of 10-unit system.



**Figure 3.** Total cost comparison chart of 7-unit Indian utility system.

**Table 12.** Comparison of CPU times among DP, FDP, and GA.

METHOD	Average time (s)
Ref [14] – DP	180
Ref [14] – FDP	158
Ref [29] – GA	677
MDP – NTPS 7-unit	18.67
MDP – IEEE 10-unit	115

**Table 13.** Final best new schedule of MDP in 10-unit 24-h case.

Load [MW]/unit	1	2	3	4	5	6	7	8	9	10	Unit schedule	GC (\$)	SUC (\$)	TFC (\$)
700	455	150	95	0	0	0	0	0	0	0	1110000000	14,326.85	550	14,876.85
750	455	150	62.27	82.73	0	0	0	0	0	0	1111000000	15,832.72	1120	16,952.72
850	455	150	115	130	0	0	0	0	0	0	1111000000	17,527.91	0	17,527.91
950	455	235	130	130	0	0	0	0	0	0	1111000000	19,261.5	0	19,261.5
1000	455	285	130	130	0	0	0	0	0	0	1111000000	20,132.56	0	20,132.56
1100	455	375	130	130	0	0	0	10	0	0	1111000100	22,623.99	60	22,683.99
1150	455	425	130	130	0	0	0	10	0	0	1111000100	23,499.39	0	23,499.39
1200	455	455	130	130	0	0	0	10	10	10	1111000111	25,911.37	120	26,031.37
1300	455	455	130	130	85	0	25	0	10	10	1111101011	28,319	2320	30,639
1400	455	455	130	130	162	0	25	0	33	10	1111101011	30,541	0	30,541
1450	455	455	130	130	162	0	53	0	55	10	1111101011	31,923.69	0	31,923.69
1500	455	455	130	130	162	0	85	0	55	28	1111101011	33,316.26	0	33,316.26
1400	455	455	130	130	162	0	25	0	33	10	1111101011	30,541	0	30,541
1300	455	455	130	130	85	0	25	0	10	10	1111101011	28,319	0	28,319
1200	455	440	130	130	25	20	0	0	0	0	1111110000	24,605.73	340	24,945.73
1050	455	420	0	130	25	20	0	0	0	0	1101110000	21,363.4	0	21,363.4
1000	455	370	0	130	25	20	0	0	0	0	1101110000	20,488.16	0	20,488.16
1100	455	455	0	130	25	0	25	10	0	0	1101101100	23,252.55	580	23,832.55
1200	455	455	0	130	115	0	25	10	0	10	1101101101	26,023.74	60	26,083.74
1400	455	455	0	130	162	0	70.33	55	55	17.67	1101101111	31,825.92	60	31,885.92
1300	455	455	130	130	85	0	25	0	10	10	1111101011	28,319	1100	29,419
1100	455	375	130	130	0	0	0	10	0	0	1111000100	22,623.99	60	22,683.99
900	455	305	130	0	0	0	0	10	0	0	1110000100	18,540.37	0	18,540.37
800	455	215	130	0	0	0	0	0	0	0	1110000000	16,052.85	0	16,052.85
												575,172	6370	581,542

GC- Generation cost, SUC- startup cost, TFC- total fuel cost.

## 6. Conclusion

In this paper a MDP approach for solving the UCP was proposed. MDP for the UCP is proved to be economical in terms of cost and CPU time. MDP yielded the optimal value of the schedule after many stages of refining and checking procedures. The problem also considers the unit ramping constraint. The proposed algorithm refined the divergence in the case of the 10-unit system and it was observed that for a 7-unit Indian utility system the final UC passed through all stages without any divergence. The basic advantage of the proposed algorithm is the high speed of convergence. Results reveal that the proposed MDP approach is very effective in reaching the optimal schedule for the short-term unit commitment problem. Attempts are also being made to hybridize the FDP approach with intelligent techniques, which could improve the solution quality and computational time.

## References

- [1] Wood AJ, Wollenberg B. Power Generation Operation and Control. New York, NY, USA: John Wiley, 1983.
- [2] Baldick R. The generalized unit commitment problem. IEEE T Power Syst 1995; 10: 465-475.

- [3] Tong SK, Shahidehpour SM, Ouyang Z. A heuristic short-term unit commitment. *IEEE T Power Ap Syst* 1991; 6: 1210-1216.
- [4] Baldwin CJ, Dale KM, Ditttrich RF. A study of the economic shutdown of generating units in daily dispatch. *AIEE Transactions on Power Apparatus and Systems* 1960; 78: 1272-1284.
- [5] Happ HH, Johnson RC, Wright WJ. Large scale hydro-thermal unit commitment method and results. *IEEE T Power Ap Syst* 1971; 90: 1373-1383.
- [6] Merlin A, Sandrin P. A new method for unit commitment at Electricite de France. *IEEE T Power Ap Syst* 1983; 102: 1218-1225.
- [7] Muckstadt JA, Wilson RC. An application of mixed-integer programming duality to scheduling thermal generating systems. *IEEE T Power Syst* 1968; 87: 1968-1978.
- [8] Cohen AI, Yoshimura MA. Branch and bound algorithm for unit commitment. *IEEE T Power Ap Syst* 1983; 102: 444-451.
- [9] Liu C, Shahidehpour M, Wu L. Extended Benders decomposition for two-stage SCUC. *IEEE T Power Syst* 2010; 25: 1192-1194.
- [10] Bellman RE. *Dynamic Programming Book*. Princeton, NJ, USA: Princeton University Press, 1957.
- [11] Lowery PG. Generating unit commitment by dynamic programming. *IEEE T Power Ap Syst* 1966; 5: 422-426.
- [12] Snyder WL, Powel HD, Rayburn JC. Dynamic programming approach to unit commitment. *IEEE T Power Syst* 1987; 2: 339-350.
- [13] Juste KA, Kita H, Tanaka E, Hasegawa J. An evolutionary programming solution to the unit commitment problem. *IEEE T Power Syst* 1999; 14: 1452-1459.
- [14] Senthil Kumar S, Palanisamy V. A hybrid fuzzy dynamic programming approach to unit commitment. *Journal of the Institution of Engineers (India)* 2008; 88: 3-9.
- [15] Senthil Kumar S, Palanisamy V. A dynamic programming based fast computation Hop?eld neural network for unit commitment and economic dispatch. *Electr Pow Syst Res* 2006; 77: 917-925.
- [16] Sasaki H, Watanabe M, Yokoyama R. A solution method of unit commitment by artificial neural networks. *IEEE T Power Syst* 1992; 7: 974-981.
- [17] Ouyang Z, Shahidehpour SM. A hybrid artificial neural network – dynamic programming approach to unit commitment. *IEEE T Power Syst* 1992; 7: 236-242.
- [18] Mantawy AH, Abdel-Magid YL, Selim SZ. A simulated annealing algorithm for unit commitment. *IEEE T Power Syst* 1998; 13: 197-204.
- [19] Xiaomin B, Shahidehpour SM, Erkeng Y. Constrained unit commitment by using tabu search algorithm. In: *Proceedings of the International Conference on Electrical Engineering*; 1996. pp. 1088-1092.
- [20] Mantawy AH, Abdel-Magid YL. Unit commitment by tabu search. *IEE P-Gener Transm D* 1998; 145: 56-64.
- [21] Kazarlis SA, Bakirtzis AG, Petridis V. A genetic algorithm solution to the unit commitment problem. *IEEE T Power Syst* 1996; 11: 83-92.
- [22] Salam S. Unit commitment solution methods. *Proc Wrld Acad Sci E* 2007; 27:2070-3740.
- [23] Sheble GB, Fahd GN. Unit commitment literature synopsis. *IEEE T Power Syst* 1994; 9:128-135.
- [24] Padhy NP. Unit commitment-a bibliographical survey. *IEEE T Power Syst* 2004; 19: 1196-1205.
- [25] Ayoub AK, Patton AD. Optimal thermal generating unit commitment. *IEEE T Power Ap Syst* 1971; 90: 1752-1756.
- [26] Van den Bosch PPJ, Honderd G. A solution of the unit commitment problem via decomposition and dynamic programming. *IEEE T Power Ap Syst* 1985; 104: 1684-1690.
- [27] Pang CK, Sheble GB, Albuyeh F. Evaluation of dynamic programming based methods and multiple area representation for thermal unit commitments. *IEEE T Power Ap Syst* 1981; 100: 1212-1218.

- [28] Hobbs WJ, Hermon G, Warner S, Sheblé GB. An enhanced dynamic programming approach for unit commitment. *IEEE T Power Syst* 1988; 3: 1201-1205.
- [29] Swarup KS, Yamashiro S. Unit commitment solution methodology using genetic algorithm. *IEEE T Power Syst* 2002; 17: 87-91.
- [30] Ganguly D, Sarkar V, Pai J. A new genetic approach for solving the unit commitment problem. In: *International Conference on Power System Technology-POWERCON 2004*; 21–24 November 2004; Singapore.
- [31] Marifield TT, Sheble GB. Genetic based unit commitment algorithm. *IEEE T Power Syst* 1996; 11: 1359-1370.