Nonlinear acoustic echo cancellation using an adaptive Hammerstein block structure based on a generalized basis function

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Abstract: We investigated adaptive algorithms for a Hammerstein block structure in which a static nonlinear block and dynamic linear block are cascaded. The approach considered here is to use generalized orthonormal basis functions in a Hammerstein block structure by using fixed pole filter banks. We applied the normalized least mean square approach to the developed adaptive algorithm in order to acquire Hammerstein block structure parameters. Performance comparison of the proposed approach was investigated considering convergence speed and parametric complexity for acoustic echo cancellation application. The results indicated that in the developed algorithm along with appropriate selection of fixed poles, the algorithm convergences faster and less parametric complexity is provided when compared to direct adaptive Hammerstein algorithms with IIR and FIR linear blocks.

Key words: Nonlinear adaptive filters, Hammerstein block structure, generalized orthonormal basis functions, acoustic echo cancellation, convergence analysis

1. Introduction
The identification of nonlinear systems has attracted considerable interest in system theory, control design, and signal processing [1]. The reason for such an interest is that many natural systems or man-made systems are inherently nonlinear. Since there are a wide variety of nonlinear system structures, several approaches have been developed in order to model the most appropriate input–output relation. Two of these approaches are black box and gray box techniques. The block structured models such as the Hammerstein model (a nonlinear memoryless system is followed by a linear dynamic one) and the Wiener model (the same subsystems are connected in reverse order) have been popular in the gray box approach. The black box approach includes Volterra model representation [2]. In addition to model selection, many nonlinear applications require adaptive algorithms such as acoustic echo cancellation [3]. In the present study we developed a new adaptive Hammerstein filter algorithm by using generalized basis functions and investigated its performance in an acoustic echo cancellation application.

Echo cancellation is a widespread technology in communication systems. The echo path is not well approximated by a linear filter because it has a mixture of linear and nonlinear characteristics. Therefore, over the years the problem of echo cancellation has been addressed by means of multichannel [4,5] and cascade structures [6] based upon memoryless Hammerstein modeling. An acoustic echo canceller electrically models the path from the loudspeaker to the microphone by a nonlinear adaptive filter. By exciting the adaptive filter with

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the signal from the far-end, an echo replica is generated. The echo replica is subtracted from the microphone signal, resulting in echo-cancelled speech for transmission. The reflection of acoustic signals inside an enclosed environment is almost linearly distorted, but a loudspeaker introduces nonlinearities. These nonlinearities limit the performance of any linear cancellation algorithm [3].

Some methods have been studied for nonlinear echo cancellation. Volterra series-based filters [7] can represent a large class of nonlinear systems but imply a high computational complexity. Birkett and Goubran proposed a neural network-based approach with a cascade structure [8]. Stenger and Kellerman reported a new faster perspective for estimating high order nonlinearity that does not need an extra reference microphone [9]. A NARMAX structure method was also proposed for nonlinear echo cancellation that is a general parametric model but needs a pre-identification procedure [10]. Nollet and Jones proposed a nonpolynomial Wiener–Hammerstein model with a saturation nonlinear memoryless function [11]. However, this kind of function cannot model a large class of nonlinearities. In [12], a functional link adaptive filter was introduced to model nonlinear echo path based on a Hammerstein structure. Vaernbergh et al. used kernel-based identification of the Hammerstein system [13].

In this study, we developed an adaptive Hammerstein algorithm by using generalized orthonormal basis functions in Section 2, expanding the preliminarily results presented in [14]. We also adopted an adaptive algorithm for Hammerstein system as in [15] to apply it to the nonlinear acoustic echo cancellation problem. We note that in this algorithm nonlinear block output is not available for identification but one can estimate that adaptively. Section 3 presents adaptive Hammerstein algorithms. In addition to the results in [14], we aimed to make a comparison about convergence, parametric complexity, and algorithm simplicity given as an example in Section 4. We also aimed to see the behavior of adaptive algorithms for nonlinear acoustic echo cancellation application in Section 5. Conclusions are drawn in Section 6.

2. Hammerstein block structure model using a generalized basis function

A Hammerstein block structure as shown in Figure 1 consists of a static nonlinearity connected by a dynamic linear block. Nonlinear block, $N[.]$, is defined by polynomial coefficients and linear block is defined by either FIR or IIR filter coefficients. In Figure 1, $u(n)$ is the nonlinear block input, $x(n)$ is the input of the linear block, and $y(n)$ is the output of the linear block. In addition, $v(n)$ is the additive noise of the system.

![Figure 1](image_url)

We can model a memoryless nonlinear block by

$$x(n) = \sum_{l=1}^{L} \gamma_l u^l(n), \quad (1)$$
where \( \gamma_l, l = 1, \ldots, L \) are polynomial nonlinearity coefficients of order \( L \). The transfer function of linear block \( H(q^{-1}) \) is given by

\[
H(q^{-1}) = \frac{1 + \sum_{k=1}^{M} b_k q^{-k}}{1 + \sum_{k=1}^{N} a_k q^{-k}} = \sum_{k=1}^{N} c_k G_k(q^{-1})
\]  (2)

and \( a_k, k = 1, \ldots, N \) and \( b_k, k = 1, \ldots, M \) are linear block numerator and denominator polynomial coefficients, respectively. \( q^{-1}, q \) is the shift operator, i.e. \( q^{-1} x(n) = x(n-1), qx(n) = x(n+1) \). Here in (2) \( c_k, k = 1, \ldots, N \) are weight parameters for generalized orthogonal basis functions of \( G_k(q^{-1}) \) defined in Definitions 2.1–2.2. Using (1) and (2), output of the Hammerstein system is given as

\[
y(n) = - \sum_{k=1}^{N} a_k y(n-k) + x(n) + \sum_{k=1}^{M} b_k x(n-k)
\]  (3)

\[
y(n) = - \sum_{k=1}^{N} a_k y(n-k) + \sum_{l=1}^{L} \gamma_l u^l(n) + \sum_{k=1}^{M} b_k x(n-k)
\]  (4)

In matrix form (4) can be written as

\[
y(n) = \varphi^T(n) \theta,
\]  (5)

where the regressor vector \( \varphi(n) \) of dimension \((L + M + N) \times 1\) and the parameter vector \( \theta \) of dimension \((L + M + N) \times 1\) are given respectively by

\[
\varphi(n) = [u(n), \ldots, u^L(n), x(n-1), \ldots, x(n-M), -y(n-1), \ldots, -y(n-N)]^T \]  (6)

\[
\theta = [\gamma_1, \ldots, \gamma_L, b_1, \ldots, b_M, a_1, \ldots, a_N]^T. \]  (7)

The regressor vector is dependent on both input–output and output of the nonlinear block structure. The parameter vector contains linear and nonlinear block parameters. Note that \( x(n) \) is not available for identification but we may estimate its previous values using adaptive approaches as in [7].

Linear block \( H(q^{-1}) \) has an IIR structure as given in (2). If \( a_k \) are all zero in (2), then it will be FIR structured. As a result in [1,16–18], we may represent or well approximate \( H(q^{-1}) \) rational function by using weighted sum of sequences of fixed pole basis functions, \( G_k(q^{-1}) \). These subsystems have generalized orthonormal basis functions composed of real and/or complex-conjugate fixed poles [16]. We specify the orthonormal basis functions in [19] using the approach taken in [17,18] for either real poles or complex conjugate pole pairs.

**Definition 2.1** The sequence of a fixed real poles basis function is defined via

\[
G_k(q) = \sqrt{\frac{1 - \alpha_k^2}{(q - \alpha_k)}} \prod_{i=1}^{k-1} \frac{1 - \alpha_i q}{(q - \alpha_i)}
\]  (8)

for \( k = 1, \ldots, N \) when the poles are real numbers inside the unit circle \( (G_0(q) = 1) \).
The sequence of a fixed complex pole basis function is defined via

\[ G_{2k-1}(q) = \frac{\sqrt{(1 + \beta_k)(1 + \beta^*_k)(1 - \beta_k\beta^*_k)}}{2} \frac{(q - 1)}{(q - \beta_k)(q - \beta^*_k)} \prod_{i=1}^{k-1} \frac{(1 - \beta_i q)(1 - \beta^*_i q)}{(q - \beta_i)(q - \beta^*_i)} \]  

(9)

\[ G_{2k}(q) = \frac{\sqrt{(1 - \beta_k)(1 - \beta^*_k)(1 - \beta_k\beta^*_k)}}{2} \frac{(q + 1)}{(q - \beta_k)(q - \beta^*_k)} \prod_{i=1}^{k-1} \frac{(1 - \beta_i q)(1 - \beta^*_i q)}{(q - \beta_i)(q - \beta^*_i)} \]  

(10)

for \( k = 1, \ldots, N/2 \), where the complex conjugate pole pairs, \((\beta_k, \beta^*_k)\), are inside the unit circle \((G_0(q) = 1)\).

Note that if \( \alpha_k \) are a simple real pole in (8) then \( G_k(q) \) are called Laguerre functions [17] and also if \( \beta_k \) are a simple complex-conjugate pole pair in (9)–(10), then \( G_k(q) \) are called Kautz functions [18].

We now write input and output relations of Hammerstein block structure by using generalized orthonormal basis functions defined with fixed pole filter banks such as

\[ y(n) = \sum_{k=1}^{N} c_k G_k(q^{-1}) x(n) = \sum_{k=1}^{N} \sum_{l=1}^{L} \gamma_l c_k G_k(q^{-1}) u_l(n). \]  

(11)

In the matrix form, we may write this relation as

\[ y(n) = \theta_\gamma^T \varphi^T(n) \theta_c, \]  

(12)

where the parameter vectors \( \theta_\gamma \) and \( \theta_c \) include nonlinear polynomial coefficients and weighted fixed pole filter bank coefficients and are given by

\[ \theta_\gamma = [\gamma_1, \gamma_2, \cdots, \gamma_L]^T, \quad \theta_c = [c_1, c_2, \cdots, c_N]^T, \]  

(13)

and \( \varphi(n) \) is the regressor matrix of dimension \( N \times L \), which includes the filtered input by using fixed pole filters given by

\[ \varphi(n) = \begin{bmatrix} G_1(q^{-1})u(n), & G_1(q^{-1})u^2(n) & \cdots & G_1(q^{-1})u^L(n) \\ G_2(q^{-1})u(n), & G_2(q^{-1})u^2(n) & \cdots & G_2(q^{-1})u^L(n) \\ \vdots & \vdots & \ddots & \vdots \\ G_N(q^{-1})u(n), & G_N(q^{-1})u^2(n) & \cdots & G_N(q^{-1})u^L(n) \end{bmatrix}. \]  

(14)

Note that both representations of the nonlinear Hammerstein block structure in (3)–(7) and in (11)–(14) are linear in the parameters. Hence, one can apply a gradient descent-type NLMS [20] approach to estimate adaptively not only nonlinear block parameters but also weight parameters of fixed pole filter banks.

3. Adaptive Hammerstein algorithms

We developed adaptive algorithms using a Hammerstein block structure based on the least mean squares (or normalized least mean squares) approach. This approach is widely used due to its simple structure [20]. We first show the development of the adaptive approach in [7] that we called a direct adaptive Hammerstein algorithm with IIR linear block (HamIIR) and with FIR linear block (HamFIR). Then we developed an adaptive
Hammerstein algorithm with generalized orthonormal basis function (HamOBF). We also suggest basis function selection or fixed pole selection.

**Algorithm 1 (HamIIR, HamFIR):** We can apply the NLMS approach to the Hammerstein block structure given by (3)–(7) so that

\[
\hat{\theta}(n+1) = \hat{\theta}(n) + \frac{\mu}{\varepsilon + \hat{\varphi}^T(n)\hat{\varphi}(n)}\hat{\varphi}^T(n)e(n)
\]

(15)

\[
e(n) = y(n) - \hat{y}(n) = y(n) - \hat{\varphi}^T(n)\hat{\theta}(n)
\]

(16)

\[\hat{\varphi}(n) = [u(n), \ldots, u^L(n), \hat{x}(n-1), \ldots, \hat{x}(n-M), -y(n-1), \ldots, -y(n-N)]^T,\]

(17)

where \(\hat{\varphi}(n)\) is the estimate of the regressor vector in (6), \(\hat{\theta}(n)\) is the estimate of the parameters vector in (7) at time \(n\), \(\mu\) is the step-size parameter, and \(\varepsilon\) is a positive small constant to deal with the problems of regressor vector normalization. This adaptive algorithm aims to obtain linear block numerator and denominator parameters and polynomial nonlinear block parameters directly. The HamFIR algorithm does not include previous value of output \(y(n)\) in (17) since \(a_k\) are all zero in (2).

**Algorithm 2 (HamOBF)** In this study, considering slow adaptation, we derive a generalized basis function-based adaptive Hammerstein algorithm using (11)–(14). We apply the NLMS approach to get the nonlinear coefficients and weighted parameters of generalized orthonormal basis functions such that

\[
\hat{\theta}_c(n+1) = \hat{\theta}_c(n) + \frac{\mu_c}{\varepsilon + \hat{\varphi}_c^T(n)\hat{\varphi}(n)}\phi^T(n)\hat{\theta}_c(n)e(n)
\]

(18)

\[
\hat{\theta}_\gamma(n+1) = \hat{\theta}_\gamma(n) + \frac{\mu_\alpha}{\varepsilon + \hat{\varphi}_\gamma^T(n)\hat{\varphi}(n)}\phi(n)\hat{\theta}_c(n)e(n)
\]

(19)

\[e(n) = y(n) - \hat{y}(n)
\]

(20)

\[\hat{y}(n) = \phi^T(n)\begin{bmatrix} \hat{\theta}_c(n) \\ \hat{\theta}_\gamma(n) \end{bmatrix}, \quad \hat{\varphi}(n) = \frac{1}{2} \begin{bmatrix} \phi(n)\hat{\theta}_\gamma(n) \\ \phi^T(n)\hat{\theta}_c(n) \end{bmatrix}\]

(21)

and \(\hat{\theta}_\gamma(n), \hat{\theta}_c(n)\) are estimates of the parameter vectors given the values of estimated nonlinear and linear parameters at time \(n\), respectively.

Selection of the positions of fixed poles is important for the performance of the algorithm developed by (18)–(21). According to this, an adaptive algorithm can be used as in [16] or fixed poles can be selected by the Prony approach [1,20] if the prior knowledge about the linear block impulse response is known (or well estimated).

**4. Convergence example for adaptive Hammerstein algorithms**

To evaluate the performance of the adaptive Hammerstein algorithms, we present simulation results obtained for a nonlinear Hammerstein structure as in Figure 1. Here we first investigate all three algorithms’ performance considering speed of convergence and parametric complexity. In the first experiment, the Hammerstein system has been modeled by the cascade of a third-order memoryless polynomial, followed by a second-order IIR linear
The input signal $u(n)$ of the Hammerstein system that is white Gaussian with zero mean is applied to a nonlinear block to get internal signal $x(n)$ as

$$x(n) = 0.84u(n) + 0.01u^2(n) - 0.02u^3(n).$$  \hspace{1cm} (22)

The output $y(n)$ of the Hammerstein system is expressed as

$$y(n) = 1.2y(n-1) - 0.5y(n-2) + x(n) + 0.8x(n-1).$$  \hspace{1cm} (23)

We also add zero mean i.i.d. random noise to output data to make SNR 60 dB. 25,000 samples are used for all of the algorithms. We aimed to see the convergence of the adaptive algorithms and so all the step-size parameters are chosen as 0.05.

In all approaches, third-order polynomial nonlinearity ($L = 3$) was selected for the nonlinear block. For the linear block, both HamOBF and HamIIR have a second-order IIR filter but HamFIR has a 25th order FIR filter. For the selection of the two fixed poles in the HamOBF, the Prony approach is used and fixed poles are obtained by the impulse response knowledge of the linear block. Here parameter complexity becomes that there are 7 parameters in both the HamOBF and HamIIR approaches and 28 parameters in the HamFIR approach.

To show how the estimates behave, we calculated the squared error that is the average of 50 Monte Carlo simulations. The error is plotted in Figure 2. This figure shows that the squared error goes to additive noise level (nearly $10^{-5}$) in all adaptive algorithms. Although both HamOBF and HamIIR have the same parametric complexity, HamOBF converges at nearly 3000 samples while HamIIR converges at nearly 9000 samples. Moreover, HamFIR converges at nearly 13,000 samples, which suffers slow convergence due to parametric complexity. Even though the bigger step-size parameters increase convergence speed, we aim here to compare all adaptive algorithms. Figure 3 shows input–output relations of the memoryless nonlinear block and impulse responses of the linear block for the true Hammerstein system and estimates of the adaptive algorithms. Parameter estimates are calculated after convergence of the adaptive algorithms. Then they are averaged for 50 Monte Carlo simulations. Impulse response of HamOBF and HamIIR is calculated in order to compare HamFIR. The estimates are very close to true values. We selected the number of parameters for HamOBF and HamIIR based on the true Hammerstein system but we used the memory of the dynamic linear system for HamFIR.

5. Acoustic echo cancellation

We conducted a performance comparison of utilized adaptive Hammerstein algorithms with an application of a room acoustic echo problem encountered during a teleconference as shown in Figure 4. The echo effect will be cancelled by obtaining the acoustic echo path model parameters using a nonlinear adaptive echo canceller. By exciting the adaptive echo canceller with the signal from the far-end, an echo replica is generated and is subtracted from the microphone signal. Finally, we get echo-cancelled speech for transmission (near-end speech). The reflection of acoustic signals inside the room is linearly distorted. On the other hand, the loudspeaker introduces nonlinearities and so we use algorithms based on the Hammerstein block structure given in (15)–(17) (HamIIR) and suggested algorithm given in (18)–(21) (HamOBF). In the developed algorithms, HamFIR has all poles fixed at zero and HamOBF has real and/or complex conjugate fixed poles. Then we apply these for the echo cancellation application.
**Figure 2.** Convergence of the adaptive Hammerstein algorithms; performance comparison in terms of squared error using HamOBF, HamIIR, and HamFIR.

**Figure 3.** True and estimated input vs. outputs of nonlinear block as well as linear block impulse response coefficients using HamOBF, HamIIR, and HamFIR algorithms.
In Figure 4, nonlinear block $N[.]$ is defined by a sigmoid function as

$$x(n) = N[u(n)] = \left( \frac{2}{1 + \exp(-\nu u(n))} - 1 \right) \chi,$$

where $\nu = 1$ and $\chi = 2$. The acoustic impulse response $h(n)$ of linear block $H(q^{-1})$ was randomly generated by an exponentially decayed random signal filtered with a second-order lowpass Butterworth filter that has 0.2 cut-off frequency. Impulse response $h(n)$ was limited to 512 taps considering sampling frequency of 8 kHz. Near-end and far-end speech signals were used from real telephone communication. Background noise is generated as zero mean white noise and its variance is set to obtain a signal to noise ratio of 60 dB at the microphone’s location. We aimed to see the echo cancellation behavior of the adaptive algorithms; all the step-size parameters are chosen as 0.005.

In all approaches, third-order polynomial nonlinearity ($L = 3$) was selected for the nonlinear block. For the linear block, both HamOBF and HamIIR used a 10th order IIR filter but HamFIR used a 512th order FIR filter. For the selection of the two fixed poles in the HamOBF, the Prony approach is used and fixed poles are obtained by room acoustic impulse response knowledge. Here parameter complexity becomes that there are 23 parameters in both the HamOBF and HamIIR approaches and 515 parameters in the HamFIR approach.

Figures 5a–5c show far-end, near-end, and noisy microphone signals, respectively. Cancelled echo signals of HamOBF, HamIIR, and HamFIR can be seen in Figures 5d–5f, respectively. In the HamOBF algorithm, the echo effect is cancelled until 8 s. In the HamIIR algorithm, it takes only nearly 15 s but in the HamFIR algorithm it takes the longest time due to the parametric complexity. It is possible to obtain faster convergence by choosing appropriate fixed pole positions. The convergence speed gets slower when the parametric complexity increases.

The performance measure is the echo return loss enhancement (ERLE), defined as

$$ERLE(n)_{dB} = 10 \log_{10} \left( \frac{d^2(n)}{e^2(n)} \right).$$

Figure 6 shows the performance comparison of HamOBF, HamIIR, and HamFIR for an almost 33 s teleconference conversation in terms of ERLE. HamOBF has better performance than the other algorithms. Randomly
Figure 5. Acoustic echo cancellation (a) far-end speech, (b) near-end speech, and (c) microphone signal. Performance comparison between (d) HamOBF, (e) HamIIR, and (f) HamFIR algorithms in terms of acoustic echo canceller outputs.

Figure 6. Performance comparison in terms of ERLE between HamOBF, HamIIR, and HamFIR algorithms in the case of microphone input.
generated acoustic room impulse response and its estimates using HamOBF, HamIIR, and HamFIR can be seen in Figure 7. The estimates are very close to the true values.

![Figure 7](image)

**Figure 7.** True and estimated impulse response coefficient of the room impulse response using HamOBF, HamIIR, and HamFIR algorithms.

6. Conclusion

In this study, an adaptive Hammerstein algorithm was developed by using generalized orthonormal basis functions. In the developed algorithm, fixed pole basis functions were used and it is possible to obtain these real and/or complex poles by virtue of a priori knowledge about the system. We used Hammerstein block structure-based adaptive algorithms to cancel the echo effect for teleconference application. Since the signal between nonlinear and linear block is not available for echo cancellation application we can only estimate that adaptively. We on the other hand developed an adaptive algorithm not necessary to estimate the mid-signal. It can be concluded that the suggested Hammerstein algorithm provides advantages in parametric complexity and convergence rate according to the other adaptive Hammerstein algorithms. Improvement in performance can be easily seen by choosing appropriate fixed pole positions. From the analysis of this paper, it is demonstrated that HamOBF has the ability of faster convergence, produces equal or better quality, and accounts for equal or less parameter complexity compared to HamIIR and HamFIR. Thus, the developed HamOBF algorithm should be given priority when identifying a nonlinear Hammerstein system.

References


