Markovian approach applied to reliability modeling of a wind farm

Mazaher HAJI BASHI*, Akbar EBRAHIMI
Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran

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Abstract: The exploitation of wind energy as a resource for generating electricity is going to make a leap forward. Large-scale wind farms (WFs) will be engaged with power systems in the near future and this will cause changes in the supply pattern. Wind speed has an intermittent behavior and the output power of a WF is highly variable, consistent with this fact. The Markov process could consider this characteristic of wind speed using the state transition rate probability matrix, an issue that is neglected when a probability distribution function is established to model the wind speed data. The adequacy indices of a power system describe the ability of the generation units to meet the load demand. When a WF is going to be added to a generation system or an existing WF is subjected to expansion study, in addition to the probability of occurrence for the wind speed states, which is considered in the probability distribution function of the wind speed, temporal relations between the wind speed states may affect the adequacy indices. In this paper, the effect of considering transition rates on adequacy indices is studied using a Markovian approach. A WF is modeled with real wind speed data, and then it is added to the generation system of Roy Billinton test system. The transition rate matrix of the WF is modified by definition of the scenarios and the effect of this modification on the loss of load expectation (LOLE) and loss of load frequency (LOLF) is studied. Next, the LOLF index, in comparison with the LOLE, is assessed in expansion strategies and the results enhance the importance of using a Markovian approach and considering the temporal relation of wind speed states in adequacy studies.

Key words: Wind farms, adequacy assessment, wind farm expansion, reliability evaluation, Markovian approach

1. Introduction

Wind farms (WFs) are going to have a great contribution in generating electricity. Large-scale WFs like the North Sea project are indications of this fact [1,2]. As a generation unit, WFs improve the ability of power systems to meet the demand in a certain manner. To quantify the effect of WFs on the ability of power systems to meet the load demand, reliability studies, and especially adequacy assessment, are performed. The reliability evaluation of a power system falls into 3 hierarchical levels in which 1, 2, or 3 main parts of the power system, in addition to the system load, are studied. In the first hierarchical level, the generation system is modeled and combined with the load model to derive the adequacy indices. In the second level, the transmission system is also considered in the studies and the effects of failure for this part of the power system are also included in the results. In the third level, the generation, transmission, and distribution system is covered [3]. Mainly, failure of the power system elements causes the contingencies that lead to the loss of the system supply. These contingencies that affect the entire behavior of the system and its reliability are usually modeled by stochastic methods. The first hierarchical level of reliability studies considers the power system as the generation system
and the load, and the capability of the generation system to supply the load adequately is studied. In other words, the principle concern of the first level is to determine or estimate the required capacity of the generation system to meet the load sufficiently. In this level, the generation system is modeled by states corresponding to the failure and operating modes of the generating units. However, modeling a WF distinctly differs from an ordinary generation unit (OGU). The variable output power of WFs consistent with the intermittent behavior of wind speed is the reason for the significant difference between them and OGUs in reliability modeling [4–9]. Several methods have been proposed to cover this difference, which mainly fall into 2 categories: analytical and simulation methods. Simulation circumstances like the Monte Carlo method [7,10] and the autoregressive and moving average [5] are used to model WFs for reliability assessment purposes. These methods need long-term data of wind speed, and this kind of data may not be available for certain wind sites. The method proposed in [5] tried to simplify the modeling process of wind speed and WF output power, but it was limited to the wind sites that had similar annual averages and standard deviations as those data that were used in [5]. Analytical methods were also used to model WFs for many years [11,12]. Using analytical methods reduces the computational efforts in comparing the simulation methods. The reliability models of OGUs originate from the reliability data of the failures and repair times, and the 2 state models cover all of the aspects of OGUs. The multistate model of output power has been used for reliability analysis of WFs [13]. The probability of occurrence for discrete output power states is considered using the probability distribution function of wind speed data [8,14]. The wind speed clusters into some discrete states, a histogram regarding the wind speed data is established, and the probability distribution function is fitted to the data. Using this method, just the probabilities of occurrence for each state are evaluated, while the probability of changes between these states could be effective on the result of adequacy studies. The Markov process considers the probability of transition between the output power states in addition to the probability of occurrence, and so it covers the intermittent behavior of wind speed with more details as compared with the probability distribution function (PDF). In [15], the Markov process was used for analyzing wind resources, emphasizing the advantage of considering temporal relations between wind speed states using the transition probability matrix, an issue that is neglected when a PDF is used to model the wind speed. In [16,17], the Markov method was used to model WFs for reliability assessment studies. Using the proposed method in [16,17], a large number of states are obtained just for the WF and those must be added to the other states of the generation units to model the entire generation system, and so this method is not applicable to model large-scale WFs. In [18], an adequacy study of WFs was conducted using Markov chains and the method used was efficient to model large-scale WFs. However, similar to [16,17], the authors did not pay attention to the ability of the Markov process to consider the probability of transition between wind speed states, while as is shown in this paper, the probability of transition between wind speed states affects the results of adequacy studies. In [19], the Markovian approach was used to determine the appropriate wind turbine-rated power and wind resources based on the adequacy indices. In [20], the wind speed pattern variation was considered as a factor modifying the temporal relation between the output power states of a WF, but the probability of occurrence for the wind speed states was obtained using fundamental probability theory. Using this method, the temporal relations of wind speed states inherently do not affect the probability of occurrence, and so the loss of load expectation (LOLE) is obviously independent of this characteristic of wind speed, while it could be effective on this index. In this paper, the effect of considering transition rates on the adequacy indices of a power system containing a WF is studied and the importance of this fact is determined. Both the steady-state probabilities and fundamental probability theory are used to obtain the probability of occurrence for wind speed states and the LOLF and loss of load frequency (LOLE) are considered as the adequacy indices.
of the power system. The results obtained using both methods are compared and interesting conclusions are made. First, the WF is modeled with the real wind speed data of Khaf, a city in eastern Iran, and then the transition rates of the wind speed are modified to scrutinize the effect of this characteristic of the wind speed on the adequacy indices of a power system that contains a WF. This is done while taking into consideration that the transition rate manifests itself as an important factor in power system planning when a WF is going to be added to the generation system with a particular wind speed behavior. This paper is organized as follows: Section 2 explains the procedure of modeling a WF with the Markov process. In Section 3, the adequacy indices used for risk analysis are determined. Section 4 provides the case study and analysis of the results. Finally, in Section 5, the conclusions are made.

1.1. Modeling method
The first step in the adequacy assessments of a power system is to model the generation units. A generation unit has a failure rate $\lambda$ and a repair rate $\mu$ based on the reliability data that have been collected in the field. When a generation unit fails, the state of the system will change by the rate of $\lambda$ from the operating mode to the failure mode, and its probability, called unavailability, is derived using Eq. (1). Similarly, when a generation unit is going to be repaired, the state of the system will change by the rate of $\mu$ from the failure to operating mode and its probability, called availability, is derived from Eq. (2).

$$A = \mu/(\lambda+\mu)$$

$$U = A/(\lambda+\mu)$$

In this 2-state model, OGUs are assumed to produce 0 MW in the failure mode and to produce the rated power in the operating mode. However, when a WF is going to be modeled for reliability studies, the 2-state model will not work properly. This is due to the very different behavior of WFs compared with OGUs, having a nonlinear output power curve in different wind speeds as shown typically in Figure 1 [21]. For example, as is depicted in Figure 1, production of 0 MW occurs not only in the failure state, but the output of the WF is also 0 MW below the cut-in and after the cut-out speed of the wind turbine. Thus, the output power of a WF is highly dependent on the wind speed and a proper multistate model should be used.

![Figure 1. Power curve of the 3-MW wind turbine.](image)

1.2. Single wind turbine
Considering the intermittent behavior of wind speed is the most important part of the reliability modeling for a WF. In addition to the probabilities of occurrence for wind speed states, which are thoroughly considered using
PDF-based methods, the temporal relation of the wind speed states could also be effective on the reliability evaluation of the power systems containing WFs. The advantage of the Markov process, when compared with the PDF-based method, is the ability of the Markov method to consider the probability of transition between each discrete state. The wind speed is clustered into some discrete states that establish the states of the Markov process. These states correspond to the levels of the output power states. For deducing the state transition probability matrix, first the counting matrix is determined [3].

\[ C_{ij} = \text{number of observed transition from state } i \text{ to } j \]
\[ i = 1, ..., n \]
\[ j = 1, ..., n \]

Here, \( n \) is the number of discrete states. To deduce the \( S \) matrix, each row of the counting matrix (\( i \)th row) is divided by the total number of time steps in which the system settles in state \( i \).

\[ S_{ij} = \frac{C_{ij}}{T_i} \]

As is seen from the literature, there are 2 choices for deducing the probability of occurrence for each state [22,23]. The first is to use the steady-state behavior of the Markov process.

\[ P^{(t)} = P^{(0)} S^t \]

Here, \( P^{(0)} \) is the vector of primary probabilities and \( P^{(t)} \) is the vector of probabilities at time \( t \). The vector of the steady-state probabilities is deduced as follows:

\[ \lim_{t \to \infty} S^t = P_j \]

where \( P_j \) is the vector of the steady-state probabilities. As is seen from Eq. (6), the steady-state probability vector is independent from the primary probability vector and this is known as the Markovian property [22]. In contrast, the steady-state probability vector is dependent on the \( S \) matrix. In [18], the basic probability equation was used to obtain the probability of occurrence for each state. This method is called the second method in this paper.

\[ P_i = \frac{T_i}{\sum_{i=1}^{n} T_i} i = 1, 2, 3, ..., n \]

\[ P_j = [P_1, P_2, ..., P_n] \]

Here, \( T_i \) is the entire time in which the wind speed data correspond to state \( i \), and \( n \) is the total number of discrete states. The transition rates are obtained from Eq. (9).

\[ \lambda_{+i} = \sum_{j=1}^{n} S_{ij} \]  

\[ j > i \]
\[ \lambda_{+i} = \sum_{j=1}^{n} S_{ij} \]  
(9b)

Here, \( \lambda_{+i} \) is the transition rate to the upper state (the states with higher output power) and \( \lambda_{-i} \) is the transition rate to the lower states (the states with lower output power). Using Eqs. (3)–(9), the single wind turbine reliability model could be obtained as is illustrated in Figure 2.

The arrows indicate the probability of transition between output power states in a single wind turbine, which happen due to the variation of the wind speed. It is possible to have an arrow between each of the 2 states that depends on the wind speed data, but all possible states are not illustrated intentionally to prevent the figure from being obscure. In fact, \( S \) is a matrix with dimensions of \( n \times n \), in which its elements are obtained from Eq. (4) and strongly depend on the wind speed pattern.

1.3. WF model

Based on the method employed to obtain the probabilities, the WF model can be established. In the Markov chain method, the failure and repair rates for a wind turbine are used to form the \( S \) matrix for the entire WF. There are 2 parameters determining the state of the Markov model for the WF. The first is the wind speed and the latter is the number of wind turbines operating properly (not in failure mode). As is illustrated in Figure 3, the letter \( k \) in each row indicates the number of wind turbines that are in operating mode. Moving to lower rows, the number of unfailed turbines decreases and vice versa.

In the first row of Figure 3, all of the wind turbines are available and each one could fail in the next time step. If \( \lambda_t \) indicates the failure rate of a wind turbine, the probability of transition from the states that are in the first row to the states that are the in the second row is equal to \( k \lambda_t \). Similarly, if \( \mu_t \) indicates the repair rate of a wind turbine, the probability of transition from the second row to the first row is equal to \( \mu_t \).
because only one of the wind turbines is in failure mode in the states of the second row. The transition rate between all the vertical states could be determined in a similar manner. The probability of transitions between the horizontal states is determined based on the intermittent behavior of the wind speed, as was explained in the previous subsection.

To establish the $S$ matrix for a WF, it is necessary to consider the failure and repair rates between the WF output power states, as illustrated above. For this purpose, the zero matrices are defined as follows.

$$ z = [] \quad i = 1 $$

$$ z(n, n(i - 1)) \quad i \neq 1 $$

If $N_t$ is the total number of wind turbines constituting the WF, $i = 1, 2, \ldots, N_t$ is considered as the loop counter and $ii = N_t$, and the $S$ matrix for the WF ($S_{wf}$) is deduced as is illustrated in Figure 4.

Figure 4. Obtaining the $S$ matrix of the WF.

Here, $I(n)$ is an $n \times n$ identity matrix. Establishing the $S$ matrix with respect to Figures 3 and 4, the probability of occurrence for each state is obtained using the Markovian property. If the second method is used to obtain the probabilities, the $S$ matrix is used only to obtain the transition rates and Eq. (12) gives us the probability of occurrence for each state from the basic probability rules.

$$ P_i(N_{avi}, n) = \left( \begin{array}{c} N_t \\ N_{avi} \end{array} \right) (1 - avi)^{N-t-N_{avi}} avi^{N_{avi}} P_j(1, n) $$

Here, $N_{avi}$ is the number of wind turbines operating properly in each state, $N_t$ is the total number of wind turbines consisting the WF, $avi$ is the availability of a single wind turbine, and $n$ is the number of considered output power states for a single wind turbine. Independent of the method used for obtaining the probabilities, some similar output power states will exist in the WF model. Such similar states should be merged. The merging equations are found in [18]. The final step toward the WF model is to obtain the frequency of each output power state, as follows.

$$ f_i = P_i(\lambda_{+i} + \lambda_{-i}) $$

Finally, the WF model is determined, which contains some discrete levels of the output power, corresponding probability of occurrence, transition rate to the upper and lower states, and frequency of occurrence for each output power level.

For the sake of defining the wind speed pattern variation and to unveil the difference between the 2 probability calculation methods, a basic example is advanced in this section. Two separate systems are illustrated in Figure 5. The systems are assumed to have 3 symbolic states, such as 1, 2, and 3. The states take place in 2 different patterns. In fact, the time sequence of settling the system in the different states is not similar.
The counting matrices $C_a$ and $C_b$ for systems a and b are obtained as follows.

$$C_a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad C_b = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ \quad (14)

The $S$ matrices are also equal to the following.

$$S_a = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}, \quad S_b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \end{pmatrix}$$ \quad (15)

As seen from Figure 5 and Eqs. (14) and (15), the 2 systems have different $S$ matrices. The probability of occurrence for systems a and b is calculated using Eqs. (6)-(8), and the results are illustrated in Table 1.

**Table 1.** Probability of occurrence for the simple system of Figure 4.

<table>
<thead>
<tr>
<th>Markov method</th>
<th>Basic probability method</th>
<th>The states of system a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3333</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0.3333</td>
<td>2</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3333</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Markov method</th>
<th>Basic probability method</th>
<th>The states of system b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.3333</td>
<td>1</td>
</tr>
<tr>
<td>0.25</td>
<td>0.3333</td>
<td>2</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3333</td>
<td>3</td>
</tr>
</tbody>
</table>

As seen from Figure 5 and Eqs. (14) and (15), the 2 systems have different $S$ matrices. The probability of occurrence for systems a and b is calculated using Eqs. (6)-(8), and the results are illustrated in Table 1.

In fact, this condition is too contingent on the wind velocity. The severity of the variation in the wind speed pattern will not be detected if the PDF-based method is used for modeling the wind speed. On the other hand, if the Markov chain is used to model the wind speed, not only will the probabilities be attainable, but the probable differences between the wind speed patterns will also be detectable.

The transition between the states is clearly illustrated in Figure 6 if each interval (5 m/s) is considered as a wind speed state (these data are available in [24]). The aim of this paper is to determine the effect of considering the effect of transition rates for wind speed data on the adequacy indices of a generation system that contains a WF.
In this paper, it is assumed that all wind turbines elicit similar kinetic energy from the wind source and the wake effect is not considered in the calculations. The spatial arrangement of wind turbines constituting the WF affects the output power of the wind turbines, especially those downstream. The kinetic energy of the wind decreases due to the energy harvesting of the wind turbines and this effect could be considered using the efficiency factor, which also covers other losses, equal to 90% to 95%, as is done in the literature [7]. Recently a method was proposed to thoroughly evaluate the impacts of the wake effect on the output power of WFs using the Jensen model [25].

2. Adequacy indices

In this paper, the WF is added to the Roy Billinton reliability test system (RBTS) generation system [26] and the adequacy indices are derived by convoluting the load model with the generation system model. The RBTS is a test system that is frequently used for reliability studies. The RBTS load is fed by 240 MW of ordinary generation units. The pick load is equal to 185 MW. After obtaining the output capacity table (OCT) of the WF, it could be added to the RBTS generation system by a recursive algorithm [3]. The new system is called the modified RBTS (MRBTS) and this study is developed to deduce the adequacy indices of this system. The load margin method is selected for convoluting the load and generation system model. The frequency and duration indices are derived with this method, such as the LOLE (hours per year) and LOLF (occurrences per year). Establishing the cumulative load margin table, the LOLE and LOLF are obtained as follows [22].

\[
\text{LOLE} = 8760 \times P_{fn} \quad (16)
\]

\[
\text{LOLF} = 8760 \times F_{fn} \quad (17)
\]

Here, \(P_{fn}\) and \(F_{fn}\) are the probability of occurrence and frequency of the first negative margin in the cumulative load margins table, respectively [3].

3. Case study

The real wind speed data of Khaf (a city in eastern Iran), as a wind resource, are used to model the WF [24]. The hourly average data of the wind speed are the inputs of the wind turbine power curve for obtaining the output power. A 3-MW GE wind turbine is used for the modeling and the power curve is illustrated in Figure 1 [21].

As stated previously, clustering the output power of a wind turbine is the first step of the modeling procedure. States of 0, 0.75, 1.5, 2.25, and 3 MW are considered as the discrete states for the 3-MW wind
turbine. Next, through Eqs. (3)–(9), the single wind turbine model is obtained. The output power states for the WF are considered to be the multiplications of the rated power for the wind turbine, so the output power states of the WF are dependent on the number of wind turbines that it contains. As an example, the OCT of the 18-MW wind turbine with the Khaf wind speed data is illustrated in Table 2.

### Table 2. OCT of the 18-MW WF.

<table>
<thead>
<tr>
<th>Capacity in MW</th>
<th>( \lambda_+ )</th>
<th>( \lambda_- )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2491</td>
<td>0.1068</td>
<td>0.0266</td>
</tr>
<tr>
<td>3</td>
<td>0.2630</td>
<td>0.2350</td>
<td>0.2009</td>
</tr>
<tr>
<td>6</td>
<td>0.1596</td>
<td>0.2537</td>
<td>0.2157</td>
</tr>
<tr>
<td>9</td>
<td>0.1655</td>
<td>0.2470</td>
<td>0.2212</td>
</tr>
<tr>
<td>12</td>
<td>0.1605</td>
<td>0.2187</td>
<td>0.2030</td>
</tr>
<tr>
<td>15</td>
<td>0.1811</td>
<td>0.1506</td>
<td>0.1800</td>
</tr>
<tr>
<td>18</td>
<td>0.0666</td>
<td>0</td>
<td>0.1130</td>
</tr>
</tbody>
</table>

### 3.1. Comparing the results of the 2 methods

In this subsection, 4 different sizes of WFs are modeled and added to the RBTS generation system individually. For each modified generation system, the adequacy indices are obtained and illustrated in Table 3.

### Table 3. Adequacy indices of the MRBTS using the 2 methods.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Markov method</th>
<th>Second method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LOLE</td>
<td>LOLF</td>
</tr>
<tr>
<td>9 MW</td>
<td>0.7382</td>
<td>0.3574</td>
</tr>
<tr>
<td>18 MW</td>
<td>0.5232</td>
<td>0.2627</td>
</tr>
<tr>
<td>27 MW</td>
<td>0.4189</td>
<td>0.2202</td>
</tr>
<tr>
<td>36 MW</td>
<td>0.3536</td>
<td>0.1851</td>
</tr>
<tr>
<td>45 MW</td>
<td>0.3240</td>
<td>0.1720</td>
</tr>
<tr>
<td>54 MW</td>
<td>0.3165</td>
<td>0.1694</td>
</tr>
</tbody>
</table>

As is seen from Table 3, the results are almost similar. Now, to study the effects of wind speed pattern variation on the adequacy indices of a power system, the wind speed pattern of Khaf is modified with 3 predefined scenarios.

### 3.2. Modification of transition rates

The most probable and severe variation in the wind speed pattern is the daily pattern variation, such as how the wind speed decreases in daylight and increases at night [27]. To modify the transition rates of the wind speed data in Khaf, random numbers are generated using uniform distribution. The random numbers correspond to 24 h of a day, sequentially. Each hour is then replaced with the wind speed that is registered for the same hour in the original wind speed data. In this manner, a new pattern is generated for the wind speed using the original wind speed data. The inherently random nature of the generated wind speed pattern causes us to consider 3 generated patterns for this scenario, named R1, R2, and R3.

Different rated power WFs are modeled in the conditions that the wind turbines encounter with the R1, R2, and R3 wind speed patterns. Next, each WF is considered as a generation unit and added to the RBTS generation system. The adequacy indices of the generation system, which contain a WF, are obtained and the results are illustrated in Tables 4 and 5.
As is seen from Tables 4 and 5, the results are nearly the same and the error between the results is less than 4%. Moreover, the LOLEs in Tables 4 and 5 are similar to the LOLEs in Table 3, regardless of the modified wind speed pattern and the method used for obtaining the probabilities. Using the second method, the modification of the transition rates does not affect the LOLE. This is similar to PDF-based methods that only consider the probability of occurrence for each discrete state. However, the second method used in this paper has a feature to study these probable effects on both the LOLF and LOLE, while the first and PDF-based method does not have this feature. Although Markov chains could study the probable effects of the transition rates of the wind speed on the LOLE index, the results from both methods show that the wind speed patterns do not affect the LOLE. As is seen from Tables 4 and 5, modification of the transition rates affects the LOLF index regardless of the method used for the WF modeling and has no influence on the LOLE. However, as stated previously, the modification of the transition rates affects the $S$ matrix and the probability of occurrence for each output power state, but why does the LOLE remains constant? The probability of occurrence for each output state is modified smoothly due to the modification of the transition rates. For obtaining the reliability
indices, the load and generation model are convoluted. This means that the probability of occurrence for the load margins is equal to the multiplication of the probabilities of the load model and generation system states. Hence, the small probability changes in the OCT that are observed due to the modification of the transition rates become smaller because of 2 multiplications: the first in establishing the generation system model and the other in obtaining the load margin probabilities. Thus, the LOLE, which is the expectation of hours in which the load exceeds the generation level and is obtained by summation of negative load margin probabilities, is not greatly affected by the modification of the transition rates. Knowing these results, it is possible to say that the transition rates affect the LOLF regardless of the mathematical model that is used for the WF modeling.

For obtaining the effective load carrying capability (ELCC) as an adequacy index, both the LOLF and LOLE could be considered as the system risk indices. In [28], it was shown that the resulting ELCC will be different using the LOLF or the LOLE if the WF is part of a generation system. Although the modifications of the transition rates performed in this paper are fictitious, they show the feature of the Markov chain method to study the effect of the transition rates of wind speed on the adequacy indices of a power system containing a WF.

In this section, the distinction between the LOLF and LOLE is studied. For this purpose, 3 scenarios are defined for the annual load growth (LG). The minimum satisfying risk indices are considered to be equal to the obtained index for the RBTS, while a 6-MW WF is added to the generation system.

Increasing the annual peak load along with the LG scenarios, the rated power of the existing WF increases to maintain the risk level of the entire system, higher than or equal to a predefined level. It is necessary to remodel the whole generation system, including the WF, while the size of the WF is increased since the WF is fed with a certain wind regime regardless of its rated power. This means that when the size of the WF is selected to be 18 MW, it is not correct to model 2 separate 9-MW WFs and add them to the generation system of the RBTS because the output power states for these WFs are not independent events according to the basic probability rules.

In this case study, the rated power of the WF is increased by 9-MW steps to satisfy the predefined risk index of the system. The LOLF and LOLE are considered as the risk indices. It is assumed that the physical environment of the site limits the maximum number of wind turbines to 78. Thus, the maximum rated power of the WF could be a 234-MW ($78 \times 3$) capacity with 3-MW wind turbines.

Figure 7 illustrates the appropriate rated power of the WF to satisfy the LOLE as the system risk index along with the LG scenarios. As is seen from Figure 7, in the third scenario, the WF could not satisfy the risk index due to a physical space limit. The profiles of the LOLE during the 5 years of study are illustrated in Figure 8.

![Figure 7. WF expansion based on the LOLE.](image)

![Figure 8. LOLE profile during the expansion program.](image)
Figure 9 illustrates the appropriate WF rated power that is needed to preserve the LOLF as the system risk index. The LOLF profile during the 5 years study is also illustrated in Figure 10.

![Figure 9](image1)  
**Figure 9.** WF expansion based on the LOLF.

![Figure 10](image2)  
**Figure 10.** LOLF profile during the expansion program.

Now the LOLF is selected as the risk index for the system and similar assessments are conducted. As is seen from Figure 10, the appropriate rated power of the WF for satisfying the LOLF is higher than or equal to this in comparison with the condition where the LOLE is considered as the risk index of the system. Hence, the results of this case study show that if the LOLF is considered as the risk index, the LOLE will also be satisfied, but if only the LOLE is considered as the risk index, the LOLF index will not achieve the desired level. For instance, in the condition of 1.5% LG, the appropriate rated powers of the WF for satisfying the LOLE and LOLF are compared in Figure 11.

![Figure 11](image3)  
**Figure 11.** Comparing the appropriate WF-rated power during the expansion program based on the LOLE and LOLF.

As is seen from Figure 11 that in the first and third year the appropriate rated power for satisfying both the LOLF and LOLE is the same, and in all of the remaining years, the LOLF dictates more WF rated power to achieve the desired risk level. Thus, the factors modifying the LOLF are as important as the factors modifying the LOLE. Wind speed pattern variation has the capability of modifying the LOLF. The reliability indices should be considered in the expansion and planning of the system because these indices describe the ability of the power system to meet the load demand. If this is done, the importance of the points mentioned in this paper will be more distinguished. It is also obvious from the results that if the probability distribution function is used for modeling the wind speed behavior, the effect of the variable characteristics of the wind speed on adequacy indices and the planning strategies will be neglected because the probability of occurrence
for the discrete states of a multistate model could not cover this important aspect and it is essential to consider the temporal relation of wind speed states in the reliability modeling of a WF.

In fact, this sensitivity analysis, which is performed on the LOLE and LOLF, unveils the importance of the LOLF in the planning of a power system from an adequacy point of view. Hence, the factors that have probable effects on the LOLF, such as the transition rates of the wind speed, also have great importance. In particular, if the power system that contains a WF is assigned to feed an industrial area, it will have a great influence on what the frequency of failure is in the generation system. For some special fields like the plastic industry, in which plastic is melted in a melting machine, the frequency of failure is more important than the amount of energy that is not delivered to the customer.

4. Conclusions

In this paper, the procedure of modeling WFs for reliability assessment, using the Markov process, was explained. In addition to the probability of occurrence for the wind speed states, the temporal relation between the wind speed states in a multistate model for WFs could be considered using the Markov process, while using probability distribution functions, these data are neglected and, consistent with this fact, the effect of these data on the adequacy studies of power systems containing WFs remains concealed. In this paper, the effect of the transition rates between the output power states of a WF on adequacy indices such as the LOLE and LOLF was determined. Two methods were used for obtaining the probability of occurrence of the output power states for a WF. Using the Markov steady-state probabilities to model the WF output power gives us the feature of considering the probable effects of the transition rates on the LOLF and LOLE, but using the fundamental probability theory could only allow us to study the probable effects of the transition rates on the LOLF. It is shown that, regardless of the method used to drive the probability of occurrence (the steady-state probabilities of the Markov process or fundamental probability theory), the LOLE is not affected by the transition rates and the temporal relation of the wind speed states only affects the LOLF. This shows the advantage of using the Markov chain method to model the output power of WFs for adequacy study purposes. The LOLE and LOLF were employed as the risk indices in a planning program and the results show that the appropriate rated power of the WF, which is needed to preserve the risk level, is higher than the condition in which the LOLE is to be preserved. This issue enhances the importance of the factors modifying the LOLF, such as the transition rates between the output power and the importance of using the Markovian approach, which could unveil the effect of these factors on the adequacy assessment results.

Nomenclature

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG</td>
<td>Load growth</td>
</tr>
<tr>
<td>LOLE</td>
<td>Loss of load expectation</td>
</tr>
<tr>
<td>LOLF</td>
<td>Loss of load frequency</td>
</tr>
<tr>
<td>MRBTS</td>
<td>Modified Roy Billinton reliability test system</td>
</tr>
<tr>
<td>OCT</td>
<td>Output capacity table</td>
</tr>
<tr>
<td>OGU</td>
<td>Ordinary generation unit</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability distribution function</td>
</tr>
<tr>
<td>RBTS</td>
<td>Roy Billinton reliability test system</td>
</tr>
<tr>
<td>WF</td>
<td>Wind farm</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>The number of observed transitions from state $i$ to $j$</td>
</tr>
<tr>
<td>$S$</td>
<td>State transition probability matrix</td>
</tr>
<tr>
<td>$i$</td>
<td>Subscript denoting the dimension of $S$</td>
</tr>
<tr>
<td>$j$</td>
<td>Subscript denoting the dimension of $S$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Probability of occurrence vector</td>
</tr>
<tr>
<td>$P_{ij}$</td>
<td>Probability of occurrence vector</td>
</tr>
<tr>
<td>$P_{j}$</td>
<td>Probability of occurrence vector</td>
</tr>
<tr>
<td>$S_{ij}$</td>
<td>The elements of $S$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time step (1 h)</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Duration of system settling in state $i$</td>
</tr>
<tr>
<td>$\lambda_{+i}$</td>
<td>Transition rate from $i$ to upper states</td>
</tr>
<tr>
<td>$\lambda_{-i}$</td>
<td>Transition rate from $i$ to lower states</td>
</tr>
<tr>
<td>$a_{vi}$</td>
<td>Availability of a wind turbine</td>
</tr>
<tr>
<td>$N_i$</td>
<td>The number of wind turbines constituting the WF</td>
</tr>
<tr>
<td>$K$</td>
<td>The number of wind turbines operating properly</td>
</tr>
</tbody>
</table>
$z(a, b) a \times b$ zero matrix
$I(n) n \times n$ identity matrix
$z^T$ Transpose of $z$ matrix
$f_i$ Frequency of occurrence for state $i$
$P_{fn}$ Probability of first negative margin
$F_{fn}$ Frequency of first negative margin

References


