A neural-based electromagnetic inverse scattering approach to the detection of a conducting cylinder coated with a dielectric material

Senem MAKAL∗, Ahmet KIZILAY
Department of Electronics and Communications Engineering, Faculty of Electrical and Electronics Engineering, Yıldız Technical University, 34349 İstanbul-TURKEY
e-mails: smakal@yildiz.edu.tr, akizilay@yildiz.edu.tr

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Abstract
In this study, a radial basis function neural network approach is applied for the estimation of the localization and the radius of a conducting cylinder with a circular cross-section coated with a dielectric material. A set of features, the radar cross-section (RCS) values, are derived from scattered fields, which are calculated using the surface equivalence principle and the method of moment. RCS values are obtained using 10 different scattering angles that are fed into the network. The outputs of the network are the location \((x_0, y_0)\) and the radius \(r_p\) of the conducting cylinder. This is an application of the electromagnetic inverse scattering of the objects embedded in a material based on the use of a neural network.

Key Words: Electromagnetic scattering, the method of moment, neural network, inverse scattering

1. Introduction
Because of their significant applications in geophysics, seismology, ground-penetrating radar, biomedical fields, and the localization of cables and pipes, electromagnetic inverse problems attract great interest [1–5]. Different approaches to the localization of a target have been proposed in past years, but the solutions relating to this problem are complex, time-consuming, and computationally intensive.

Recently, neural network-based algorithms have been proposed for the extraction of information about a target [6–11]. They are applied to electromagnetic inverse problems such as the detection of mines, the classification of radar targets, microwave imaging, or pattern recognition of subsurface images [12–16].

Neural network approaches have previously been used for inverse scattering problems related to the detection of the localization, radius, or electrical parameters of a cylindrical object of a circular cross-section without any coating material [8–10]. In this paper, a neural network approach is applied to the inverse scattering

∗Corresponding author: Department of Electronics and Communications Engineering, Faculty of Electrical and Electronics Engineering, Yıldız Technical University, 34349 İstanbul-TURKEY
problem of a conducting cylinder coated with a dielectric material having different boundaries. As a result, the effects of the thickness, the permittivity, and the boundary shape of the coating material on the neural network performance are investigated.

2. Calculation of the scattered electric field

The data set utilized in the neural network is obtained using the surface equivalence principle and the method of moment (MoM) [17], because the boundary of the coating material is not circular.

The scattering problem is shown in Figure 1, consisting of a perfectly electrically conducting (PEC) cylinder and a dielectric layer. External impressed sources produce the incident transverse magnetic wave (TM\(\_z\)) and the total fields at an arbitrary point outside the dielectric cylinder are \(\vec{E}\) and \(\vec{H}\). The surfaces of the dielectric coating and the conducting cylinder are \(S_d\) and \(S_c\), respectively.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{scattering_problem.png}
\caption{The scattering problem.}
\end{figure}

The surface equivalence principle is used to find the total field at an external point. It has 2 steps, containing 2 equivalent and simpler problems. Figure 2 shows the external equivalence problem. The equivalent surface electric current \(\vec{J}_d\) and the equivalent surface magnetic current \(\vec{M}_d\) radiate into the unbounded medium \((\varepsilon_0, \mu_0)\) [18].

\begin{align*}
\vec{J}_d &= \hat{n}_d \times \vec{H}(S_d^+) \\
\vec{M}_d &= \vec{E}(S_d^+) \times \hat{n}_d
\end{align*}

\(S_d^+\) represents the surface just outside \(S_d\) and \(\hat{n}_d\) is the outward unit normal vector to \(S_d\). The total field is zero inside \(S_d\), denoted as \(S_d^-\):

\[\vec{E}^* \left( \vec{J}_d, \vec{M}_d \right) = -\vec{E}^i \text{ on } S_d^-.
\]

Figure 3 shows the internal equivalence problem. The total field is zero for both \(S_d^+\) and \(S_c^-\):

\[\vec{E}^* \left( -\vec{J}_d, -\vec{M}_d, \vec{J}_c \right) = 0 \text{ on } S_d^+,
\]

\[\vec{E}^* \left( -\vec{J}_d, -\vec{M}_c, \vec{J}_c \right) = 0 \text{ on } S_c^-.
\]
\[ \vec{E}^s \left( -\vec{J}_d, -\vec{M}_d, \vec{J}_c \right) = 0 \text{ on } S_c. \] (5)

Three integral equations, Eqs. (3) through (5), are solved using the MoM for 3 unknown surface currents \((\vec{J}_d, \vec{M}_d, \vec{J}_c)\), and surfaces \(S_d\) and \(S_c\) are, respectively, approximated by linear segments:

\[
\vec{J}_c \left( \vec{\rho} \right) = \hat{z} \sum_{i=1}^{N_c} I_{c,i} P_c \left( \vec{\rho} \right),
\]

\[
\vec{P}_d \left( \vec{\rho} \right) = \hat{z} \sum_{i=1}^{N_d} I_{d,i} P_d \left( \vec{\rho} \right),
\]

\[
\vec{M}_d \left( \vec{\rho} \right) = \sum_{i=1}^{N_d} \hat{\tau}_i K_d \left( \vec{\rho} \right),
\]

where \(\vec{\rho}\) represents the source points; \(N_d\) and \(N_c\) are the numbers of segments on \(S_d\) and \(S_c\), respectively; \(I_{c,i}\) and \(I_{d,i}\) are the unknown values of the electric currents on the \(i\)th segment of \(S_c\) and \(S_d\), respectively; and \(K_d^i\) denotes the value of the magnetic current on the \(i\)th segment of \(S_d\). Pulse functions \((P_c, P_d)\) are chosen as the expansion functions. The unit vector in the circumferential direction tangent to the \(i\)th segment of \(S_d\) is denoted by \(\hat{\tau}_i\) and the unit vector in the \(z\)-direction is denoted by \(\hat{z}\). After the surface currents are calculated, the far scattered field can be computed using only \(\vec{J}_d\) and \(\vec{M}_d\) [19,20]:

\[
E^s_z = \frac{-\omega \mu_0}{4} \sqrt{\frac{2}{3}} \frac{e^{-jk_0 \rho}}{\sqrt{\rho}} \sum_{i=1}^{N_d} I_{d,i} e^{-jk_0 (x' \cos \phi_s + y' \sin \phi_s)} \Delta_{d,i}^i, \]

\[
+ \frac{4k_0}{4} \sqrt{\frac{2}{3}} \frac{e^{-jk_0 \rho}}{\sqrt{\rho}} \sum_{i=1}^{N_d} K_d^i e^{-jk_0 (x' \cos \phi_s + y' \sin \phi_s)} \Delta_{d,i}^i,
\] (9)

where \(k_0\) is the free-space wave number, \(\phi_s\) is the scattering angle, \(\rho\) is the distance measured from the origin, and \(\Delta_{d,i}\) is the length of the segment on \(S_d\).

**Figure 2.** The external equivalence problem.

**Figure 3.** The internal equivalence problem.
3. Neural-based electromagnetic inverse scattering approach

After determining the scattered field values, the inputs of the neural network are obtained by calculating radar cross-section (RCS) values at 10 equally spaced observation points [19]:

\[
\sigma (\varphi_s) = \lim_{\rho \to \infty} 2\pi \rho \left| \frac{E_z^s(\rho, \varphi_s)}{E^s_z(\rho, \varphi_s)} \right|^2.
\] (10)

In this study, the radial basis function (RBF) network is used to find the location and the radius of the conducting cylinder. RBFs are embedded into a 2-layer feed-forward neural network characterized by a set of inputs and a set of outputs. This network has a hidden layer between the input and output layers, as shown in Figure 4. Each hidden unit implements a RBF [22].

The training parameters \( \lambda_{ij} \) and \( c_{ij} \) are used to calculate the total input to the \( i \)th hidden neuron, \( \gamma_i \):

\[
\gamma_i = \sqrt{\sum_{j=1}^{n} \left( \frac{x_j - c_{ij}}{\lambda_{ij}} \right)^2}.
\] (11)

For \( i = 1, 2, \ldots, N \), \( N \) is the number of hidden neurons. \( z_{ij} \) is the output of the \( i \)th hidden neuron.

\[
z_{ij} = \sigma (\gamma_i)
\] (12)

Here, \( \sigma (\gamma) \) is a RBF. Hidden neurons are used to calculate the outputs of the network.

\[
y_k = \sum_{i=0}^{N} \omega_{ki} z_i
\] (13)

Here, \( \omega_{ki} \) is the weight between the \( i \)th neuron of the hidden layer and the \( k \)th neuron of the output layer for \( k = 1, 2, \ldots, m \) [23].

The input layer of the RBF is designed to have 10 neurons, including the RCS values, and the output layer has 3 neurons corresponding to the position coordinates and the radius of the conducting cylinder seen in Figure 5. The MATLAB “newrbe” command is used with the spread value of 0.9.
4. Numerical results

The geometry of inverse scattering problems is shown in Figures 6 and 7. In Figure 6, the boundary of the coating is given by:

\[ x^2 + y^2 = r_c^2 + (0.01 \lambda_0)^2 \pm 0.02r_c\lambda_0 \sin \theta, \]

where \( 0 \leq \theta \leq 2\pi \), \( \lambda_0 \) is the wavelength in free space, and \( r_c \) is the radius of the circular boundary. In Figure 7, \( s_c \) is the half-length of the side of the square-shaped boundary. Permittivity of the coating is chosen to be 4, and \( r_c \) and \( s_c \) are chosen to be 1.3\( \lambda_0 \). By changing the coordinates of position \((x_0, y_0)\) by varying \( 0 \leq x_0 < 0.17\lambda_0 \), \( 0 \leq y_0 < 0.17\lambda_0 \) and changing the radius of the conducting cylinder \( r_p \) by varying \( 0.03\lambda_0 \leq r_p < 0.16\lambda_0 \), 7000 samples of RCS values are obtained.

![Figure 6](image6.png) Figure 6. The inverse scattering problem of an object coated with a dielectric material having a slightly perturbed circular boundary.

![Figure 7](image7.png) Figure 7. The inverse scattering problem of an object coated with a dielectric material having a square-shaped boundary.

The performance of the neural network is found by using half of the dataset as a test group for the 2 cases considered in Figures 6 and 7. The other half of the dataset is used as a training group. The testing and training accuracies of the network are found by calculating the mean square error (MSE) and standard deviation (SD) in Tables 1 and 2 for an object coated with a dielectric material having a slightly perturbated circular boundary and square-shaped boundary, respectively.

Table 1. Mean square error (MSE) and standard deviation (SD) of a) testing and b) training results for the object coated with the dielectric material having a slightly perturbated circular boundary.

<table>
<thead>
<tr>
<th></th>
<th>Testing results</th>
<th>Training results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_0 )</td>
<td>( y_0 )</td>
</tr>
<tr>
<td>MSE</td>
<td>3.38E-10</td>
<td>8.96E-11</td>
</tr>
<tr>
<td>SD</td>
<td>1.7E-5</td>
<td>8.78E-6</td>
</tr>
<tr>
<td>MSE</td>
<td>6.37E-14</td>
<td>3.26E-14</td>
</tr>
<tr>
<td>SD</td>
<td>1.67E-7</td>
<td>1.21E-7</td>
</tr>
</tbody>
</table>

The results in Tables 1 and 2 show that the estimation of the localization and the radius of the scatter-coated dielectric material by the RBF neural network is sufficiently accurate. Moreover, Figures 8 and 9 prove the reliability of the stability and accuracy of the network. Although the results of the neural network for the problem of coating with the slightly perturbated circular boundary are better than the results for coating with...
a square-shaped boundary, the network is accurate in finding the location and the radius of the conducting cylinder regardless of the boundary shape of the coating.

Table 2. Mean square error (MSE) and standard deviation (SD) of a) testing and b) training results for the object coated with the dielectric material having a square-shaped boundary.

<table>
<thead>
<tr>
<th></th>
<th>Testing results</th>
<th></th>
<th>Training results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_0$</td>
<td>$y_0$</td>
<td>$r_p$</td>
<td>$x_0$</td>
</tr>
<tr>
<td>MSE</td>
<td>3.29E-9</td>
<td>1.66E-9</td>
<td>1.88E-9</td>
<td>2.9E-9</td>
</tr>
<tr>
<td>SD</td>
<td>3.85E-5</td>
<td>2.89E-5</td>
<td>3.03E-5</td>
<td>3.49E-5</td>
</tr>
</tbody>
</table>

Figure 8. Estimated and real values of the location coordinate and radius of the object coated with the dielectric material having a slightly pertubated circular boundary: a) $x_0/\lambda_0$, b) $y_0/\lambda_0$, and c) $r_p/\lambda_0$. 1254
After analyzing the performance of the network for the estimation of the localization and the radius of the scatterer, the effect of the coating radius on finding the location and the radius of the PEC cylinder is investigated. The location error is calculated by:

\[
MSE_L = \frac{1}{N} \sum_{n=1}^{N} \left( \sqrt{(x_r - x_e)^2 + (y_r - y_e)^2} \right)^{1/2} ,
\]

where \((x_r, y_r)\) and \((x_e, y_e)\) are the real and estimated coordinates of the conducting cylinder, respectively. The effect of the coating thickness on the MSE of the location \(MSE_L\) and the radius \(MSE_{r_p}\) of the conducting cylinder is shown in Figures 10 and 11 for an object coated with a dielectric material having a slightly perturbed circular boundary and a square-shaped boundary, respectively. It is seen that as \(r_c\) and \(s_c\) increase, the difference in the backscattering field with and without the conducting cylinder becomes bigger, and thus \(MSE_L\) and \(MSE_{r_p}\) decrease.
Figure 10. Effects of the coating radius on the MSE results for the object coated with the dielectric material having a slightly perturbated circular boundary.

Figure 11. Effects of the coating radius on the MSE results for the object coated with the dielectric material having a square-shaped boundary.

The effect of the relative permittivity of the coating material on the recognition of the location and the radius of the PEC cylinder is analyzed in Figures 12 and 13 for an object coated with a dielectric material having a slightly perturbated circular boundary and a square-shaped boundary, respectively. It is seen that as the relative permittivity of the coating material increases, the contrast between the impedances of the coating material and the conducting cylinder decreases. Naturally, the difference in the backscattering field with and without the conducting cylinder becomes smaller, and thus $MSE_L$ and $MSE_{rp}$ increase.

Figure 12. Effects of the relative permittivity of the coating material on the MSE results for the object coated with the dielectric material having a slightly perturbated circular boundary.

Figure 13. Effects of the relative permittivity of the coating material on the MSE results for the object coated with the dielectric material having a square-shaped boundary.
5. Conclusion

In this study, an application of a neural-based electromagnetic inverse scattering approach has been presented. According to the MSEs and SDs of the testing and training results and the figures showing the estimated versus the real values of the location coordinates and the radius, the RBF network has successful results in determining the position and radius of the conducting cylinder, and the performance of the network was tested for various permittivities and the thicknesses of the coating material having different boundaries. As a result, the RBF neural network approach is an encouraging technique for the electromagnetic inverse scattering of buried objects.

References


