Analysis of reconstruction performance of magnetic resonance conductivity tensor imaging (MRCTI) using simulated measurements

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Abstract: Magnetic resonance conductivity tensor imaging (MRCTI) was proposed recently to produce electrical conductivity images of anisotropic tissues. Similar to magnetic resonance electrical impedance tomography (MREIT), MRCTI uses magnetic field and boundary potential measurements obtained utilizing magnetic resonance imaging techniques. MRCTI reconstructs tensor images of anisotropic conductivity whereas MREIT reconstructs isotropic conductivity images. In this study, spatial resolution and linearity of five recently proposed MRCTI algorithms are evaluated using simulated measurements gathered from three different computer models. The results show that all five algorithms have quite similar reconstruction performances. Since the AB²S algorithm is easier to apply compared to the other four algorithms it can be said to be the best algorithm among the five algorithms.

Key words: Anisotropic conductivity, electrical impedance, imaging, magnetic resonance, reconstruction

1. Introduction

In magnetic resonance electrical impedance tomography (MREIT), the object to be imaged is probed by an electrical current synchronized with a magnetic resonance (MR) pulse sequence [1]. The probing current is applied via surface electrodes. Then the induced magnetic field distribution throughout the field of view (FOV) is measured by using magnetic resonance imaging (MRI) techniques. Surface potential values are also measured. Distribution of the current density can be reconstructed by means of Ampere’s law using magnetic field measurements induced by at least two orthogonal current injection patterns. Since only the parallel component of the induced magnetic field to the main MR imaging field can be measured, to measure the required orthogonal magnetic field components the object is rotated inside the MR scanner to align two different axes with the main MR field. These magnetic field measurements or the current density distribution calculated from the measured magnetic fields together with boundary potential measurements are given to MREIT algorithms as input and conductivity images are reconstructed uniquely. Various MREIT algorithms have been proposed in the literature [1–9]. If an MREIT algorithm uses magnetic field measurements, it is a Type-1 algorithm. On the other hand, Type-2 algorithms use current density measurements for reconstruction.

In the biomedical field, electrical conductivity has great importance for the accurate solutions of bioelectric field problems and proper modeling of biological tissues. MREIT reveals the electrical conductivity distribution within a conducting region if the region has MRI active nuclei [10]. However, it is important to note that

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several biological tissues such as brain, heart muscle, and other striated muscle tissues have anisotropic electrical conductivities [11]. De girmenci and Eyüboğlu [12] investigated the effect of reconstructing an object having anisotropic conductivity by using an isotropic MREIT algorithm and showed the erroneous results. Seo et al. [13] introduced a Type-1 algorithm to reconstruct anisotropic conductivity images where conductivity is expressed as a $3 \times 3$ positive-definite symmetric matrix and the solution of this matrix is obtained iteratively. Another algorithm for magnetic resonance conductivity tensor imaging (MRCTI) was proposed by De girmenci and Eyüboğlu [14]. This algorithm is based on construction of equipotential lines using measured current density distribution in the FOV; therefore, it is a Type-2 algorithm. Recently, De girmenci and Eyüboğlu proposed several algorithms for MRCTI [15].

In this study, five MRCTI algorithms proposed by De girmenci and Eyüboğlu [14,15] are evaluated based on their imaging properties such as spatial resolution and linearity using simulated measurements from numerical phantoms. Results of all algorithms are also compared with each other.

2. Methods
2.1. Reconstruction algorithms
In MRCTI reconstruction, the aim is to reconstruct the conductivity tensor from measured magnetic flux density or current density distributions. In two dimensional (2-D) MRCTI, the conductivity tensor is expressed as $\overline{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$, where $\overline{\sigma}$ is a positive-definite symmetric matrix. Both current density-based (Type-2) and magnetic flux density-based (Type-1) reconstruction algorithms are used in this study. Brief explanations of the five algorithms are given here. Detailed descriptions of these algorithms can be found in [14] and [15].

The anisotropic equipotential projection (AEPP) algorithm constructs equipotential lines first and projects boundary potentials inside the FOV along these lines to obtain the potential distribution. Similarly, the other current density-based algorithm, namely the anisotropic J-substitution (AJS) algorithm, obtains the potential distribution using the finite element method (FEM) instead of equipotential lines. Then both algorithms calculate anisotropic conductivity components by forming a minimization equation system between measured and calculated current density vectors and solving this equation system iteratively. The third Type-2 algorithm is the hybrid J-substitution (AHJS) algorithm. This algorithm uses the result of the AEPP algorithm as the initial conductivity distribution to the AJS algorithm. The anisotropic harmonic $B_2$ (AHB$_2$) algorithm constitutes a nonlinear equation between one component of magnetic flux density and anisotropic conductivity components and it calculates anisotropic conductivity distribution recursively. The other Type-1 MRCTI algorithm, called the anisotropic $B_2$ sensitivity (AB$_2$S) algorithm, constitutes a sensitivity matrix using the relation between the change in the normal component of the magnetic flux density and the change in anisotropic conductivity components. Then it calculates anisotropic conductivity from the inverse of the sensitivity matrix. Type-2 MRCTI algorithms require the measurement of three magnetic flux density components and therefore rotation of the object inside the MR scanner, whereas for Type-1 MRCTI algorithms only one component of magnetic flux density is sufficient for reconstruction.

2.2. Numerical models
In order to evaluate the imaging properties of the algorithms in [14] and [15], three different 2-D numerical phantoms are designed. Dimensions of all models are designated as $9 \times 9$ cm. The outer geometry of the
models together with the electrode placements are given in Figure 1. Four different 20 mA current injection profiles are simulated using a pair of electrodes at each injection. These electrode pairs are given in Table 1.

![Figure 1. Geometry and dimensions of the numerical model and placement of the electrodes.](image)

Table 1. Electrodes used as current injection and sink for current patterns $I_1$, $I_2$, $I_3$, and $I_4$.

<table>
<thead>
<tr>
<th>Injection electrode (+20 mA)</th>
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<td>E2</td>
<td>E8</td>
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<td>Sink electrode (-20 mA)</td>
<td>E6</td>
<td>E4</td>
<td>E5</td>
<td>E7</td>
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All computer models are discretized into $40 \times 40$ square pixel elements. Therefore, measurements of 2-D magnetic field and current density distributions are simulated as $40 \times 40$ matrices. Two potential measurements from the current injecting electrodes are generated for each current injection profile. Background conductivity is selected as 0.2 S/m (isotropic) for all models to reflect blood conductivity.

The first model proposed in this study is designed to evaluate spatial resolution properties of the algorithms throughout the FOV. A small square object having the size of a pixel ($2.25 \times 2.25$ mm$^2$) is modeled to obtain the point spread function (PSF) and hence the spatial resolution of the algorithms. The geometry of the model is given in Figure 2a. The one pixel-sized perturbation mimics a point anisotropic perturbation. The point perturbation is placed at 19 different positions having $d$ distance from the left edge of the model in which $d$ is selected to be $k \times 2.25$, for $k = 1, 3, 5, \ldots, 37$ on a horizontal line passing through the middle of the model. Anisotropic conductivity of the element is defined as $\sigma_{xx} = 2$ S/m and $\sigma_{yy} = 0.02$ S/m. Other components of its anisotropic conductivity are selected as zero. To express the spatial resolution, the full width at half maximum (FWHM) value of the PSF is obtained. In this study, FWHM values for 19 different point perturbation locations are computed to obtain a measure of spatial resolution at different locations in the FOV.

The second numerical model is developed to evaluate the spatial resolution properties of the proposed reconstruction algorithms throughout the FOV, diagonally. Similar to the previous model, one pixel-sized perturbations are used, but this time 9 square elements are placed along the main diagonal line of the FOV. There is about 9.5 mm of distance between each element. The general model geometry can be seen in Figure 2b. Anisotropic conductivity values of the point perturbations are selected to be the same as the ones in the first model. Together with the first model, these two models reveal full information about the spatial resolution and distinguishability properties of the algorithms throughout the FOV.
The last numerical model contains a square object with 22.5 mm side length positioned at the center of the model. The geometry of the model is given in Figure 2c. Nineteen different anisotropic conductivity values are assigned to the central perturbation. These nineteen contrast levels were generated to investigate the linearity properties of the algorithms under changing conductivities. Nine of these are for perturbations having higher conductivity than the background conductivity. The other nine simulations are for perturbations with lower conductivity than the background conductivity. The remaining simulation is for the uniform case in which square object conductivity is assigned as 0.2 S/m in both directions. This value is selected as the reference value because it corresponds to average tissue conductivity for the human body. Conductivity values of the perturbation for all cases are given in Table 2. Off-diagonal components of the conductivity tensor are assigned to be zero for all 19 simulations.

Table 2. Anisotropic conductivity values for the cases of more conductive and less conductive perturbation in model 3. Values are in S/m.

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<td>More conductive</td>
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<td>( \sigma_{xx} )</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td>1.8</td>
<td>2</td>
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<tr>
<td>( \sigma_{yy} )</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td>Less conductive</td>
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<tr>
<td>( \sigma_{xx} )</td>
<td>0.1</td>
<td>0.067</td>
<td>0.05</td>
<td>0.04</td>
<td>0.033</td>
<td>0.028</td>
<td>0.025</td>
<td>0.022</td>
<td>0.02</td>
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<tr>
<td>( \sigma_{yy} )</td>
<td>0.2</td>
<td>0.2</td>
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3. Results and discussions

In this section, reconstruction results of the algorithms will be given. Results together with related discussions are grouped under each model and finally a general discussion is given.

3.1. Results for model 1

In order to explore the spatial resolution properties of the algorithms, FWHM values of reconstructed impulsive objects in computer model 1 are calculated. Since the one pixel-sized object reflects an impulsive conductivity in the x direction and impulsive resistivity in the y direction, two separate plots are prepared for both directions. Figures 3a and 3b give the FWHM plots for the impulsive conductor in the x direction for current density-based and magnetic flux density-based algorithms, respectively. Similarly, Figures 4a and 4b show the FWHM plots for the impulsive resistivity in the y direction.
As seen from Figure 3a, the AEPP algorithm has a PSF that is about two times the pixel size at every point in the FOV in the case of a conductor perturbation with respect to background. This means that this
algorithm cannot distinguish the objects when the distance between them is less than one pixel side length of the imaging grid. On the other hand, the AJS and the AHJS algorithms have FWHM values in the x direction, which is almost the same as the pixel size, which provides distinguishability for one pixel-spaced perturbations. Similarly, in the y direction, all three current density-based algorithms have almost the pixel-sized FWHM values. When the conductivity in the x direction is high and conductivity in the y direction is low for a perturbation as in model 1, current flows mainly in the x direction inside this perturbation. Therefore, equipotential lines are forced to be almost in the y direction in the object. Since the AEPP algorithm constructs equipotential lines first in order to obtain the potential field, sparse equipotential lines along the x direction results in coarse sampling of the potential field along the x direction.

When the FWHM graphs of magnetic flux density-based algorithms in Figures 3b and 4b are investigated, it is seen that the AHB$_z$ algorithm has FWHM values that are about two times the pixel side length in both directions. This means that the AHB$_z$ algorithm could not distinguish the impulsive objects in the y direction and can poorly distinguish in the x direction when the distance between them is less than a pixel side length of the imaging grid. The AHB$_z$ algorithm has an inherent sweeping effect in the y direction for the reconstructed $\sigma_{xx}$ image and in the x direction for the reconstructed $\sigma_{yy}$ image. This situation can be seen in reconstructed images of the AHB$_z$ algorithm given in Section 3.2. Therefore, this behavior affects the PSF significantly and causes nondistinguishability when two objects are one pixel size apart from each other. Similarly, the AB$_z$S algorithm has FWHM values greater than the pixel size length in both directions and this reduces the spatial resolution of the AB$_z$S algorithm. The AB$_z$S algorithm is based on an assumption of small conductivity changes. On the other hand, anisotropic conductivity values of one pixel-sized square are selected as ten times more conductive with respect to background in the x direction and ten times less conductive with respect to background in the y direction. These high-level perturbations cause oscillatory behaviors in the reconstructed images in the vicinity of perturbations, as seen in reconstructed images of the AB$_z$S algorithm in Section 3.2.

### 3.2. Results for model 2

Figures 5a–5d show the reconstruction results of the AEPP algorithm for computer model 2 for noise-free simulations. Following that, in Figures 6a–6d and 7a–7d, results of other two current density-based algorithms, namely the AJS and the AHJS algorithms, will be given. Finally, in Figures 8a–8b and 9a–9c, results of magnetic flux density-based algorithms for the second computer model are given.

![Figure 5](image_url)

**Figure 5.** Reconstruction results for computer model 2 using the AEPP algorithm (SNR = $\infty$): a) $\sigma_{xx}$, b) $\sigma_{yy}$, c) $\sigma_{xy}$, and d) $\sigma_{yx}$.

Computer model 2 is prepared to investigate spatial resolution of the proposed reconstruction algorithms throughout the FOV, diagonally. Therefore, nine impulsive objects whose x-directed and y-directed conductivities are respectively ten times more conductive and ten times less conductive than the background are placed in
the model. When the reconstruction results given in the figures are reviewed, it is seen that all five algorithms can reconstruct nine impulse object conductivities in both directions independent of position. The AEPP algorithm results show a background artifact caused by equipotential lines. The AHJS algorithm removes these artifacts to some extent and increases the background conductivity reconstruction accuracy. Furthermore, the AHJS algorithm increases the reconstructed conductivity accuracies of the impulsive elements. In the case of the ABzS algorithm, since the theory of the algorithm is constructed on an assumption of small conductivity change, nine impulsive objects cause some artifacts at their neighboring pixels. In the case of spatial resolution, it can be said that all five algorithms have position-independent spatial resolution.

**Figure 6.** Reconstruction results for computer model 2 using the AJS algorithm (SNR = ∞): a) $\sigma_{xx}$, b) $\sigma_{yy}$, c) $\sigma_{xy}$, and d) $\sigma_{yx}$.

**Figure 7.** Reconstruction results for computer model 2 using the AHJS algorithm (SNR = ∞): a) $\sigma_{xx}$, b) $\sigma_{yy}$, c) $\sigma_{xy}$, and d) $\sigma_{yx}$.

**Figure 8.** Reconstruction results for computer model 2 using the AHBz algorithm (SNR = ∞): a) $\sigma_{xx}$, b) $\sigma_{yy}$.

### 3.3. Results for model 3

In order to explore the linearity properties of the algorithms, two separate linearity plots, one for the perturbations more conducting than the background and one for the perturbations less conducting than the background,
are prepared. In these plots, ten different x-directed true conductivity values of the square object versus the corresponding reconstructed x-directed mean conductivity values of the square object are shown for each algorithm. Linearity plots for the higher and lower conductive perturbations are given in Figures 10a and 10b, respectively.

**Figure 9.** Reconstruction results for computer model 2 using the ABzS algorithm (SNR = ∞): a) $\sigma_{xx}$, b) $\sigma_{yy}$, c) $\sigma_{xy} = \sigma_{yx}$.

**Figure 10.** Linearity plot for the algorithms for a) more conductive x-directed conductivity with respect to background, b) less conductive x-directed conductivity with respect to background.

As seen from the linearity plot for the more conductive case, all five algorithms confront a problem in reconstructing high contrasts. However, this is not the case when the conductivity becomes less conductive with respect to background. The reason for this situation could be that, when the conductivity value of a region is increased, the current passing through this region will increase. However, this increment in current will not be as many times as the increment in conductivity since the total current in the FOV is constant and some of the current will continue to pass through the background. Because the reconstruction algorithms of both types use information originating from current flow in an object or background, they will converge to a lower conductivity value than the true conductivity, but when the conductivity value of a region is decreased, the current passing through that region will also decrease by almost the same amount and the remainder of the current will pass through the background. In this case, the conductivity value of that region could be calculated accurately.

When the individual reconstruction accuracies of the algorithms are investigated for computer model 3, it is seen that the AHJS algorithm gives the best results among all. For higher and lower conductivity
perturbations with respect to background, the $AB_2S$ algorithm shows the poorest results. This is because the underlying theory of that algorithm assumes small conductivity changes; therefore, increasing the conductivity contrast causes more erroneous results.

3.4. Final discussions

Individual imaging properties of each algorithm can be expressed as follows in light of the results obtained in this study. For the AEPP algorithm it is seen that it spreads the object boundaries in the direction of the high conductivity component with respect to background, whereas this is not the case in the direction of the low conductivity component with respect to background. The AEPP algorithm shows this characteristic at every point of the FOV; therefore, it has a position-independent spatial resolution. The AEPP algorithm also shows a nonlinear behavior for increasing object conductivity components with respect to background but it is linear for decreasing object conductivity components. The AJS and AHJS algorithms show quite similar imaging properties. They both reconstruct objects without spreading the boundaries with position independently. Their linearity properties show the same characteristics as the AEPP algorithm such that they are not linear in the case of increasing conductivity components but they are linear for decreasing ones. In the case of the $AHB_2$ algorithm, it inherently spreads the object boundaries in the $y$ direction for $x$-directed conductivity component and in the $x$ direction for $y$-directed conductivity component position independently. The $AB_2S$ algorithm can reconstruct sharp edges at object boundaries and it has a position-independent spatial resolution. Linearity properties of both Type-1 algorithms are similar to those of the other algorithms.

4. Conclusions

In this study five recently proposed MRCTI algorithms are evaluated for their imaging properties such as spatial resolution and linearity. It is important to note that, in most works, this kind of study is skipped and imaging properties of the proposed algorithms are underestimated. The results of the study show that all five algorithms have position-independent spatial resolution although some of the algorithms have point-spreading characteristics. For the linearity property, it can be said that all algorithms exhibit nonlinear behavior for increasing conductivity components but they are linear in reconstruction for decreasing conductivity components.

All five algorithms have quite similar imaging properties and, among them, the AHJS algorithm shows slightly better reconstruction performance. However, considering its ease of application, the $AB_2S$ algorithm is the best algorithm among the five algorithms. This is because the $AB_2S$ algorithm does not require object rotation inside the MRI scanner to measure three components of the magnetic field and instead it reconstructs anisotropic conductivity using only one component of magnetic flux density. This increases the potential of the MRCTI technique to be a diagnostic imaging technique. The major obstacle in this way is the injected current. The amount of the injected current for MRCTI reconstruction is still far beyond the secure limits for humans. Therefore, future studies should be on this topic and, if they are successful, the MRCTI technique would be a diagnostic imaging technique.

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