Gain and coherence collapse condition for a laser diode with optoelectronic feedback using Volterra series

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Abstract: Four-tone small signal analysis was performed for a nonlinear optoelectronic feedback laser diode system. In the analysis, Volterra power series up to second-order opening, the second kernel for the output intermodulation distortion (IMD) analysis was performed. The components of the alternative IMD frequency were selected for analysis. These are the four IMD frequency components. The variation of IMD frequency component amplitudes was investigated under various values of delay-time ($t_0$) and the feedback gain constant ($K$). The feedback values and the critical frequencies, at which the coherence collapse or chaos occur, were also determined.

Key words: Laser diode, Volterra series, intermodulation, harmonic distortion, nonlinear distortion, electronics feedback, chaos, gain, nonlinear optoelectronic feedback

1. Introduction

Laser systems, which are the optical source for fibre-optic and hybrid communication systems, employ various laser diode applications. Laser diodes have been increasingly used both commercially and industrially due to their optical characteristics, small size, and ruggedness. Laser diodes have three main characteristic quantities: gain [1–3], refractive index change [4–6], and linewidth enhancement factor [7–9]. These quantities have been studied many times theoretically [10–15], experimentally [16–24], and intelligently [22–42]. In this current study, the characteristic quantity gain and coherence collapse condition with optoelectronic feedback are analysed and simulated with the use of Volterra series.

In optical communication systems, different optical feedback schemes are used based on the structure of the laser diode. Hybrid (analogue/digital) communication system structures for metropolitan areas consist of fibre-optic and coaxial cables and use feedback for different purposes [43]. In recent years, data communication is widely used due to extensive use of the Internet. However, the communication systems infrastructure has reached its limits to meet this fast increasing demand for data transfer. To overcome this problem, the bandwidth of systems can be increased with the use of subcarrier systems. The prominent property of these systems is the use of the wideband subcarrier multiplexing (SCM) technique. SCM systems are classified according to the employed analogue and digital modulation techniques. Asymmetric digital subcarrier line (ADSL), digital subcarrier line (DSL), very-high speed digital subcarrier line (VDSL), discrete multitone (DMT), discrete wavelet multitone (DWMT), and wavelength division multiplexing (WDM) are some examples of SCM systems. Due to the nonlinear behaviour of laser diodes, intermodulation distortion (IMD) frequency components are generated.

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The systematic analysis of Volterra kernels is carried out in [44]. Several authors presented results on harmonic distortion, optimised intermodulation [45–49,50,51], and various aspects of the issue [52,53].

The use of an optic filter system in optical feedback provides the stable operation of the dynamic system by controlling the nonlinear transmission function through the filter [54]. Moreover, the operation of the dynamic system can be adjusted by means of the feedback systems [55]. Feedback is also used in controlling the dynamic systems with various modulation types [56–58]. Another use of feedback is to achieve wavelength stability [59] of wavelength division multiplexing (WDM) systems through the adjustment of cross-correlation level by setting up the DC bias current [60].

In high speed fibre-optic systems, chaotic laser systems are used. The properties of chaotic communication systems change depending on bandwidth, performance, bit error rate, and the length of the fibre-optic line [61]. Rotated polarisation and inverted polarisation optical feedback systems are used in chaos-based safe fibre-optic communication systems [62]. Adjustable time delayed optoelectronic feedback is used in safe chaos-based multiplexed communication systems [63,64]. The feedback between nonlinear elliptical polarisation and laser source is another optical feedback method [65]. Feedback is used for steady data transmission in commercial communication systems [66]. For high resolution two-tone separation and two-tone signal quality, an optical filter/feedback system is used [67]. Feedback methods are also used for setting the quality factor, gain, and loss [68]. For dynamic systems, time delayed polarisation mode switching in semiconductor lasers polarisation rotating coupling methods are applied [69]. In nonlinear dynamic delayed feedback optoelectronic oscillator systems, dual fibre optical feedback is used between the hybrid system and the oscillator [70]. In another study, the stability of the feedback system is achieved by the relaxation oscillation frequency feedback current [71]. Periodical feedback oscillation is obtained by high injection current [72]. The coherence collapse regime is used to focus the optical output power and the reflected optical power under test [73].

In this work, the analysis of alternative IMD frequency components of a nonlinear laser diode with optoelectronic feedback under four-tone small signal input is performed. The Volterra operators $Z_1$, $Z_2$ and the Volterra kernels $H_1$, $H_2$ are found. The analysis is carried out for the second kernel output.

The main contribution this study is that alternative IMD frequency components are employed in order to increase the bandwidth of the fibre-optic system.

2. The basic single-mode laser diode

Hassine [74], Tucker [75], and Olshansky [76] modelled the basic single mode laser diode using the following dynamic equations:

$$\frac{dp(t)}{dt} = \Gamma A \left[ n(t) - N_{tr} \right] \left[ 1 - \hat{\varepsilon} p(t) \right] p(t) - \frac{1}{\tau_p} p(t) + \frac{\beta \Gamma}{\tau_n} n(t) \tag{1}$$

$$\frac{dn(t)}{dt} = \frac{1}{q} I(t) - \frac{1}{\tau_n} n(t) - \Gamma A \left[ n(t) - N_{tr} \right] \left[ 1 - \hat{\varepsilon} p(t) \right] p(t), \tag{2}$$

where $p(t)$ and $n(t)$ denote the photon and carrier numbers, respectively, and $\Gamma$ the compression factor ($\Gamma = 0.3$), $A$ gain constant ($A = 1.83 \ \text{times} \ 10^4 \text{s}^{-1}$), $N_{tr}$ the number of carriers at the threshold ($N_{tr} = 10^7$), $\tau_p$ photon life ($\tau_p = 1.6 \ \text{times} \ 10^{-12} \text{s}$), $\tau_n$ carrier life ($\tau_n = 2.2 \ \text{times} \ 10^{-9} \text{s}$), $\beta$ spontaneous emitting factor ($\beta = 10^{-4}$), $I(t)$ total current, $q$ elementary charge ($q = 1.6 \ \text{times} \ 10^{-19} \text{C}$), $\hat{\varepsilon} = \varepsilon/V = 1 \times 10^{-6}$ dimensionless gain factor, and $V$ the volume of the active region.
Using (1) and (2), the Volterra kernels \((H_1, H_2)\) are determined. The interested reader is referred to [66–70] for details of Volterra and the nonlinear system theories and to [44] for the detailed solutions of a single-mode laser diode.

In [45,46], the analysis of the system with two-tone and three-tone inputs for the harmonic distortion, the bandwidth, and the optimisation is given with the results. The general four-tone small signal input \(i(t)\) is given as

\[
i(t) = u_o \cos(\omega_o t + \delta_0) + u_1 \cos(\omega_1 t + \delta_1) + u_2 \cos(\omega_2 t + \delta_2) + u_3 \cos(\omega_3 t + \delta_3) + u_4 \cos(\omega_4 t + \delta_4),
\]

where \(u_i, \omega_i,\) and \(\delta_i\) are the amplitude, frequency, and phase of the \(i\)th \((i = 1,2,3,4)\) input, respectively. The carrier signal is represented by \(u_o \cos(\omega_o t + \delta_o)\). The number of photons, \(P(t)\), at the output is given as

\[
P(t) = p_1(t) + p_2(t) + p_3(t) + \ldots + p_n(t) = \sum_{i=1}^{n} p_i(t),
\]

where \(p_i(t) (i = 1,2,\ldots,n)\) is the estimated photon numbers of the \(i\)th Volterra kernel [44,45].

3. The first-order Volterra operator

The transfer function \(Z_1(j\omega)\) of the first order Volterra operator is given as [44]

\[
Z_1(j\omega) = \frac{H_1(j\omega)}{1 + H_1(j\omega)G_1(j\omega)},
\]

where \(H_1(j\omega)\) and \(G_1(j\omega)\) is defined as

\[
H_1(j\omega) = \frac{B_o}{q} \left( \frac{1}{(D_o - \omega^2) + jD_1\omega} \right),
\]

\[
G_1(j\omega) = ge^{-j\omega t_0},
\]

where \(B_o, D_0\) and \(D_1\), and \(g\) are constants and \(t_0\) is the normalised time. The detailed definitions can be found in [44].

The input current, \(i(t)\), is selected as in (3) excluding carrier component as follows:

\[
i(t) = u_1 \cos(\omega_1 t + \delta_1) + u_2 \cos(\omega_2 t + \delta_2) + u_3 \cos(\omega_3 t + \delta_3) + u_4 \cos(\omega_4 t + \delta_4).
\]

The photon output of the first order Volterra kernel is defined as

\[
p_1 = u_1 |Z_1(j\omega_1)| \cos(\omega_1 t + \delta_1 + \angle Z_1(j\omega_1)) + u_2 |Z_1(j\omega_2)| \cos(\omega_2 t + \delta_2 + \angle Z_1(j\omega_2)) + u_3 |Z_1(j\omega_3)| \cos(\omega_3 t + \delta_3 + \angle Z_1(j\omega_3)) + u_4 |Z_1(j\omega_4)| \cos(\omega_4 t + \delta_4 + \angle Z_1(j\omega_4)),
\]

where \(|Z_1(j\omega_i)|, \angle Z_1(j\omega_i)\) and \(\omega_i (i = 1,2,3,4)\) denote the amplitude, phase, and the angular frequencies of the input currents, respectively.
4. The second-order Volterra operator

Using the block diagrams given in [44], the block diagram of the second order Volterra operator, \( p_2(t) \), can be obtained as given in Figure 1.

![Block diagram of the second-order Volterra operator](image)

Figure 1. Block diagram of the second-order Volterra operator.

In Figure 1, the \( R_1 \) and \( H_1 \) are linear time invariant (LTI) systems. The \( R_1 \) block is defined in terms of \( G_1 \) and \( H_1 \) given in (6) and (7), respectively, as

\[
R_1 (j\omega) = \frac{1}{1 + H_1 (j\omega) G_1 (j\omega)} \tag{10}
\]

Since \( R_1 \) and \( H_1 \) blocks are connected in series, the equivalent block has the following transfer function:

\[
R_1 (j\omega) H_1 (j\omega) = \frac{H_1 (j\omega)}{1 + H_1 (j\omega) G_1 (j\omega)} \tag{11}
\]

The photon output is in the following form:

\[
p_1(t) = A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2) + A_3 \cos(\omega_3 t + \alpha_3) + A_4 \cos(\omega_4 t + \alpha_4), \tag{12}\]

where

\[
A_i = u_i |Z_1 (j\omega_i)| (i = 1, 2, 3, 4) \tag{13a}
\]

\[
\alpha_i = \delta_i + \angle Z_1 (j\omega_i) (i = 1, 2, 3, 4). \tag{13b}
\]

The carrier density, \( n_1(t) \), has the following form:

\[
n_1(t) = B_1 \cos(\omega_1 t + \beta_1) + B_2 \cos(\omega_2 t + \beta_2) + B_3 \cos(\omega_3 t + \beta_3) + B_4 \cos(\omega_4 t + \beta_4) \tag{14}\]

The \( K_1 \) block has the following transfer function [44]:

\[
K_1 (j\omega) = \frac{1}{B_o} (B_1 + j\omega), \tag{15}\]

where

\[
B_i = A_i |K_1 (j\omega_i)| (i = 1, 2, 3, 4) \tag{16a}
\]

\[
\beta_i = \alpha_i + \angle K_1 (j\omega_i) (i = 1, 2, 3, 4). \tag{16b}
\]

The block diagram for \( H_2 \) is given in [44]. To find the output of \( H_2 \), \( p_2^2(t) \) and \( n_1(t)p_1(t) \) should be available. From (12),

\[
p_2^2(t) = [A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2) + A_3 \cos(\omega_3 t + \alpha_3) + A_4 \cos(\omega_4 t + \alpha_4)]^2 \tag{17}\]
and from (12) and (14)
\[ n(t) p(t) = (A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2) + A_3 \cos(\omega_3 t + \alpha_3) + A_4 \cos(\omega_4 t + \alpha_4)) \\
(B_1 \cos(\omega_1 t + \beta_1) + B_2 \cos(\omega_2 t + \beta_2) + B_3 \cos(\omega_3 t + \beta_3) + B_4 \cos(\omega_4 t + \beta_4)) \]
(18)
are obtained. In [44], \( f_2(t) \) is given as
\[ f_2(t) = A_1 [n_1(t) p_1(t)] - \Gamma A [N_o - N_{ts}] p_1^2(t). \]
(19)
The above expression for \( f_2(t) \) is obtained by substituting for \( p_1^2(t) \) and \( n_1(t) p_1(t) \) from (17) and (18).

The function \( M(j\omega) \) is given as [44]
\[ M(j\omega) = \frac{\frac{1}{\Gamma - \frac{1}{\tau_o}}}{(D_o - \omega^2) + jD_1\omega} R_1(j\omega). \]
(20)
Substituting for \( R_1 \)
\[ M(j\omega) = \left[ \frac{j\omega - \frac{1}{\Gamma - \frac{1}{\tau_o}}}{(D_o - \omega^2) + jD_1\omega} \right] \left[ \frac{D_o - \omega^2 + jD_1\omega}{D_o - \omega^2 + jD_1\omega + \frac{2B_o}{g} e^{j\omega t_o}} \right] \]
(21)
is obtained. The kernel number of photon output \( p_2(t) \) is defined as
\[ p_2(t) = f_2(t) M(0). \]
(22)
Substituting \( \omega = 0 \) in (21), \( M(0) = \frac{(1-\Gamma)}{(D_o q + g B_o) \tau_o} \). Using this value for \( M(0) \), one can obtain the expression for \( p_2(t) \) from (22).

The carrier density transfer function is defined as
\[ N_2(s) = \frac{1}{B_o} \left[ \frac{(s + B_1) \left( s - \frac{1}{\Gamma - \frac{1}{\tau_o}} \right)}{s^2 + D_1 s + D_o} - 1 \right] F_2(s). \]
(23)
The interested reader is referred to [44,45] for details of the analysis.

5. Results
In this study, the input current, \( I_0 \), is chosen such that \( I_0 = 3.33 I_{th} \). The alternative IMD frequency components, which are obtained from the second kernel output, are chosen as
1. \( (\omega_0 + \omega_1), (-\omega_0 + \omega_1), \)
2. \( (\omega_0 + \omega_2), (\omega_0 - \omega_2), \)
3. \( (\omega_0 + \omega_3), (\omega_0 - \omega_3), \)
4. \( (\omega_0 + \omega_4), (\omega_0 - \omega_4). \)
The results are given in Figures 2–4. The line types indicated in Figure 4 are also valid for Figures 2 and 3.
In Figures 2, 3, and 4, the gain curves are given for the feedback gains $K = 1.231$, $K = 1.2972$, and $K = 1.182$ with the delay-time $t_0 = 1 \times 10^{-10}$ s. From Figure 2, the bandwidth becomes narrower compared with the analogue and digital communication specifications. However, the gain increases. Here, the normalised collapse frequency value and gain are 1.77 and -21 dB, respectively.

In Figure 3, the normalised collapse frequency and gain are 1.132 and -18.5 dB, respectively, under the feedback gain $K = 1.2972$. 
In Figure 4 under the feedback gain $K = 1.182$, the normalised collapse frequencies of minimum gain (37.71 dB) are 1.212 and 1.823, and the maximum gain is 79 dB. That is, the optoelectronic system with feedback behaves like a bandpass filter. The bandpass characteristics are found as follows:

$$\omega_L = 1.212_{\text{norm}} = 12.12 \text{ GHz}$$

and

$$\omega_H = 1.823_{\text{norm}} = 18.23 \text{ GHz},$$

where the \text{norm} denotes that the value is the normalised frequency value.

This leads the centre frequency to be

$$\omega_0 = \sqrt{\omega_L \omega_H} = 1.486_{\text{norm}} = 14.86 \text{ GHz}.$$ 

Since the bandwidth is

$$BW = \omega_H - \omega_L = 18.23 - 12.12 = 6.11 \text{ GHz},$$

the quality factor or the selectivity $Q = \frac{\omega_0}{BW}$ of the bandpass filter is found as

$$Q = \frac{14.86}{6.11} = 2.432[71].$$

Available bandwidths are found as 17.90 GHz and 7.5 GHz for digital and analogue communication, respectively.

6. Conclusion

In this study, an optoelectronic feedback system with adjustable gain and time delay is applied to a laser diode. The following is obtained by changing the gain $K$:

The input current is selected as $I_0 = 3.33I_{th}$. Different $I_0$ values may lead to different results. The normalised frequency at the collapse or chaos corresponding to the critical feedback is found as 1.212 and 1.823. The critical frequency depends on the input current, delay-time, and the feedback gain. Small variations in the feedback can result a chaotic behaviour. The optical system is highly sensitive to variations in the feedback.
The two collapse frequency ranges changed by the ratio of 81.69%, while the change in the input current is 11%. This clearly shows that the system is nonlinear.

An interesting result arises for $K = 1.182$. In this case, the gain curve is very similar to a bandpass filter with low and high bandpass frequencies 12.12 GHz and 18.23 GHz (Figure 4). The centre frequency is estimated as 14.86 GHz with a quality factor larger than 2. However, the gain (37.71 dB) at the low and high frequencies is much higher than $-3$ dB. Additionally, these values change with changing DC injection current.

The critical feedback value for chaos seems impossible to predict. Around the chaos frequencies, the modulation should not be performed. The laser diode systems should be tested for stability at the critical feedback and frequency. Then the system can be employed in fibre-optic subcarrier communication systems.

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