

## Stochastic congestion management considering power system uncertainties: a chance-constrained programming approach

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**Abstract:** Considering system uncertainties in developing power systems, algorithms such as congestion management (CM) are vital in power system analysis and studies. This paper proposes a new model for power system CM by considering power system uncertainties based on chance-constrained programming (CCP). In the proposed approach, transmission constraints are taken into account by stochastic, instead of deterministic, models. The proposed approach considers network uncertainties with a specific level of probability in the optimization process, and then an analytical approach is used to solve the new model of stochastic congestion management. In this approach, the stochastic optimization problem is transformed into an equivalent deterministic problem. Moreover, an efficient numerical approach based on a real-coded genetic algorithm and Monte Carlo technique is proposed to solve the CCP-based congestion management problem in order to make a comparison to the analytical approach. The effectiveness of the proposed approach is evaluated by applying the method to the IEEE 30-bus test system. The results show that the proposed CCP model and the analytical solving approach outperform the existing models.

**Key words:** Congestion management, system uncertainties, chance-constrained programming, Monte Carlo simulation, stochastic optimization

### 1. Introduction

In the past decade, open access to transmission networks in restructured power systems has resulted in the emergence of bilateral contracts. This trend, together with the growth in electricity consumption, has increased the possibility of congestion in transmission networks. Congestion is essentially referred to as the violation of the physical, operational, and policy constraints of the network. Both vertically integrated and unbundled power systems have experienced such problems [1,2]. Congestion generally arises from 2 main sources: occurrence of system contingencies and ignorance of the generation location in the market-clearing mechanism [3]. Congestion may occur in day-ahead, hour-ahead, and real-time dispatch. The system operator is responsible for the necessary preventive or remedial actions to prevent or relieve congestion. The set of remedial activities performed to relieve violated limits is referred to as congestion management (CM). In order to alleviate network congestion, cheaper generators may be replaced by more expensive ones in primary market dispatch. Therefore, managing network congestions may impose additional cost on the operation of the system. Transmission congestion may result in market power for some participants or may endanger the stability of the system [4]. Therefore, preventive or corrective actions are necessary to relieve congestion and decrease system risks.

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Recently, many researchers have investigated CM techniques. The basic differences among CM approaches arise from the modeling of the power market, available controls to relieve congestion, and algorithms applied to the proposed CM problems. Kumar et al. [1] categorized CM approaches into 4 distinct methods: sensitivity factor-based, redispatch-based, auction-based, and pricing-based methods. In [1], a wide range of literature on the mentioned approaches was reviewed. A unified framework for different CM schemes was presented in [2]. In the framework presented in [2], 2 distinct stages were considered for the operation of an electricity market: market dispatch and congestion redispatch. The second stage will only be performed if the first stage cannot achieve a feasible operating state with no constraint violation. In most CM methods, the general concept of the congestion redispatch problem implies the minimization of the market rearrangement cost to alleviate congested lines [5–7].

Network uncertainties may be related to different sections of the power system, including generation, transmission, and distribution [8]. Therefore, considering the system uncertainties in power system operation and control actions, such as CM, is of prime importance in power system analysis and studies. Applying the uncertainties to CM models and algorithms may significantly improve the feasibility of the resulted operating point and the power system security level. For example, Esmaili et al. proposed a stochastic CM technique in [9], using a scenario-based approach. In their technique, scenarios are produced for the power system operating point to deal with power system uncertainties. Moreover, they reduced the generated scenarios to include the most probable and nonrepetitive ones. Finally, CM is performed for the reduced scenarios. The final solution to the CM problem was obtained by computing the expectation of the solutions associated with each scenario.

In this paper, we propose a new formulation for CM considering network uncertainties. In our proposed method, probability density functions (PDFs) of line flows are employed to model the stochastic CM problem. The new model of stochastic congestion management is formulated based on chance-constrained programming (CCP), in which deterministic transmission constraints are replaced by stochastic ones with respect to PDFs of line flows. Therefore, the proposed method can have better accuracy compared to scenario-based methods, which generally select a set of most important scenarios and eliminate the others. In our proposed approach, a stochastic optimization problem based on a PDF of line flows will be formulated to handle the system uncertainties in the CM process. The PDF of line flows can be obtained using a Monte Carlo simulation considering the uncertain model of the system variables. Moreover, an analytical approach is used to solve the new model of stochastic CM. In this method, the stochastic optimization problem will be transformed into an equivalent deterministic problem. The effectiveness of the proposed approach is evaluated by applying the method to the IEEE 30-bus test system.

The rest of this paper is organized as follows. Section 3 presents the deterministic CM model. The stochastic CM model, based on CCP, is described in Section 4. The results of the simulation on the IEEE 30-bus test system are presented in Section 5. Finally, the concluding remarks are summarized in Section 6.

## 2. Deterministic CM model

In this paper, a day-ahead pool electricity market is used as the framework for implementing the proposed CM algorithm. In the market, suppliers and consumers submit their bids to the market operator, who is responsible for the clearing procedure [10]. The time framework for the market-clearing procedure is 24 h. On the other hand, CM is performed on an hourly basis if necessary.

In this environment, the deterministic CM can be formulated as follows:

$$\text{Min} \sum_{i=1}^{N_G} C_i(\Delta P_{Gi}) + \sum_{j=1}^{N_L} C_j(\Delta P_{Lj}), \quad (1)$$

$$\text{s.t.} \quad \left| P_{line_l} + \sum_{i=1}^{N_G} a_{l,i} \Delta P_{Gi} - \sum_{j=1}^{N_L} a_{l,j} \Delta P_{Lj} \right| \leq P_{line_l}^{\max} \forall l \in N_{Line} \quad , \quad (2)$$

$$\Delta P_{Gi}^{\min} \leq \Delta P_{Gi} \leq \Delta P_{Gi}^{\max} \forall i \in N_G, \quad (3)$$

$$\Delta P_{Lj}^{\min} \leq \Delta P_{Lj} \leq \Delta P_{Lj}^{\max} \forall j \in N_L, \quad (4)$$

$$\sum_{i=1}^{N_G} (\Delta P_{Gi}) = \sum_{i=1}^{N_L} (\Delta P_{Lj}). \quad (5)$$

The objective function of the CM problem is the minimization of the total cost of the redispatch step in the day-ahead market. The objective function in Eq. (1) consists of 2 distinct terms related to the costs imposed by generators and the loads for congestion redispatch. In Eq. (1),  $C_i(\Delta P_{Gi})$  is the redispatch cost function related to the generators, which can be expressed as:

$$C_i(\Delta P_{Gi}) = C_i^{up} \times \Delta P_{Gi}^+ + C_i^{down} \times \Delta P_{Gi}^-, \quad (6)$$

where  $C_i^{up}$  and  $C_i^{down}$  are the bids of generator  $i$  for the power increment and decrement, respectively. The redispatch cost function related to the loads is similar to that of the generators.

The constraint of Eq. (2) explains the changes in line flows, where  $a_{l,i}$  stands for transmission congestion distribution factor (TCDF), described in detail in [11]. TCDF can be defined as the changes in flow of transmission line  $l$  due to the unit increment in the power injection at bus  $i$ .

$$a_{l,i} = \frac{\Delta P_{line_l}}{\Delta P_{Gi}} \quad (7)$$

Constraints of Eqs. (3) and (4) express the maximum-allowed changes in the power of generators and loads. Moreover, the constraint of Eq. (5) models the power balance equation. In this paper, transmission losses in CM formulation are ignored.

### 3. Stochastic CM model

Power system behavior involves various types of uncertainties, which should be considered in the modeling process of power system problems in order to perform dependable analyses and studies on power systems [12]. Power system uncertainties may be categorized into loads forecasting errors, availability of equipment, and price uncertainties in the power market. The modeling of power system uncertainties is the first step in stochastic CM.

The main steps for the proposed stochastic CM can be expressed as follows:

- Determining PDFs of output variables, such as line flows, using a primary Monte Carlo simulation considering the model of network uncertainties in Section 3.1.
- Formulating the stochastic CM based on CCP in Section 3.2 with respect to PDFs of line flows.
- Identifying the equivalent deterministic problem for CCP-based stochastic CM, as will be explained in Sections 3.3 and 3.4.

### 3.1. Modeling of power system uncertainties

Power system loads follow a random pattern; hence, load forecasting errors are inevitable. As a result, a load in power systems is modeled as a random variable ( $\xi$ ) with normal distribution as the PDF [12]. These uncertain loads may be correlated with each other [13]. The normal PDF of the loads can be expressed as:

$$\rho(\xi) = \frac{1}{\sqrt{(2\pi)^s |\Sigma|}} \exp\left(-\frac{1}{2} (\xi - \mu)^T \Sigma^{-1} (\xi - \mu)\right), \quad (8)$$

where  $\mu$  and  $\Sigma$  have the following mean vector and covariance matrix representation, respectively.

$$\mu = [\mu_1, \mu_2, \dots, \mu_s]^T \quad (9)$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1 \sigma_2 r_{12} & \cdots & \sigma_1 \sigma_q r_{1s} \\ \sigma_2 \sigma_1 r_{21} & \sigma_2^2 & \cdots & \sigma_2 \sigma_q r_{2s} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_q \sigma_1 r_{s1} & \sigma_q \sigma_2 r_{s2} & \cdots & \sigma_s^2 \end{bmatrix} \quad (10)$$

In this paper, short-term load forecasting is considered. Therefore, the order of  $\sigma_i/\mu_i$  should be smaller than 0.15 [12].

Availability of system equipment, including generators and transmission lines, can be modeled using forced outage rate (FOR) [14].

$$FOR = \frac{MTTR}{MTTR + MTTF} \quad (11)$$

The uncertainties related to market prices have been neglected in this paper. In order to model system uncertainties, we have used a Monte Carlo simulation utilizing the PDF of input variables. Using Monte Carlo simulation, the PDFs of output variables, such as line flows ( $P_{line_i}$ ), are achieved. In each Monte Carlo iteration, the samples of uncertain variables are generated. The primary market dispatch is generated using these samples and, as a result, the values of line flows are obtained. The PDFs of line flows are extracted from the obtained values in Monte Carlo iterations. These PDFs are effectively used to formulate the stochastic CM.

### 3.2. Formulating the stochastic CM based on CCP

CCP is a special type of optimization problem, which is useful for problems involving uncertain variables in their objective function or constraints. In this type of optimization, the fulfillment of constraints is guaranteed with

a specific level of probability, instead of being treated as hard constraints. A typical CCP can be formulated as follows [15]:

$$\begin{aligned} & \text{Min. } f(x) \\ & \text{Pr} \{g_i(x, \xi) \leq 0\} \geq \alpha_i \forall i \end{aligned} \quad (12)$$

where  $x$  is the decision vector of the optimization problem and  $\xi$  stands for a set of uncertain variables. Furthermore,  $\alpha_i$ , which identifies the level of constraint satisfaction, is referred to as confidence level.

Based on the CCP formulation in Eq. (12), the probabilistic version of the CM problem in Eqs. (1)–(5) can be written as follows:

$$\text{Min} \sum_{i=1}^{N_G} C_i(\Delta P_{Gi}) + \sum_{j=1}^{N_L} C_j(\Delta P_{Lj}), \quad (13)$$

$$\begin{aligned} & \text{s.t.} \\ & \text{Pr} \left[ \left| P_{line_l} + \sum_{i=1}^{N_G} a_{l,i} \Delta P_{Gi} - \sum_{j=1}^{N_L} a_{l,j} \Delta P_{Lj} \right| \leq P_{line_l}^{\max} \right] \geq \alpha_l \forall l \in N_{Line} \end{aligned} \quad (14)$$

$$\Delta P_{Gi}^{\min} \leq \Delta P_{Gi} \leq \Delta P_{Gi}^{\max} \forall i \in N_G, \quad (15)$$

$$\Delta P_{Lj}^{\min} \leq \Delta P_{Lj} \leq \Delta P_{Lj}^{\max} \forall j \in N_L, \quad (16)$$

$$\sum_{i=1}^{N_G} (\Delta P_{Gi}) = \sum_{j=1}^{N_L} (\Delta P_{Lj}). \quad (17)$$

The constraint of Eq. (14) express the line flow limitations that should be satisfied with the minimum probability of  $\alpha_l$ . In the CCP-based congestion management,  $\alpha_l$  identifies the probability of the constraint satisfaction for the flow in the line  $l$ . For example, for  $\alpha_l = 0.9$ , the stochastic constraints of the transmission branches must be satisfied for at least 90% of the system states, which are modeled in the primary Monte Carlo simulation.

In Eq. (14),  $P_{line_l}$  is an uncertain variable whose PDF has been determined in the primary Monte Carlo simulation. In addition,  $\Delta P_{Gi}$  and  $\Delta P_{Lj}$  are the decision variables for the optimization problem in Eqs. (13)–(17).

The stochastic optimization problem stated in Eqs. (13)–(17) is difficult to solve since it includes a set of stochastic constraints in Eq. (14). In [12], a sequential approach was proposed to solve this type of optimization problem, which includes a simulation layer as well as an optimization layer. A numerical method using a genetic algorithm (GA) and Monte Carlo technique was presented in [16] to solve the stochastic transmission expansion planning modeled by CCP. In fact, real-coded GA generates the suggested solution for stochastic CM, while Monte Carlo evaluates the satisfaction of the probabilistic constraints. Rao developed an analytical approach to solve the stochastic optimization problems in [17].

The stochastic CM problem, formulated by CCP, was solved in [18] by using a combined approach that includes real-coded GA and Monte Carlo simulation, as in [16]. The real-coded GA was incorporated into the Monte Carlo simulation to solve the proposed stochastic CM. In fact, the real-coded GA generates the suggested solution for stochastic CM, whereas Monte Carlo simulation evaluates the satisfaction of the probabilistic constraints. In this paper, we utilize the analytical approach proposed by Rao [17]. The proposed methods in [9] and [18] are also implemented to perform a comprehensive study.

### 3.3. Equivalent linear model for stochastic CM

If the variation of the shift factors under stochastic modeling are neglected ( $Var(a_{l,i}) \simeq 0$ ), the stochastic optimization problem in Eqs. (13)–(17) would be equal to the deterministic optimization problem, expressed below [17]:

$$Min \sum_{i=1}^{N_G} C_i(\Delta P_{Gi}) + \sum_{j=1}^{N_L} C_j(\Delta P_{Lj}), \quad (18)$$

$$\left| \overline{P_{line_l}} + \sum_{i=1}^{N_G} \overline{a_{l,i}} \Delta P_{Gi} - \sum_{j=1}^{N_L} \overline{a_{l,j}} \Delta P_{Lj} \right| \leq P_{line_l}^{\max} + E_l \sqrt{Var(P_{line_l})} \forall l \in N_{Line}, \quad (19)$$

$$\Delta P_{Gi}^{\min} \leq \Delta P_{Gi} \leq \Delta P_{Gi}^{\max} \forall i \in N_G, \quad (20)$$

$$\Delta P_{Lj}^{\min} \leq \Delta P_{Lj} \leq \Delta P_{Lj}^{\max} \forall j \in N_L, \quad (21)$$

$$\sum_{i=1}^{N_G} (\Delta P_{Gi}) = \sum_{i=1}^{N_L} (\Delta P_{Lj}), \quad (22)$$

where  $E_l$  is the standard normal variable obtained from the standard normal distribution  $\rho(E_l)$  with respect to the value of the confidence level  $\alpha_l$ , as expressed in Eq. (23):

$$\rho(E_l) = 1 - \alpha_l \Rightarrow E_l = \rho^{-1}(1 - \alpha_l) = -\rho^{-1}(\alpha_l). \quad (23)$$

It should be noted that in this study it is assumed that  $\alpha_l > 0.5$  and consequently  $1 - \alpha_l < 0.5$ ; therefore,  $E_l < 0$  [17]. For example, if  $\alpha_l = 0.99$ , then we have  $E_l = -\rho^{-1}(0.99) = -2.33$  using standard normal distribution with a probability of 99%. The deterministic optimization problem in Eqs. (18)–(22) is a linear problem, which can be easily solved using linear programming.

### 4. Equivalent nonlinear model for stochastic CM

Considering the variance of the shift factors, the stochastic optimization problem in Eqs. (13)–(17) would be equal to a nonlinear deterministic optimization problem, expressed below [17]:

$$Min \sum_{i=1}^{N_G} C_i(\Delta P_{Gi}) + \sum_{j=1}^{N_L} C_j(\Delta P_{Lj}), \quad (24)$$

$$\left| \overline{P_{line_l}} + \sum_{i=1}^{N_G} \overline{a_{l,i}} \Delta P_{Gi} - \sum_{j=1}^{N_L} \overline{a_{l,j}} \Delta P_{Lj} \right| \leq P_{line_l}^{\max} + E_l \sqrt{Var(h_l)} \forall l \in N_{Line}, \quad (25)$$

$$\Delta P_{Gi}^{\min} \leq \Delta P_{Gi} \leq \Delta P_{Gi}^{\max} \forall i \in N_G, \quad (26)$$

$$\Delta P_{Lj}^{\min} \leq \Delta P_{Lj} \leq \Delta P_{Lj}^{\max} \forall j \in N_L, \quad (27)$$

$$\sum_{i=1}^{N_G} (\Delta P_{Gi}) = \sum_{i=1}^{N_L} (\Delta P_{Lj}), \quad (28)$$

where:

$$h_l = P_{line_l} + \sum_{i=1}^{N_G} a_{l,i} \Delta P_{Gi} - \sum_{j=1}^{N_L} a_{l,j} \Delta P_{Lj} - P_{line_l}^{\max}, \quad (29)$$

$$Var(h_l) = \sum_{i=1}^{N_G} Var(a_{l,i}) \Delta P_{Gi}^2 + \sum_{j=1}^{N_L} Var(a_{l,j}) \Delta P_{Lj}^2 + Var(P_{line_l}). \quad (30)$$

Constraints of Eqs. (19) and (25) are the set of equivalent constraints for the stochastic constraints in Eq. (14) [17]. These equivalent constraints transform the stochastic optimization into a deterministic one. Therefore, the solution of the stochastic CM in Eqs. (13)–(17) can be obtained by solving the deterministic optimization problem in Eqs. (18)–(22) or Eqs. (24)–(30). The equivalent deterministic problem in Eqs. (24)–(30) is a nonlinear optimization that can be solved by the sequential quadratic programming algorithm.

A primary Monte Carlo simulation is carried out to identify mean and variance values of the line flows and shift factors  $(\bar{a}_{l,i}, Var(a_{l,i}))$ . The uncertain model of loads and network equipment is utilized in the primary Monte Carlo simulation. The main difference between the results produced by the linear and nonlinear methods arises from the modeling of the shift factor variations, which are considered by the nonlinear approach.

The flow chart for the solution of CCP-based stochastic CM is presented in Figure 1. The flow chart consists of 2 main steps: primary Monte Carlo simulation to model the system uncertainties and produce PDF for line flows, and solving the equivalent deterministic problem for the proposed stochastic CM.

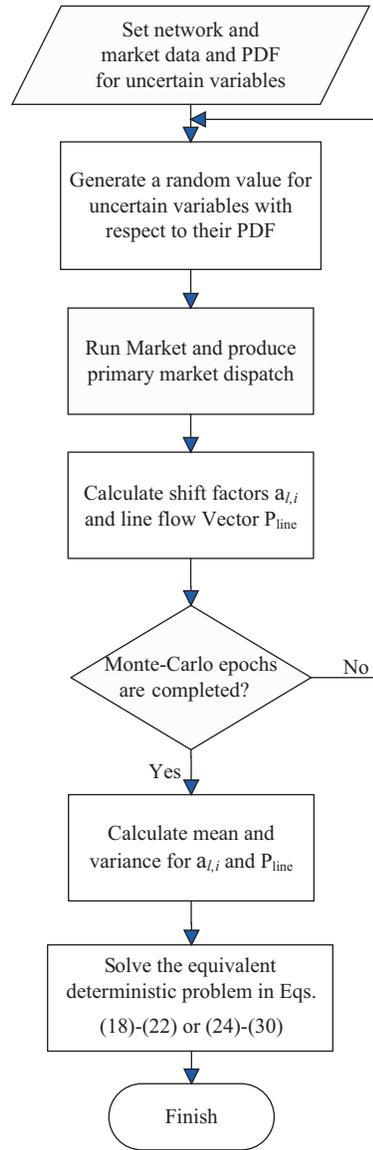
## 5. Simulation results

To evaluate the proposed probabilistic CM approach, both deterministic and probabilistic CM models have been applied to the IEEE 30-bus test system. Figure 2 shows the single-line diagram of the IEEE 30-bus test system, which includes 6 generators and 41 transmission lines.

A primary Monte Carlo simulation is performed based on the test system data presented in the Appendix (Tables A1 and A2), to model the system uncertainties and obtain the distribution function of the line flows. Mean and variance values related to the shift factors are also determined by the primary Monte Carlo simulation. The PDF for the power flow in line 14 is shown in Figure 3.

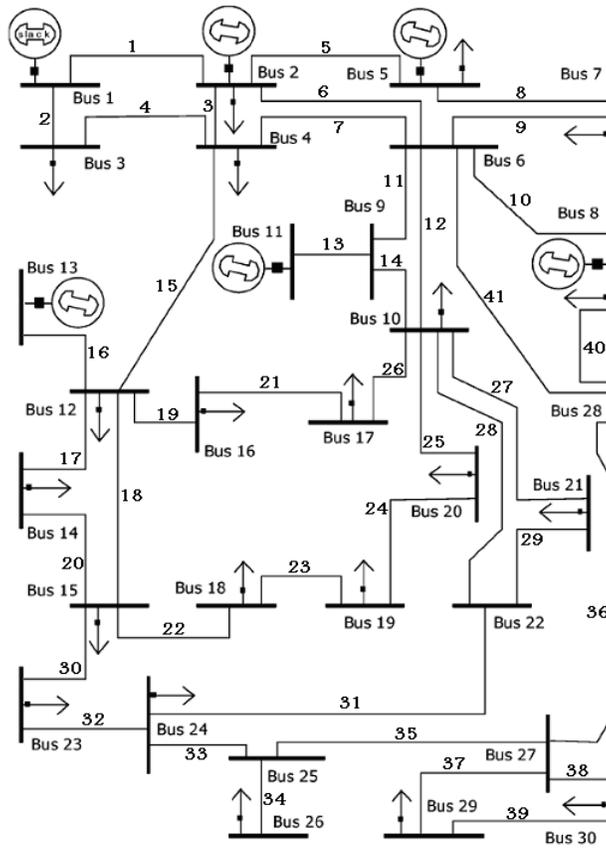
In this case study, 9 market participants, including generators installed at buses 5, 8, and 11 and dispatchable loads at buses 7, 8, 10, 15, 20, and 21 were selected to contribute to the congestion redispatch step. TCDFs for line 14, related to the market participants who contributed to the redispatch market, are shown in Figure 4. The deterministic approach and the proposed method in [9] utilize the deterministic values of TCDFs, whereas the stochastic method presented in [18] and the proposed approach in this paper use the mean values of the TCDFs. Figure 4 shows the deterministic and expected values of the TCDFs, which are used in deterministic and stochastic CM, respectively.

In this table, the total cost of redispatch, total load decrement, changes in power flow in line 14, and probability of network constraints violation are presented for each method. If the deterministic method is applied to relieve congestion, the probability of constraint violation will be approximately 50% with respect

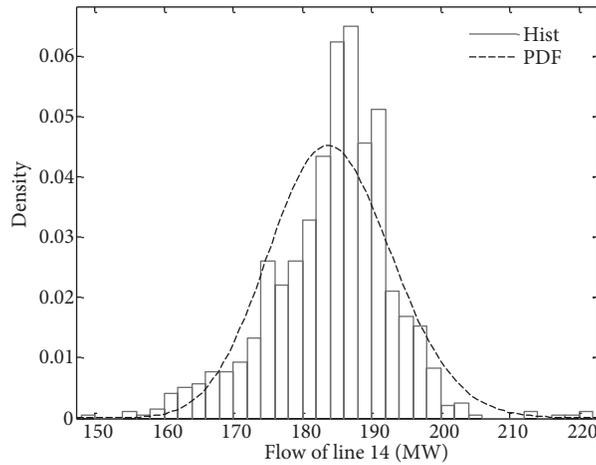


**Figure 1.** Flow chart for the solution of CCP-based stochastic CM.

to the PDF of the line flows. This figure decreases to 40% with the method presented in [9]; however, the total cost of redispatch increases by 30 \$/h to promote the probability of constraint satisfaction. To achieve the same situation,  $\alpha_l$  is set to 0.6 in the stochastic CM, which means that the maximum allowed constraint violation is 0.4. The same value is employed for  $\alpha_l$  to have a better comparison between the stochastic CM approaches. The proposed stochastic CM is solved with 3 methods: the method proposed by [18], and the linear and nonlinear analytical approaches. It is clear that the solution of the analytical method has prominence over the solution obtained by the numerical method, because the proposed method not only has the flexibility to set the confidence level, but also uses fewer approximations compared to the method in [9]. Moreover, the proposed method imposes less computational burden than the numerical method.



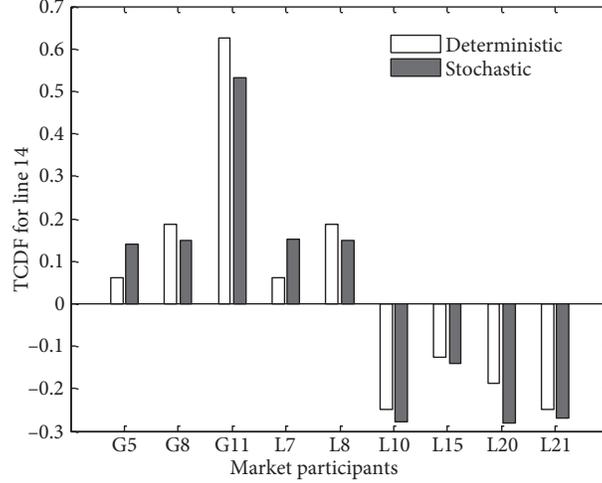
**Figure 2.** Single-line diagram of the IEEE 30-bus test system.



**Figure 3.** Probability distribution function for the power flow in line 14.

To simulate the proposed method in the case study, 5 different methods are implemented and compared. The first method is a deterministic approach that was introduced in [11], and the second approach is a stochastic CM method presented in [9]. This method uses scenario generation and reduction in order to solve the stochastic CM. The other 3 approaches are the proposed CCP-based congestion management technique with different solving algorithms, including the Monte Carlo-based real-coded genetic algorithm (RCGA) [18] and the 2

models of the analytical approach [17]. The results of managing congestion on the 30-bus test system using the mentioned methods are shown in Table 1.



**Figure 4.** TCDFs for line 14 in the IEEE 30-bus test system.

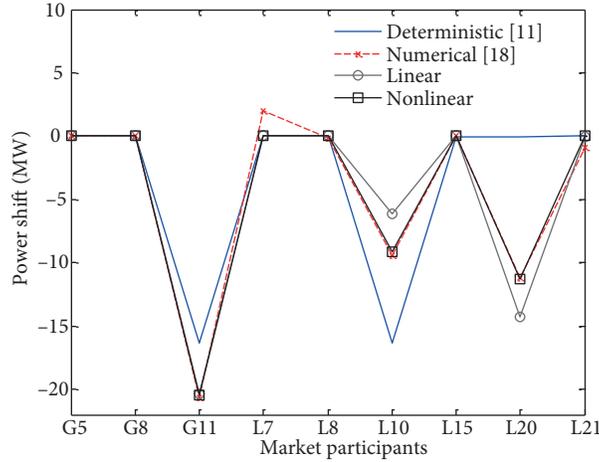
**Table 1.** Redispatch results with different methods.

Method	Algorithm	Cost (\$/h)	$\Delta P_L$ (MW)	$\Delta P_{line_{14}}$ (MW)	$\Pr(P_{line})$
Deterministic [11]	-	121.2	-16.38	-14.33	0.4897
Stochastic [9]	Scenario generation and reduction	142.8	-19.28	-16.30	0.4065
Stochastic [18]: ( $\alpha = 0.6$ )	Monte Carlo-based RCGA	159.9	-20.60	-17.26	0.3615
Proposed method: ( $\alpha = 0.6$ )	Linear model	150.8	-20.38	-16.55	0.3919
Proposed method: ( $\alpha = 0.6$ )	Nonlinear model	151.3	-20.45	-16.60	0.3896

In contrast to the scenario-based approach proposed in [9], which selects the most important scenarios and eliminates the others, our proposed approach is an analytical method that utilizes the distribution function of output variables such as line flows. The PDF of line flows can be generated using Monte Carlo simulation or any other method, such as cumulant or 2-point estimate method.

The redispatch power of each market participant for the methods presented in Table 1 is shown in Figure 5. The pattern of redispatch strategy in stochastic CM differs from the deterministic one, since the probability of the constraint violation in the stochastic approach is reduced.

Based on the comparison between the redispatch results of the deterministic and stochastic methods and the nonlinear model in Figure 5, we can conclude that the power decrement of G11 in the stochastic model is larger than that of the deterministic approach, since the mean value of the TCDF related to this generator is smaller than its deterministic value, as shown in Figure 4. Moreover, decrement in the power of L10 is smaller for the stochastic method, since the mean value of the related TCDF is larger compared to its deterministic value, as shown in Figure 4. The power shift for the other participants is equal to zero in both deterministic and stochastic methods, except for L20 in the stochastic model. This is due to the fact that this model has a smaller value of constraint violation probability.



**Figure 5.** Redispatch power of the market participants using different approaches.

The flexibility of the proposed CCP-based CM is higher than in the other methods, since the confidence level  $\alpha_l$  is introduced in this method. Change in  $\alpha_l$  causes changes in the relief strategy to meet the satisfaction level, which is defined for the stochastic constraints. The results of stochastic congestion redispatch with the proposed linear and nonlinear methods under different confidence levels are shown in Table 2.

**Table 2.** Redispatch results with different confidence levels.

Method	Cost (\$/h)	$\Delta P_L$ (MW)	$\Delta P_{line_{14}}$ (MW)	$\overline{\Pr}(P_{line})$
Linear ( $\alpha = 0.8$ )	197.7	-26.69	-21.69	0.2001
Linear ( $\alpha = 0.9$ )	232.9	-31.46	-25.54	0.1060
Linear ( $\alpha = 0.99$ )	316.0	-42.71	-34.68	0.0148
Nonlinear ( $\alpha = 0.8$ )	200.4	-27.08	-21.98	0.1925
Nonlinear ( $\alpha = 0.9$ )	238.4	-32.21	-26.15	0.0947
Nonlinear ( $\alpha = 0.99$ )	335.6	-45.36	-36.81	0.0086

As can be seen in Table 2, first, the costs of congestion redispatch rise along with increasing  $\alpha_l$  in both linear and nonlinear methods. This is due to the decrease in the probability of constraint violation, i.e.  $\overline{\Pr}(\Delta P_{line})$ . It should be noted that  $\overline{\Pr}(\Delta P_{line})$  is almost equal to  $1 - \alpha_l$ , which is quite reasonable. Second, the cost of redispatch in the nonlinear method is higher than in the linear approach, since the nonlinear method considers the variation of shift factors in stochastic optimization. In fact, in the nonlinear method, the level of system uncertainties is larger in comparison to the linear approach, and consequently the amount of redispatched power increases to provide the level of constraint satisfaction.

The redispatch power of each market participant in the nonlinear method is shown in Figure 6. Three different confidence levels,  $\alpha_l = 0.8$  or  $0.9$  or  $0.99$ , are simulated.

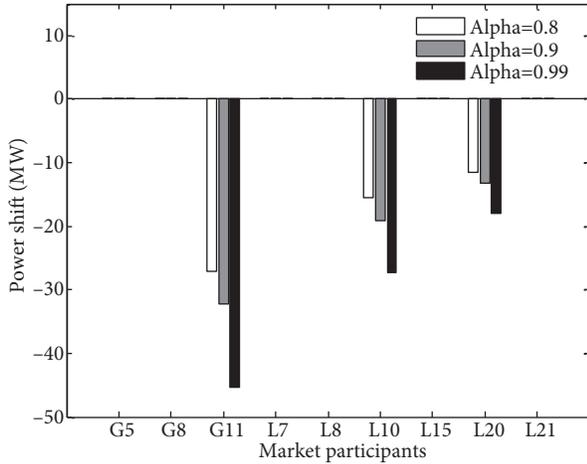
As can be seen in Figure 6, change in the confidence level  $\alpha_l$  leads to a change in the redispatch pattern. The redispatch power of the selected participants rises with an increase of  $\alpha_l$  to achieve the required satisfaction probability.

To determine the best value for  $\alpha_l$  in managing network congestion, the system operator can utilize a specific measure comparing the results of the system redispatch on the transmission network. In this paper, the interruption cost for postcontingency is applied to identify the efficiency of each proposed strategy with

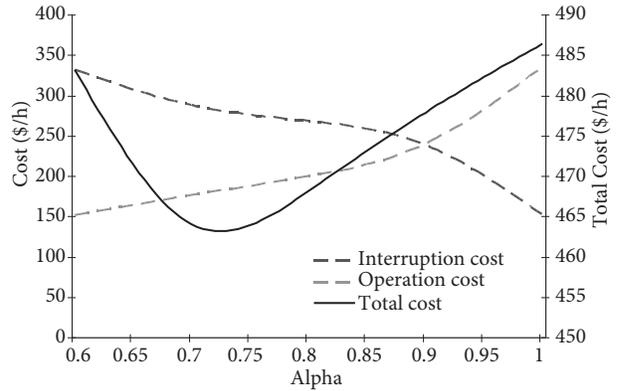
different values for  $\alpha_l$ . The expected value of interruption cost can be formulated as follows:

$$EIC = \sum_{t=1}^{N_{Cont.}} [\Pi_t \times VOLL \times \Delta P_t^{int.}]. \quad (31)$$

$N-1$  contingency analysis is performed on the network with the proposed strategy for congestion management to calculate the expected interruption cost (EIC). The amount of required load interruption ( $\Delta P_t^{int.}$ ) to remove the intense line overloads in each of the probable contingencies is the base part of the EIC. The VOLL will be paid to the loads that are curtailed to remove line overloads in postcontingencies, although it is usually much higher than the demand bids in the congestion redispatch step. Cost of market redispatch, expected interruption cost for postcontingency, and total cost of congestion management are calculated for different values of  $\alpha_l$ . Figure 7 shows the variation curve for each part of the cost described above.



**Figure 6.** Redispatch power of market participants with different  $\alpha_l$  values in the nonlinear method.



**Figure 7.** Interruption, operation, and total cost of redispatch with different values of  $\alpha_l$  in the nonlinear method.

As can be inferred from Figure 7, the best confidence level value in the studied network is approximately 0.72, which has the least total cost compared to the other values of  $\alpha_l$ . It is clear that identifying the best value for  $\alpha_l$  depends on the measure selected by the operator to compare results, the type and value of lost loads in  $N-1$  contingency analysis, and the status of the studied network. However, in this paper, an appropriate approach to evaluating the efficiency of the proposed strategies for CM and determining the best value of  $\alpha_l$  was utilized.

## 6. Conclusion

This paper proposed a new approach for probabilistic CM based on CCP. Introducing the confidence level in the congestion redispatch step promotes the flexibility of the CM approach. CCP-based probabilistic CM is a complex problem that is difficult to solve since it includes a set of stochastic constraints. The analytical approach in this paper, developed to solve the stochastic optimization problems, was employed to find a solution to the CCP-based CM problem. Moreover, a numerical method, based on a real-coded genetic algorithm, and a Monte Carlo simulation were also implemented for comparison with the analytical approach. A real-coded GA was used to find the optimum solution for the CM problem, while the Monte Carlo simulation was implemented to investigate the fulfillment level of the stochastic constraints.

The analytical method is more appropriate for real power systems, since it transforms the CCP-based CM problem to an equivalent deterministic problem. In fact, this approach not only converges in a limited time but also has an acceptable accuracy level.

The obtained simulation results show that the probability of the constraints' satisfaction identifies the redispatch strategies in different market conditions. The cost of the market redispatch increases when the system operator intends to have a higher probability of constraints fulfillment. Under such conditions, there will be larger changes in the primary arrangement of the market and more reduction in the flow of the congested lines compared to other situations with less fulfillment probabilities. In fact, the new formulation of the probabilistic CM models the probable system conditions and, consequently, the proposed strategy for congestion relief will have an acceptable confidence level, as decided by the system operator.

The main contribution of this paper is the proposition of a new formulation for stochastic CM using CCP, which allows the system operator to have a desirable level for system security and reliability, in contrast to the proposed method in [9], which uses the expectation of the selected scenarios. Furthermore, an analytical solving approach for CCP-based stochastic CM has been proposed in this paper. The analytical approach has less complexity and computation burden compared to the proposed algorithms in [12] and [18] in solving CCP-based problems.

It should be noted that the purpose of this paper was not to determine the optimal value of the confidence level in the CM process, but rather to propose a new approach to include power system uncertainties. However, the system operator can identify its desirable confidence level by evaluating the efficiency of the proposed strategy via the CCP-based approach. In this paper, a desirable level for  $\alpha_l$  using  $N-1$  contingency analysis and the expected interruption cost was computed.

## Nomenclature

### Sets

$N_G$	Set of generators
$N_L$	Set of dispatchable loads
$N_{\text{cont.}}$	Set of probable contingencies

### Functions

$C$	Redispatch function for market participants
$\rho$	Normal distribution function

### Variables

$\Delta P_{Gi}$	Redispatch power of generator $i$
$\Delta P_{Lj}$	Redispatch power of load $j$
$\Delta P_t^{\text{int.}}$	Interruption power for contingency $t$
$P_{line_l}$	Flow of line $l$
$\Delta P_{line_l}$	Change in flow of line $l$
$\Delta P_{Gi}^+, \Delta P_{Gi}^-$	Incremental and decremental redispatch power of generator $i$

**Constants**

$\overline{P_{line_l}}$	Expected value of $P_{line_l}$
$Var(P_{line_l})$	Variance of $P_{line_l}$
$P_{line_l}^{\max}$	Maximum allowed flow in line $l$
$\Delta P_{Gi}^{\min}, \Delta P_{Gi}^{\max}$	Maximum allowed redispatch power for generator $i$
$\Delta P_{Lj}^{\min}, \Delta P_{Lj}^{\max}$	Maximum allowed redispatch power for load $j$
$\alpha_l$	Confidence level of the $l$ th line flow
$a_{l,i}$	Transmission congestion distribution factor (TCDF) for line $l$ with respect to injection in bus $i$
$\overline{a_{l,i}}$	Mean value of $a_{l,i}$
$MTTR$	Mean time to repair
$MTTF$	Mean time to failure
$\sigma_i$	Standard deviation for random variable $i$
$r_{ij}$	Correlation coefficient between $i$ th and $j$ th random variables
$\mu_i$	Mean value for random variable $i$
$s$	Number of random variables in random vector $\xi$
$E_l$	Standard normal variable related to $\alpha_l$
$VOLL$	Value of lost loads
$\Pi_t$	Probability of contingency $t$

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