Turk J Elec Eng & Comp Sci, Vol.19, No.6, 2011, © TÜBİTAK
doi:10.3906/elk-1007-592

Active vibration control of seismic excited structural system using LMI-based mixed H_2/H_∞ state feedback controller*

Hakan YAZICI†, **Rahmi GÜÇLÜ**
Faculty of Mechanical Engineering, Yıldız Technical University,
34349, İstanbul-TURKEY
e-mails: {hyazici, guclu}@yildiz.edu.tr

Received: 13.07.2010

Abstract

This paper is concerned with the active vibration control of a four-degree-of-freedom structure, which is effected by earthquake. To obtain desired time history and frequency responses for solution of active vibration control problem, Linear Matrix Inequality (LMI) based state-feedback mixed H_2/H_∞ controller is designed in this study. The time history of ground motion of the Kobe earthquake in 1995, which is a disturbance input, is applied to modeled structure. At the end of the study, the time history of the storey displacements, velocities and frequency responses of both controlled and uncontrolled cases are presented and results are discussed. Performance of the designed controller has been shown for the different loads and disturbances using ground motion of the Kocaeli earthquake.

Key Words: *Mixed H_2/H_∞ control, linear matrix inequality, vibration, structure.*

1. Introduction

The recent earthquakes resulted in extensive destructive damage to structures. This situation exhibited the importance of the active vibration control of seismic excited structures. Seismic protective systems, in general, consist of two categories, namely passive control systems and active control systems [1]. In addition, semi-active vibration methods are proposed in literature [2]. In recent years, there are studies where active actuators are used for isolation systems in order to isolate the earthquake induced vibrations. Improvements in electromagnetic force sources and sensors have made this application possible [3], [4]. In active control systems, an external power supply provides the desired control forces which can significantly reduce the structural vibration. The effects of active control systems are typically better than such of passive control systems in decreasing the structural vibrations produced by earthquakes when the increase in flexibility and height of buildings are considered. Since, there are parametric uncertainties in buildings and the system parameters are not constant, robust control

*This paper has been selected for publication in "The Special Issue" from TOK'09 (The 17th National IFAC Symposium on Automatic Control)

†Corresponding author: Faculty of Mechanical Engineering, Yıldız Technical University, 34349, İstanbul-TURKEY

methods are offered for the active control of structures [5]. Guclu and Sertbas applied sliding mode and PID controllers for structures with an active-mass-damper [6]. Guclu designed sliding mode controller for an active control device considering a multi-degree-of-freedom structure against the ground motion of the earthquake [7]. Du et al. applied H_∞ control design for active vibration control of seismic excited a four-degree-of-freedom structure subject to parameter uncertainties [8]. Al-Dawod et al. applied fuzzy logic controller and Linear Quadratic Gaussian (LQG) controller for active vibration control of tall buildings under wind excitation [9]. Guclu and Yazici designed a fuzzy logic PID controller for a fifteen-degree-of-freedom structural system against earthquake and proposed controller was compared with uncontrolled cases [10]. Guclu and Yazici designed fuzzy logic controller for a non-linear structural system with ATMD against earthquake [11], [12]. In this study, LMI-based mixed H_2/H_∞ state-feedback controller has been designed to reduce the seismic responses of a real structural system against earthquakes.

2. Structural dynamics

In this study, a four-storey structure is modeled using spring-mass-damper subsystem. Since the destructive effect of earthquakes is mainly the result of horizontal vibrations, the degrees of freedom of the structure have been assumed only in this direction. During an earthquake, the maximum inter-storey shear force occurs on the first storey. Assuming equivalent storey stiffness and ultimate capacities, the destructive effect of an earthquake is expected to be the largest on the first storey. Besides, it is well known that the maximum displacements and accelerations are expected at the top storey of structures during an earthquake. Therefore, active control devices (actuators), which supplies energy to suppress seismic vibrations, are installed on the first and top storey of the structure. The modeled structural system is shown in Figure 1. Here, m_1 is movable mass of the ground storey, the mass of each storey is m_2 , m_3, m_4 respectively. x_1, x_2, x_3, x_4 are the horizontal displacements and x_0 is the earthquake-induced ground motion disturbance to the structure.The masses, damping and stiffnesses for each storey are assumed to be identical, and the realistic structural parameters are given as $m_1 = 450$, $m_2 = m_3 = m_4 = 345.6\ ton$, $c_1 = 26.170$, $c_2 = c_3 = c_4 = 293.7\ KNs/m$, $k_1 = 18.050$, $k_2 = k_3 = k_4 = 340.400$ KN/m. All springs and dampers are acting in horizontal direction. The equations of motion of the system can be obtained easily using Lagrange equations as below:

$$M_s\ddot{x} + C_s\dot{x} + K_s x = F_w + F_u \tag{1}$$

Here, F_w is the force induced by the earthquake. F_u is the control force; $[M_s]$, $[C_s]$ and $[K_s]$ are the mass, damping and stiffness matrices, respectively.

3. LMI-based mixed H_2/H_∞ state-feedback controller design

Francis and Doyle et al. developed state-space formulas that solved a standard optimal H_∞ control problem using Riccati equation thus making a significant breakthrough in optimal H_∞ control [13], [14]. Scherer, Gahinet ve Chilali applied formulations of the H_2 and H_∞ control problem in terms of LMI allow computationally effectiveness and systematic design of robust controllers [15], [16]. H_∞ control depends on minimizing the infinitive norm of transfer function matrix which is written from controlled output to disturbance input in order to avoiding the disturbance input to affect the system. Therefore, H_∞ control is very suitable control algorithm for the structural systems which are under effect of disturbance inputs with unknown magnitude as earthquakes. On the other hand, H_∞ control design is mainly concerned with frequency domain performance and does not

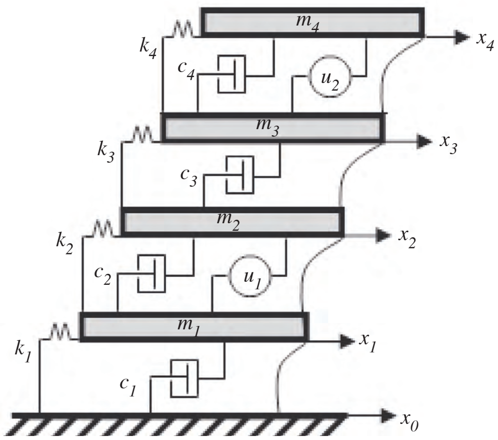


Figure 1. Physical model of a four-degree-of-freedom structural system.

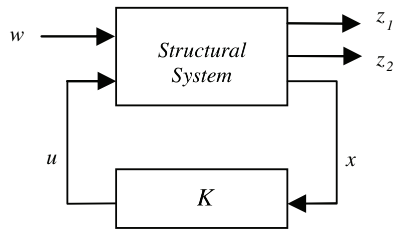


Figure 2. Block diagram of designed state-feedback control system.

guarantee good transient behaviors for the closed-loop system. H_2 control gives more suitable performance on system transient behaviors. In this study to obtain desired frequency and transient response performance, H_∞ and H_2 controller objectives have combined as a mixed control problem by the use of LMIs. The block diagram of designed state-feedback control system is shown in Figure 2. Here, structural model is considered as Linear Time Invariant (LTI) model.

The state-space representation of the system is given by

$$\begin{aligned} \dot{x} &= A + B_1 w + B_2 u \\ z_1 &= C_1 x + D_{11} w + D_{12} u \\ z_2 &= C_2 x + D_{21} w + D_{22} u \end{aligned} \tag{2}$$

where, $x \in \mathbb{R}^n$ is the state vector, $z_1, z_2 \in \mathbb{R}^{n_z}$ are the controlled output, $w \in \mathbb{R}^{n_w}$ is the disturbance input. $u \in \mathbb{R}^{n_u}$ is the control input vector. The control input is considered as a linear function of the state, *i.e.* $u = Kx$ where, $K \in \mathbb{R}^{n_u \times n}$ is the state-feedback gain. The closed-loop system can be written as below.

$$\begin{aligned} \dot{x} &= (A + B_2 K)x + B_1 w \\ z_1 &= (C_1 + D_{12} K)x + D_{11} w \\ z_2 &= (C_2 + D_{22} K)x + D_{21} w \end{aligned} \tag{3}$$

The state-space matrices and vectors of the closed-loop system are considered as below.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & 0 & 0 & -\frac{(c_1+c_2)}{m_1} & \frac{c_2}{m_1} & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{(k_2+k_3)}{m_2} & \frac{k_3}{m_2} & 0 & \frac{c_2}{m_2} & -\frac{(c_2+c_3)}{m_2} & \frac{c_3}{m_2} & 0 \\ 0 & \frac{k_3}{m_3} & -\frac{(k_3+k_4)}{m_3} & \frac{k_4}{m_3} & 0 & \frac{c_3}{m_3} & -\frac{(c_3+c_4)}{m_3} & \frac{c_4}{m_3} \\ 0 & 0 & \frac{k_4}{m_4} & -\frac{k_4}{m_4} & 0 & 0 & \frac{c_4}{m_4} & -\frac{c_4}{m_4} \end{bmatrix} \tag{4}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{k_1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c_1}{m_1} & 0 & 0 & 0 \end{bmatrix}^T, B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{m_1} & \frac{1}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_3} & \frac{1}{m_4} \end{bmatrix}^T \tag{5}$$

$$C_1 = C_2 = I_{8\times 8} \tag{6}$$

$$D_{11} = D_{12} = D_{21} = D_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{7}$$

$$x = z_1 = z_2 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 \end{bmatrix}, w = \begin{bmatrix} x_0 & \dot{x}_0 \end{bmatrix}^T, u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T. \tag{8}$$

Suppose $T_{z_1 w}$ and $T_{z_2 w}$ denote the closed-loop transfer functions, respectively from disturbance (w) to measurements (z_1 and z_2). The multi-objective state-feedback $\mathrm{H}_2/\mathrm{H}_\infty$ control may be describes as follows. Find a state-feedback control law $u = Kx$ such that minimizes $\|T_{z_2 w}\|_2$ subject to $\|T_{z_1 w}\|_\infty < \gamma$ [17]. In order to find a state-feedback control law, H_2 and H_∞ designed objectives have been separately derived by LMIs. These two constraints can be combined into one design expression as mixed $\mathrm{H}_2/\mathrm{H}_\infty$ controller. The Bounded Real Lemma plays a central role to obtain the H_∞ constraint. There exists a quadratic Lypunov function $V(x) = x^T P x$, $P > 0$, $\gamma > 0$, such that for all t. If we assume the initial conditions are zero, the Hamiltonian of the system (3) becomes

$$\dot{V} + z_1^T z_1 - \gamma^2 w^T w < 0 \tag{9}$$

and this Hamiltonian must be negative definite for all x and w. The following equation can be obtained by substituting the Equation (3) into Equation (9).

$$[(A+B_2K)x+B_1w]^T Px + x^T P[(A+B_2K)x+B_1w] + [(C_1+D_{12}K)x+D_{11}w]^T[(C_1+D_{12}K)x+D_{11}w] - \gamma^2 w^T w < 0 \tag{10}$$

Arranging the inequality (10) the following matrix inequality can be obtained as

$$\begin{bmatrix} ((A+B_2K)^T P + P(A+B_2K) + (C_1+D_{12}K)^T(C_1+D_{12}K)) & PB_1 + (C_1+D_{12}K)^T D_{11} \\ B_1^T P + D_{11}^T(C_1+D_{12}K) & -\gamma^2 I + D_{11}^T D_{11} \end{bmatrix} < 0 \tag{11}$$

The inequality (11) takes the final form using the Schur complement. Suppose P and R symmetric matrices. The condition

$$\begin{bmatrix} P & S \\ S^T & R \end{bmatrix} > 0 \tag{12}$$

is equivalent to

$$P > 0, P - SR^{-1}S^T > 0. \tag{13}$$

Pre- and post multiplying (11) by P^{-1} and using the Schur complement, inequality (14) can be written as

$$\begin{array}{c} P^{-1}(A + B_2K)^T + (A + B_2K)P^{-1} + P^{-1}(C_1 + D_{12}K)^T(C_1 + D_{12}K)P^{-1} \\ -(B_1 + P^{-1}(C_1 + D_{12}K)^T D_{11})(-\gamma^2 I + D_{11}D_{11}^T)^{-1}(B_1^T + D_{11}^T(C_1 + D_{12}K)P^{-1}) < 0 \end{array} \tag{14}$$

Applying the variable change $X_\infty = P^{-1}$ the following LMI's can be obtained.

$$\begin{bmatrix} ((A+B_2K)X_\infty + X_\infty(A+B_2K)^T X_\infty + X_\infty(C_1+D_{12}K)^T(C_1+D_{12}K)X_\infty) & B_1 + X_\infty(C_1+D_{12}K)^T D_{11} \\ B_1^T + D_{11}^T(C_1 + D_{12}K)X_\infty & -\gamma^2 I + D_{11}^T D_{11} \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} (A + B_2K)X_\infty + X_\infty(A + B_2K)^T & B_1 \\ B_1^T & -\gamma I \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} X_\infty(C_1 + D_{12}K)^T \\ D_{11}^T \end{bmatrix}\begin{bmatrix} (C_1 + D_{12}K)X_\infty & D_{11} \end{bmatrix} < 0 \tag{16}$$

By the use of the Schur complement again, the following inequality can be obtained as H$_\infty$ constraint of the closed-loop system (3) for $X_\infty > 0$,

$$\begin{bmatrix} (A + B_2K)X_\infty + X_\infty(A + B_2K)^T & B_1 & X_\infty(C_1 + D_{12}K)^T \\ B_1^T & -\gamma I & D_{11}^T \\ (C_1 + D_{12}K)X_\infty & D_{11} & -\gamma I \end{bmatrix} < 0 \tag{17}$$

The optimal H$_2$ controller can be obtained by searching the minimum η which satisfies the above mentioned LMIs for $X_2 = X_2^T$ and $Q = Q^T$. Note that the H$_2$ norm of $\|T_{z_2w}\|_2^2$ is finite if and only if $D_{21} = 0$.

$$(A + B_2K)X_2 + X_2(A + B_2K)^T + B_1B_1^T < 0 \tag{18}$$

$$\begin{bmatrix} Q & (C_2 + D_{22}K)X_2 \\ X_2(C_2 + D_{22}K)^T & X_2 \end{bmatrix} > 0 \tag{19}$$

$$Trace(Q) < \eta \tag{20}$$

The main objective of this paper is to determine a state-feedback mixed H$_2$/H$_\infty$ controller gain. To obtain proposed controller structure, it is convenient to combine the H$_2$ and H$_\infty$ designed objectives to form a mixed H_2/H_∞ controller. Here, H_2/H_∞ control problem is to minimize the H_2 norm of the over all state-feedback gains K such that also satisfies the H_∞ norm constraints [18]. Conversely, inequalities (17), (18) and (19) are not convex because of the KX_2 and KX_∞ terms. To overcome this problem a common Lyapunov matrix such that $X = X_2 = X_\infty$ with the change of the variable $W = KX$ is used. In this way, the multi-objective H_2/H_∞ control using H_2 and H_∞ performance constraints can be obtained as below.

$$\begin{bmatrix} AX + XA^T + B_2W + W^TB_2^T & B_1 & XC_1^T + W^TD_{12}^T \\ B_1^T & -\gamma I & D_{11}^T \\ C_1X + D_{12}W & D_{11} & -\gamma I \end{bmatrix} < 0 \tag{21}$$

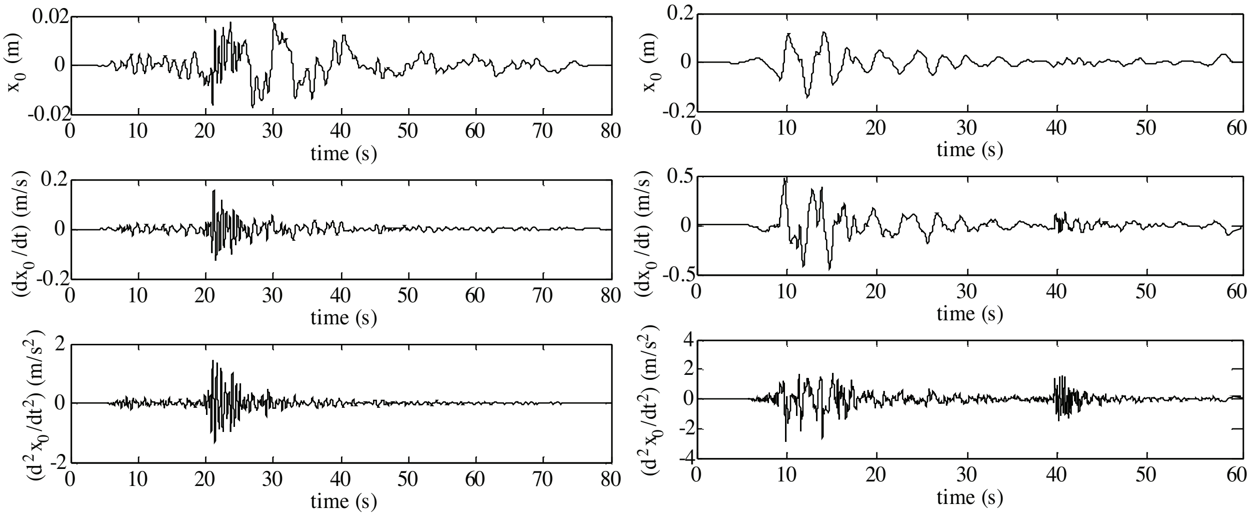
$$\begin{bmatrix} Q & C_2X + D_{22}W \\ XC_2^T + W^TD_{22}^T & X \end{bmatrix} > 0 \tag{22}$$

$$Trace(Q) < \eta \tag{23}$$

After finding a solution (X, Q, W) of this mixed control problem, the optimal state-feedback gain for the closed-loop system is obtained as

$$K = WX^{-1} \tag{24}$$

All the simulations and computations are done using Matlab with Simulink. For the solution of the resulting LMIs, Yalmip parser and LMILAB solver are used [19]. Displacements and velocities of the earthquake ground motion is used as exogenous signal and the control law is selected to be full state-feedback controller.



(a) (b)

Figure 3. (a) Kobe and (b) Kocaeli earthquake excitations input to the structure.

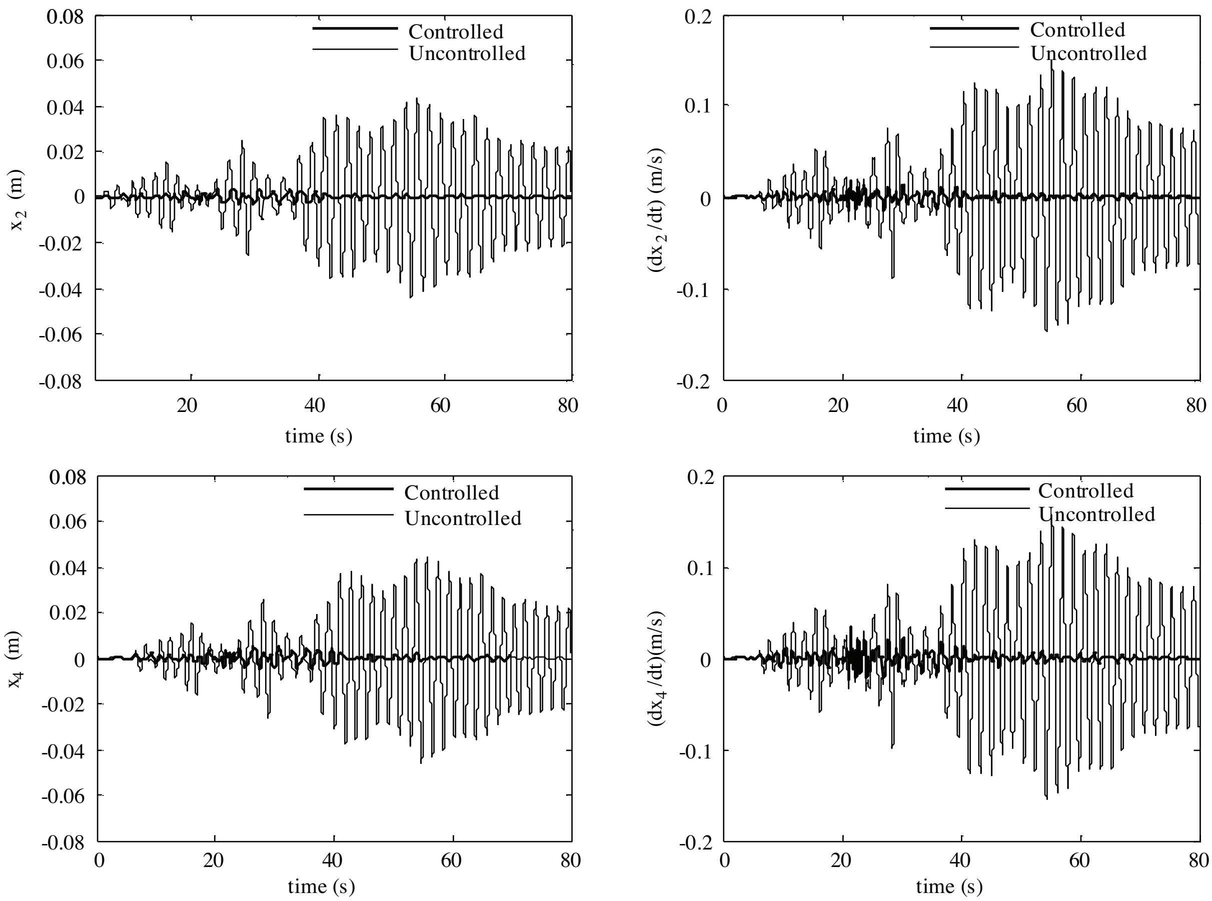


Figure 4. Time responses of second and fourth storeys against Kobe earthquake.

4. Simulation results

LMI-based mixed H_2/H_∞ state-feedback controller has been designed for the active control of Kobe and Kocaeli earthquakes excited structure. The Kobe and Kocaeli earthquake motions are shown in Figure 3 [20]. The time history of the storey displacements, velocities and frequency responses of both the uncontrolled and controlled cases of the considered structure are presented with simulation study.

Figure 4 indicates displacement and velocities of the time responses of the second and fourth storeys of the structure, respectively, for both controlled and uncontrolled cases against Kobe earthquake. As shown in Figure 4, vibration amplitudes of storeys are decreased successfully with the designed controller.

Performance of the designed controller is checked against different disturbances using ground motion of the Kocaeli earthquake in Figure 5. It is desired that the controller remains stable and effective when the structure is subjected to different disturbances. As can seen from the Figure 5, satisfactory vibration suppression is achieved under different earthquake ground motion by the proposed controller.

Figure 7 shows the frequency responses of the second and fourth storey displacements and velocities, respectively, for both controlled and uncontrolled cases. Since the system has four degrees of freedom, there are four resonance frequency points at 0.54, 3.67, 6.87, 9.16 Hz. As expected, the upper curves belong to the uncontrolled system. When the response plots of the structural systems with uncontrolled and controlled cases

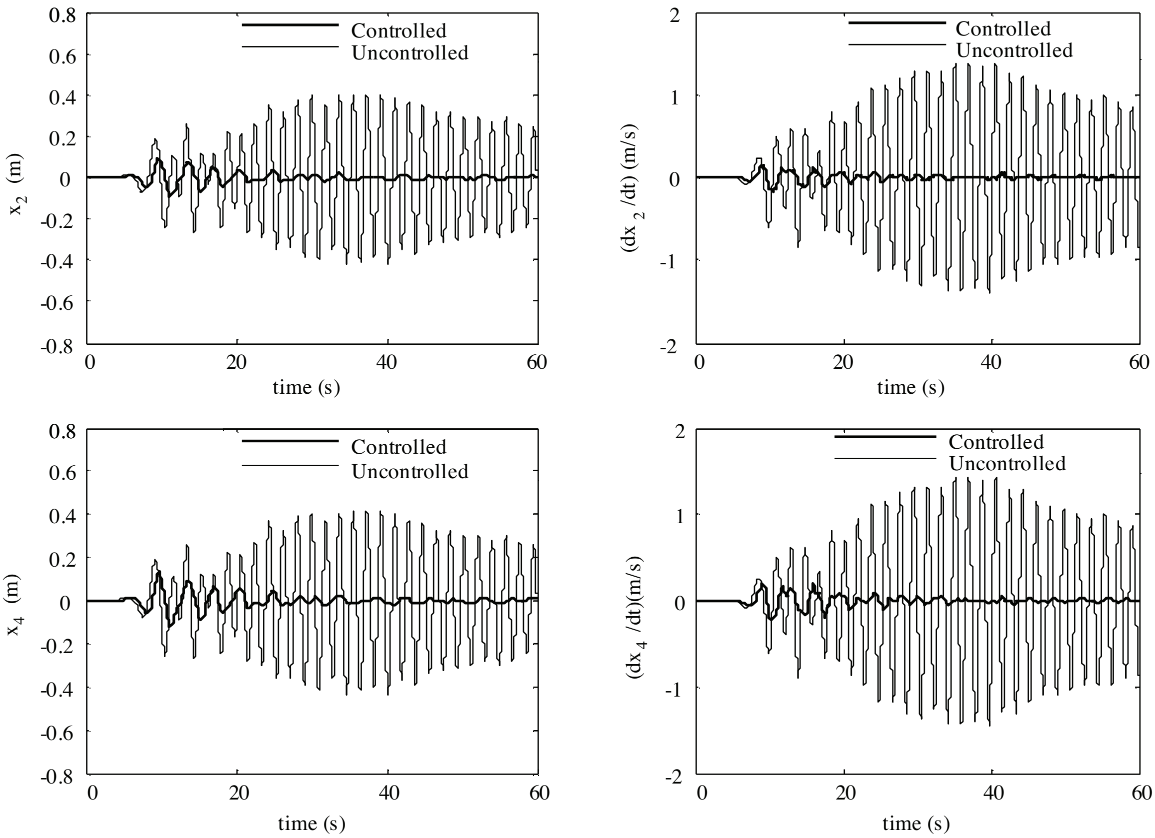


Figure 5. Time responses of second and fourth storeys against Kocaeli earthquake.

are compared, a superior improvement in the mitigation of the resonance values is observed with the proposed controller. It is well known that the first mode is the most dangerous for structures during an earthquake. However, it is obvious that this mode is successfully suppressed by the designed controller.

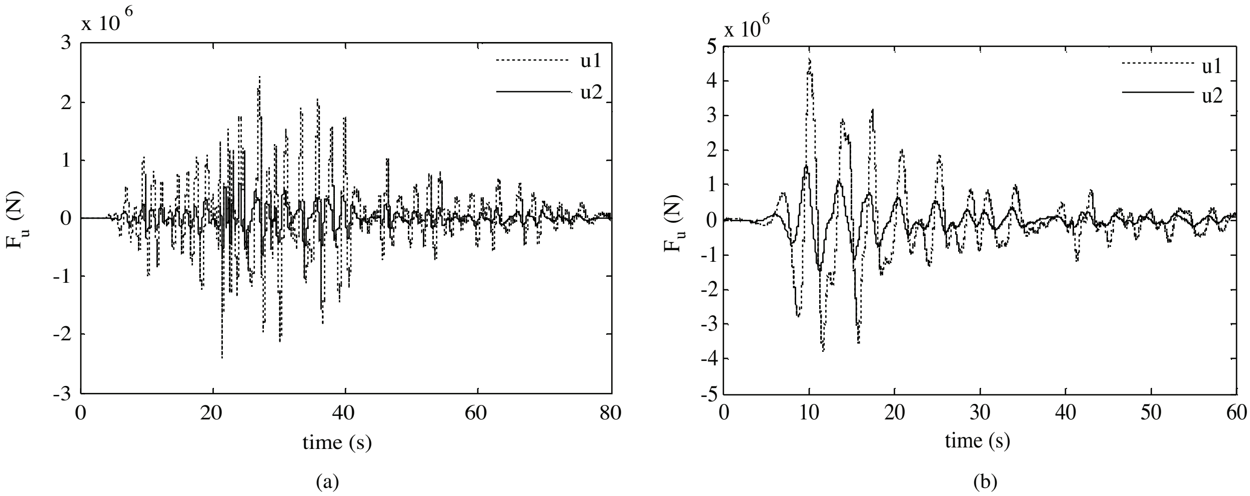


Figure 6. Time histories of the applied control forces for (a) Kobe and (b) Kocaeli earthquakes.

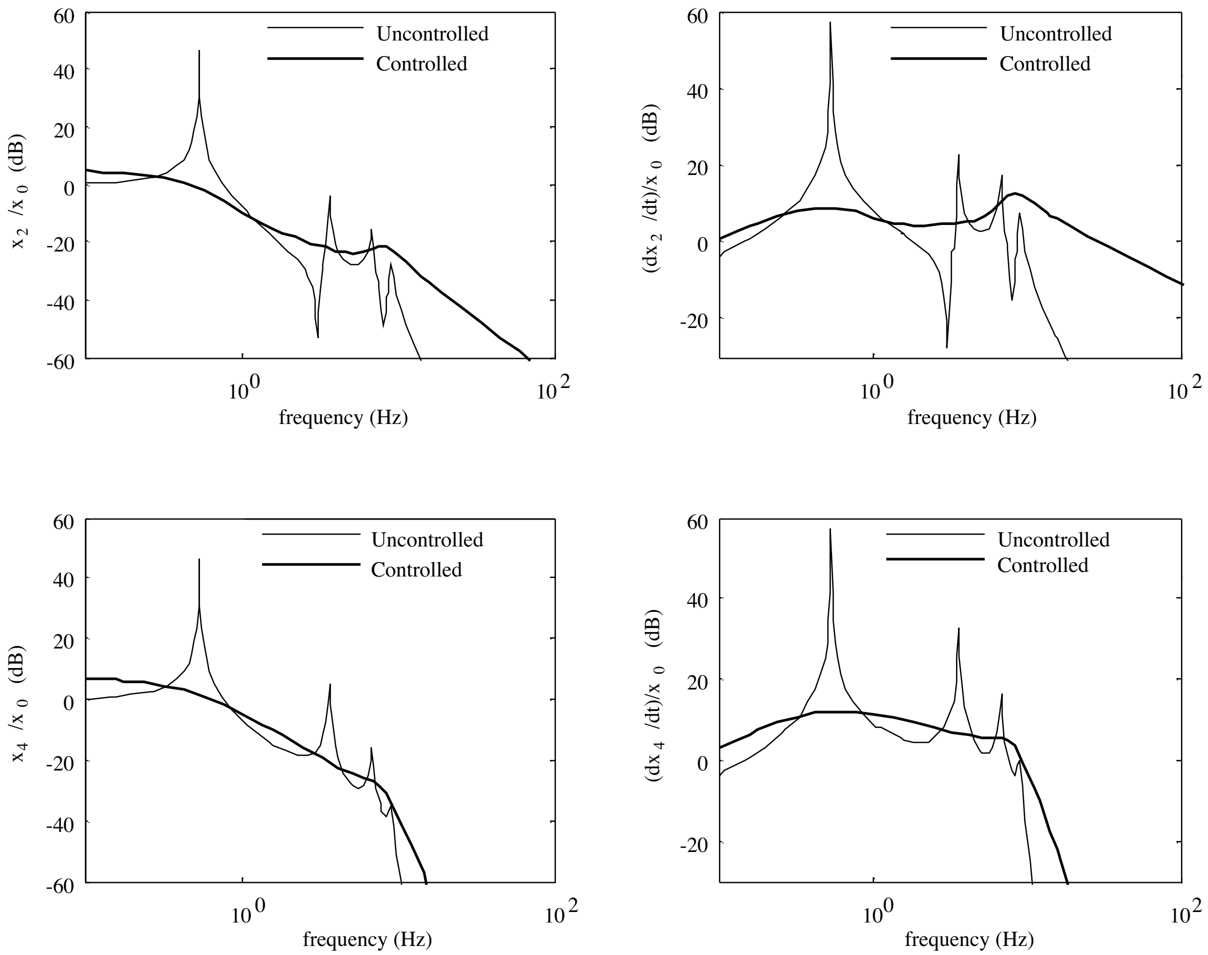


Figure 7. Frequency responses of the second and fourth storeys.

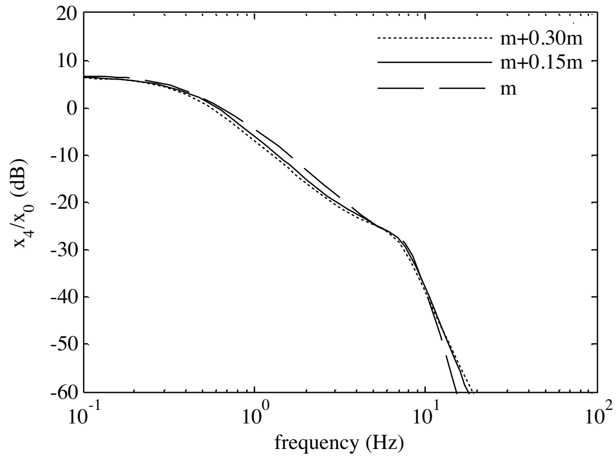


Figure 8. Frequency responses of the fourth storey for changing mass parameters.

To verify insensitiveness of the designed controller against parametric uncertainties, the values of mass parameters of each storey are increased 15% and 30% variation from initial mass of the structure. Robustness of the proposed controller has been checked using controlled displacement frequency responses of fourth storey against the uncertainties in mass parameters in Figure 8. This figure reveals that the proposed controller has a satisfactory robust character for different loads.

5. Conclusions

In this study, a four-degree-of-freedom structure is modeled for active vibration control. LMI-based mixed H_2/H_∞ state-feedback controller is applied to structural model as control algorithm and structural vibrations of seismic excited structure are analyzed for different earthquake ground motions. In order to show insensitiveness of the proposed controller against the parametric uncertainty, the values of the mass of each storey has been increased 15% and 30%. Additionally, the performance of the designed controller has been checked for different ground motion corresponding to the Kocaeli earthquakes. The simulation results demonstrate the effectiveness of the designed controller in actively suppressing structural vibrations for different earthquake ground motion, and more importantly, the robustness of the LMI-based mixed H_2/H_∞ state-feedback controller to structural mass variations. As a result, the proposed controller has great potential in active structural control for the seismic excited structural systems which are under effect of disturbance inputs with unknown magnitude as earthquakes.

References

[1] T.T. Soong, M.C. Constantinou, "Passive and Active Structural Vibration Control in Civil Engineering", CISM Courses and Lecrures No 345, Springer-Verlag, Wien, New York, 1994.

[2] K. Yoshida, and T. Fujio,"Semi-active base isolation for a building structure", Proceedings of the 1999 ASME Design Engineering Technical Conference, Las Vegas, Nevada, pp. 1-6, 1999.

[3] D. Dan Cho, "Experimental results on sliding mode control of an electromagnetic suspension", Mechanical Systems and Signal Processing Vol. 7, No. 4, pp. 283-292, 1993.

[4] M.V.C. Rao, and V. Prahlad, "A tunable fuzzy logic controller for vehicle-active suspension systems", Fuzzy Sets and Systems Vol. 85, pp. 11-21, 1997.

[5] H. Nishimura, Y. Ohkubo, K. Nonami, "Active isolation control for multi-degree-of-freedom structural system", Third International Conference on Motion and Vibration Control, pp. 82-87, Chiba, Japan, 1996.

[6] R. Guclu and A. Sertbas, "Evaluation of sliding mode and proportional-integral-derivative controlled structures with an active mass damper", Journal of Vibration and Control, Vol. 11, No. 3, pp. 397-406, 2005.

[7] R. Guclu, "Sliding mode and PID control of a structural system against earthquake", Mathematical and Computer Modelling, Vol. 44, No. (1-2), pp.210-217, 2006.

[8] H. Du, J. Lam, K.Y. Sze, "Non-fragile H_∞ vibration control for uncertain structural systems", Journal of Sound and Vibration, 273, 1031-1045, 2004.

[9] M. Aldawod, B. Samali, F. Naghdy, K.C.S. Kwok, "Active control of along wind response of tall building using a fuzzy controller", Engineering Structures, Vol. 23, pp. 1512-1522, 2000.

[10] R. Guclu, H. Yazici, “Fuzzy logic control of a non-linear structural system against earthquake induced vibration”, Journal of Vibration and Control, Vol. 13, No. 11, pp. 1535-1551, 2007.

[11] R. Guclu, H. Yazici, “Self-tuning fuzzy logic control of a non-linear structural system with ATMD against earthquake”, Nonlinear Dynamics, Vol. 56, No. 3, pp. 199-211, 2009.

[12] R. Guclu, H. Yazici, “Vibration control of a structure with ATMD against earthquake using fuzzy logic controllers”, Journal of Sound and Vibration, Vol. 318, No. 1-2, pp. 36-49, 2008.

[13] B.A. Francis, “A course in H_∞ control theory”, Lecture Notes in Control and Information Sciences 88, Edited by M. Thoma and A. Wyner, Springer-Verlag, 1987.

[14] J.C. Doyle, K. Glover, P.P. Khargonekar, B.A. Francis, “State-space solutions to standart H_2 and H_∞ control problems”, IEEE Transactions on Automatic Control, Vol. 34, No. 8, pp. 831-847, 1996.

[15] M. Chilali, P. Gahinet, C. Scherer, “Multi-objective output-feedback control via LMI optimization”, IFAC 13th Triennial World Congress, San Francisco, USA, 1996.

[16] P. Gahinet, P. Apkarian, “A linear matrix inequalities approach to H_∞ control problems”, International Journal of Robust and Nonlinear Control, Vol. 4, pp. 421-448, 1994.

[17] K. Nonami, S. Sivrioglu, “Active vibration control using LMI-based mixed H_2/H_∞ state and output-feedback control with nonlinearity”, Proc. of the 35th IEEE Conference on Decision and Control (CDC), pp. 161-166, Kobe, 1996.

[18] P.P. Khargonekar, M.A. Rotea, “Mixed H_2/H_∞ control problem: a convex optimization approach”, IEEETrans. Automatic Control, Vol.3, No.7, pp.824-837, 1991.

[19] J. Lfberg, “Yalmip: a toolbox for modeling and optimization in MATLAB”, Proceedings of the CACSD Conference, Taipei, Taiwan, 2004.

[20] A.A. Kasimzade,“ Structural Dynamics”, Istanbul, Turkey, Birsen Publication, 2004.