Fuzzy adaptive neural network approach to path loss
prediction in urban areas at GSM-900 band

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Abstract
This paper presents the results of the Adaptive-Network Based Fuzzy Inference System (ANFIS) for the
prediction of path loss in a specific urban environment. A new algorithm based ANFIS for tuning the path
loss model is introduced in this work. The performance of the path loss model which is obtained from proposed
algorithm is compared to the Bertoni-Walfisch model, which is one of the best studied for propagation analysis
involving buildings. This comparison is based on the mean square error between predicted and measured
values. According to the indicated error criterion, the errors related to the predictions that are obtained from
the algorithm are less than the errors that are obtained from the Bertoni-Walfisch Model. In this study,
propagation measurements were carried out in the 900 MHz band in the city of Istanbul, Turkey.

Key Words: ANFIS, propagation measurements, path loss, urban environment

1. Introduction
Cellular mobile communication is a field of wireless communication which gets among the most attention and in
which improvements propagate quickly. Combination of radio communication flexibility and digital transmission
quality has an important role in the success of this system. GSM (Global Systems for Mobile Communications)
has become the only global and fastest growing system standard for mobile communication in the world. It is
a system whose standards are accepted world over, is the most preferred system and have the highest number
of users. Communication between mobile unit and system is provided with base stations. One of the most
important points in system design is the need to understand through modeling the spread of the radio signal
transmitted from the transmitter antenna (which is located on the base station) to the mobile units. Quality
of the received signals is affected directly by the weakness of the transmission line, and hence affects success
of the system. As such, the basics of cellular mobile communication systems lean upon good knowledge and understanding of the principles of radio wave propagation. In the light of this, the best propagation model should consider geographic conditions and characteristics of residential areas. One way of increasing the accuracy of a model is to form the model parameters in the manner of giving minimum error, by using the measurement values of signal level and paying attention to the characteristics of the area in which they were acquired, such as residential type, height and building density.

Walfisch and Bertoni have proposed a theoretical model that encounters the effects of buildings on radio propagation. This model assumes building heights and separation between buildings are equal [1]. Chrysanthou and Bertoni [2] have improved on the Bertoni and Walfish model [2]. In the model, effects of differences in height and buildings structures to the signal spread are given. Piazza and Bertoni find the spread loss model by assuming that the buildings which have same height and distance are located on uneven land [3]. Chung and Bertoni have presented a theoretical model [4]. This model is improved by using the approach of Bertoni-Walfisch. The benefit of transmitter antenna height is also included in the model. There are different studies which use different fiber-lines for the estimation of distance loss models in the literature.

In the study of G. Cerri, feed forward neural networks for path loss prediction in urban environment was examined [5]. In the study of Ileana, neural network models for path loss prediction are compared [6]. In [7] H. H. Xia proposed a simplified analytical model for predicting path loss in urban and suburban environments. M. V. S. N. Prasad [8] offered a modification of Xia’s model, which gave better agreement with the observed results. M. McGuire [9] demonstrated how the conditional density of the location, given a measured path loss, can be approximated as a sum of kernel density functions based on radio propagation data collected from propagation surveys or estimated from computer models [9]. There are many studies on the usage of the adaptive network for parameter prediction. In the study of C. Chi-Bin and E. S. Lee studied a fuzzy adaptive network approach for fuzzy regression analysis [10]. R. J. Jhy-Shing studied an adaptive networks-based fuzzy inference system [11]. In the study of D. T. Erbay, and A. Apaydun, adaptive network is used to parameter estimations where independent variables come from an exponential distribution [12].

In this study path loss predictions are obtained by using an adaptive network-based fuzzy inference system modeled with data obtained in the Harbiye region of Istanbul, Turkey. The predictions from the network are compared with predictions from the Bertoni-Walfisch path loss model. This model is the most suitable for the Harbiye region, because this model can take advantage of the buildings database.

Remainder of this paper is organized as follows. Section 2 presents measurement details. In Section 3, path loss models are introduced. General information about fuzzy inference system and ANFIS are given in Section 4. In Section 5, which is the main focus of this article, the membership function suitable for exponential distribution is obtained and a special ANFIS and a new algorithm for path loss is given. In addition, a path loss model for real data collected from the Harbiye urban area of Istanbul, Turkey, is obtain via the proposed algorithm. In Section 6 can found a discussion and the conclusion.

2. The measurements

In this section, the steps of collecting measurements, the equipment used and the statistical analyses of the measurements are presented. The main idea of the statistical analyses is to understand the radio wave propagation behavior for the Harbiye region of Istanbul, Turkey, at 900 MHz.
To optimize the most suitable propagation model, accuracy of the digital map database and the accompanying measurements are very important. There are different studies which use different measurement equipment. Some of them used TEMS (Test Mobile System) during the measurement setup [13]. In this study, the measurement equipments consist of a transmitter and a receiver. The narrow band continuous wave (CW) transmitter, which can be tuned to a specific test frequency, was used together with an omnidirectional antenna. The output power of transmitter was tuned to 19.95 W (43 dBm). The antenna was a vertically polarized, omnidirectional Kathrein 736350 with a vertical beam width of 13 degrees. The measured antenna gain is 8 dBm at 900 MHz, so that the maximum EIRP is 51 dBm. The antenna was installed on the rooftops. In order to decrease cable losses, the transmitter was located near the antenna.

For the purpose of measurement, a narrow band CW channel is used. This ensures good frequency isolation and constant signal to avoid interference. The frequency chosen was 924.2 MHz, since neither GSM operators nor anyone else uses it. The receiver is a high speed GSM scanner, with Walkabout data collection software from Saeco Technologies. A navigation system provides both latitude and longitude information, and gives continuous data on the test vehicle’s position.

Figure 1. Map of the Harbiye region of Istanbul, showing location and strength of measured propagation.

The measurements were carried out at an approximate speed of 40 km/h, while the receiving antenna was at a height of 1.5 m from the ground. The receiver was moved through a variety of urban environments. The
measurements data was recorded every 250 m. The route length and the number of points were approximately 187.3 km and 745, respectively. The data sources of the digital map of Istanbul are satellite images and the topographic maps. The map scale is 1/10.000. DTM resolution has important effects on path loss prediction. The accuracy of elevation data is 2.5 m. The width of the roads ranges from 5 m to 50 m. Additionally, the map includes two dimensional (2-D) digitized building data.

A map of the signal level from the Harbiye regions is shown in Figure 1. The signal level enrollments are collected from along the streets which are between the base station antenna and the mobile station antenna.

3. Path loss models

A variety of experimentally or theoretically based models have been developed to predict radio propagation in land mobile system in the literature. To be able to cope with the enormous growth in GSM, radio network planning is needed in the process of planning, expanding, operating and optimizing the network. Radio planning starts from the radio cell propagation coverage. The operators with the aid of commercial planning tools are currently accomplishing cell coverage calculations. The tools are capable of computing the coverage by using the propagation models according to terrain and the building database, base station location, antenna type and azimuth. To satisfy the operator requirements for network planning and optimization, interference and traffic calculation, frequency planning and neighbors analyses, it is very important to use suitable propagation models.

The most general model of wave propagation models is Free Space Propagation. In this model, obstructions in the region are not taken into consideration and hence propagate in emptiness. The signal detected from a free space-propagated signal is dependent on only distance between antennas and frequency [14]. If \( f \) is the frequency in MHz and \( d \) is the range in kilometers, then the path loss (in dB) is

\[
PL_{FS} = 32.44 + 20 \log(f_{MHz}) + 20 \log(d).
\]

The other model is called the Exponent Path Loss Model. This model assumes that, the signal from the base antenna declines in quality by the time it reaches the mobile station antenna.

Exponential distance loss model is the model which accepted that the sign transmitted from transmitter antenna weakens until it reaches to the receiver antenna in a certain time of the logarithm of intermediate distance. Exponential distance loss model assumes the transmitted signal weakens as the logarithm of the traveled distance. This value, the distance loss base, is calculated with respect to measurements and the type of transmission area from which the measurement is taken. Distance loss (in dB) is in the form

\[
PL = 10 \log(M) - 10n \log\left(\frac{d}{d_0}\right),
\]

where \( M \) is the fixed value, \( n \) is the exponent value of path loss, \( d \) is the distance between base station antenna and mobile station antenna, and \( d_0 \) is reference distance [15].

The most popular work on experimental approach is by Okumura [16]. Okumura has published an empirical method for predicting the field strength and service area for a given terrain over the frequency ranges of 150–2000 MHz, for distances of 1 to 100 km, and for base station effective antenna height 30 to 1000 m. In order to put Okumura’s techniques into a form suitable for implementation via computer, Hata has developed an empirical formula for propagation loss based on Okumura’s results [17]. The problem with experimental
models is that the prediction expressions are based on the qualitative propagation environments such as urban, suburban and open areas. Some other models developed by theoretical approach use the classical optics based diffraction theory extended to radio propagation for terrain in which line of sight propagation is influenced by obstacles. Walfisch and Bertoni have published a theoretical model that encounters the effects of buildings on radio propagation [1].

In this study, Bertoni-Walfisch model will be used for comparison, as this model is takes into consideration the presence of buildings between antennas.

3.1. Bertoni-Walfisch model

Bertoni-Walfisch proposed a semi-empirical model that is applicable to propagation through buildings in urban environments. The model assumes building heights to be uniformly distributed and the separation between buildings are equal. Propagation is then equated to the process of multiple diffractions past these rows of buildings. Figure 2 illustrates the building geometry and parameters in the Bertoni-Walfisch model.

The Bertoni-Walfisch model consists of three main components:

1) The path loss between antennas in free space is expressed by the relation

\[ PL_{FS}(dB) = 32.44 + 20 \log(f_{MHz}) + 20 \log(d_{km}) \].

2) The reduction \( Q(\alpha) \) of the rooftop fields due to settling:

\[ L_{ms} = 10 \log(\sqrt{2}Q(\alpha))^2 \].

where,

\[ Q = \alpha \sqrt{\frac{d_c}{\lambda}} \].

3) The effect of diffraction from rooftop fields to ground level:

\[ L_{rts} = 10 \log(F^2) \].

Figure 2. Building geometry and parameters in the Bertoni-Walfisch model.
where,

\[ F \approx \left[ \frac{\lambda}{4\pi^2 r \theta'} \right], \quad (7) \]

\[ \theta' = \tan^{-1} \left[ 2(h_b - h_r)/d_c \right], \quad (8) \]

\[ r = \sqrt{(h_b - h_r)^2 + (d_c/2)^2}, \quad (9) \]

and where

- \( \lambda \) is wavelength in free space,
- \( h_r \) is mobile station antenna height,
- \( h_b \) is building height,
- \( d_c \) is the center-to-center spacing of the rows of the buildings,
- \( \alpha \) denotes the propagation angle between base station antenna and mobile station antenna in radians:

\[ \alpha = \frac{(h_t - h_b)}{d} - \frac{d}{2a_e}, \quad (10) \]

Also, \( a_e = 8.5 \times 10^3 \) km is the effective earth radius, and \( h_t \) is the base station antenna height. The expression for the total path loss in dB is

\[ PL_{B-W}(dB) = PL_{FS} + L_{rts} + L_{ms}. \quad (11) \]

Hence the excess path loss is given by

\[ PL_{B-W}(dB) = 89.5 + 21 \log f + 38 \log(d) - 181 \log(h_t - h_b) + A_b. \quad (12) \]

The influence of building geometry is contained in the term \( A_b \):

\[ A_b = 5 \log \left[ \left( \frac{d_c}{2} \right)^2 + (h_b - h_r)^2 \right] - 9 \log d_c + 20 \log \left\{ \tan^{-1} \left[ 2(h_b - h_r)/d_c \right] \right\}. \quad (13) \]

4. Fuzzy inference systems and ANFIS

4.1. Fuzzy Inference Systems

A fuzzy inference system forms a useful computing framework based on the concepts of fuzzy set theory, fuzzy reasoning, and fuzzy if-then rules. A fuzzy inference system is a powerful function approximator. The basic structure of a fuzzy inference system consists of three conceptual components: a rule base, which contains a selection of fuzzy rules; a database, which defines the membership functions used in the fuzzy rules; and a reasoning mechanism, which performs the inference procedure upon the rules to derive a reasonable output.

There are several different types of fuzzy inference systems developed for function approximation. In this study, the Sugeno fuzzy inference system, which was proposed by Takagi and Sugeno [18], will be used. From the input vector \( X = (x_1, x_2, ..., x_p)^T \) the system output \( Y \) can be determinate by the Sugeno inference system as

\[ R^L: \quad \text{If } (x_1 \text{ is } F^L_1, \text{ and } x_2 \text{ is } F^L_2, ..., \text{ and } x_p \text{ is } F^L_p), \]

then \( (Y = Y^L = c^L_0 + c^L_1 x_1 + ... + c^L_p x_p) \).
Here, \( F_j^L \) is fuzzy set associated with the input \( x_j \) in the \( L^{th} \) rule and \( Y^L \) is output due to rule \( R^L \) (\( L = 1, ..., m \)). The parameters used to define the membership functions for \( F_j^L \) is called the premise parameters, and \( c_j^L \) are called the consequence parameters. For a real-valued input vector \( X = (x_1, x_2, ..., x_p) \), the overall output of the Sugeno fuzzy inference systems a weighted average of the \( Y^L \)

\[
\hat{Y} = \frac{\sum_{L=1}^{m} w^L Y^L}{\sum_{L=1}^{m} w^L}, \tag{14}
\]

where the weight \( w^L \) is the truth value of the proposition \( Y = Y^L \) and is defined as

\[
w^L = \prod_{i=1}^{p} \mu_{F_j^L} (x_i), \tag{15}
\]

And where \( \mu_{F_j^L} (x_i) \) is a membership function defined on the fuzzy set \( F_j^L \).

### 4.2. ANFIS

Neural networks enabling the use of fuzzy inference system for prediction are known as adaptive networks. The Adaptive-Network Based Fuzzy Inference System (ANFIS) is a neural network architecture that can solve any function approximation problem.

An adaptive network is a multilayer feed forward network in which each node performs a particular function on incoming signals as well as a set of parameters pertaining to this node; and it has five layers [19–21]. The formulas for the node functions may vary from node to node and the choice of each node function depends on the overall input-output function which the adaptive network is required to carry out.

Fuzzy rule number of the system depends on numbers of independent variables and class or fuzzy sets number forming independent variables. When independent variable number is indicated with \( p \), if level number belonging to each variable is indicated with \( l_i \) (\( i = 1, ..., p \)) fuzzy rule number is indicated by

\[
L = \prod_{i=1}^{p} l_i. \tag{16}
\]

To illustrate how a fuzzy inference system can be represented by ANFIS, the special ANFIS architecture is will be given in section 5, which is suitable for path loss prediction problem.

### 5. New algorithm to path loss prediction

In this study, the path loss prediction problem has a three-dimensional input. One of them is comes from Gaussian distribution, and the others are come from exponential distribution. As such, there will be used two different membership functions, one of them is named Gaussian membership function whose parameters can be represented by the parameter set \( \{ \nu_h, \sigma_h \} \) and the other one is produced for the inputs which come from exponential distribution in this study, following the method suggested by Civanlar and Trussel [22]. This
method satisfies the theoretical need and, based on a probability density function, will be used for forming the membership function appropriate for the data cluster which comes from the exponential family. The membership function has one parameter, \( \{v_h\} \).

5.1. The membership function for exponential distribution

A membership function should provide the given conditions below for being an optimal membership function:

1. \( E \left\{ \mu(x) \bigg| x \text{ is distributed according to the underlying probability density function} \right\} \geq c; \)
2. \( 0 \leq \mu(x) \leq 1; \)
3. \( \int \mu^2(x) \, d(x) \) should be minimized.

This condition is required to obtain a selective membership function. Under these conditions optimal membership function is given in the form

\[
\mu(x) = \begin{cases} 
\lambda p(x) & \text{if } \lambda p(x) < 1 \\
1 & \text{if } \lambda p(x) \geq 1.
\end{cases} \tag{17}
\]

Here, \( p(x) \) denotes the probability density function and \( \lambda \) is a constant [22].

In the given membership function the form of \( p(x) \) is determined as the probability density function related to the interested distribution. However, the fixed element \( \lambda \) can be obtained by solving the problem, which is formed with the conditions described for optimal membership function and given by problem \( P \):

\[
P: \begin{align*}
\text{Min}_{\mu} f(\mu) & = \frac{1}{2} \int_{-\infty}^{+\infty} \mu^2(x) d(x) \\
G(\mu) & = c - E(\mu) = c - \int_{-\infty}^{+\infty} \mu(x) p(x) d(x) \leq 0 \\
\mu & \in \Omega = \{\mu(x) | 0 \leq \mu(x) \leq 1\}. \tag{18}
\end{align*}
\]

The problem given with \( P \) can be solved with the method of Lagrange multipliers for obtaining the fixed element \( \lambda \). For this, the Lagrange function is written

\[
L(\mu, \lambda) = \frac{1}{2} \int_{-\infty}^{+\infty} \mu^2(x) d(x) + \lambda \left\{ c - \int_{-\infty}^{+\infty} \mu(x) p(x) d(x) \right\}, \tag{19}
\]

where Lagrange multiplier \( \lambda \geq 0 \) and constant \( c < 1 \).

When the membership function values given in (17) are inserted into (19), the following form for the Lagrange is obtained:

\[
L(\mu^*, \lambda) = \frac{1}{2} \int_{-\infty}^{+\infty} \{I(\lambda p(x))(\lambda p(x) - 1)^2 - \lambda^2 p^2(x)\} d(x) + \lambda c, \tag{20}
\]

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where

\[
I(x) = \begin{cases} 
0 & \text{if } x \leq 1 \\
1 & \text{otherwise.}
\end{cases}
\]

By putting values \( I(\lambda p(x)) \) into the Lagrange function, the Lagrange function can be revised into the form

\[
L = -\frac{1}{2} \int_{-\infty}^{+\infty} \lambda^2 p^2(x) d(x) + \lambda c.
\]  

(21)

Holding \( \lambda \) as a constant, on taking this function’s derivative, one can obtain \( \lambda \) as

\[
\lambda = \frac{c}{\int_{-\infty}^{+\infty} p^2(x) d(x)}.
\]  

(22)

Inserting the probability density function

\[
p(x) = \frac{1}{\nu} e^{-\frac{x}{\nu}} \quad x \geq 0
\]

(23)

into equation (22), one obtains the form

\[
\lambda = 2 \nu c.
\]  

(24)

From equation (24) the general membership function given with (17) is obtained as the exponential distribution

\[
\mu(x) = 2ce^{-\frac{x}{\nu}},
\]

(25)

where \( c < 1 \) is a constant element and \( \nu \) is a distribution parameter which is called an a priori parameter.

In the data set derived from the exponential distribution, the limit of the data belonging to the cluster with one membership degree is dependent on the fixed element \( c \) and the parameter \( \nu \), which indicates the distribution. This limit, given with \( a(c) \), is described by

\[
a(c) = \max\{0, \nu \ln(2(1 - c))\}.
\]  

(26)

As a result, the optimal membership function for the exponential distribution function is obtained in the form

\[
\mu(x_i) = \begin{cases} 
2ce^{-\frac{x_i}{\nu}} & \text{if } x_i > a(c)_i \\
1 & \text{if } x_i \leq a(c)_i.
\end{cases}
\]

(27)

The process of determining parameters for the path loss prediction problem begins with determining the number of independent variables. In this study, the aim was to use a validity criterion based on fuzzy clustering as an alternative to heuristic methods in determining class numbers. There are a lot of validity criterions for fuzzy clustering in the literature. In this study, the Xie–Beni index \( S \) will be used [23]. Before giving the algorithm for path loss prediction problem, let us give the special ANFIS architecture, which is suitable for the path loss prediction problem.
5.2. ANFIS for path loss prediction

In the path loss prediction problem, the data set has three-dimensional input $X = (x_1, x_2, x_3)$. There are two fuzzy sets (or fuzzy classes) for each input. For input $x_1$, the fuzzy sets are “class1.1” and “class1.2,” for input $x_2$, the fuzzy sets are “class2.1” and “class2.2,” and for input $x_3$, the fuzzy sets are labeled “class3.1” and “class3.2”. In this case a fuzzy inference system contains the following eight rules:

$R^1$: if $x_1$ is class 1.1 and $x_2$ is class 2.1 and $x_3$ is class 3.1, then $(Y^1 = c_0^1 + c_1^1 x_1 + c_2^1 x_2 + c_3^1 x_3)$,

$R^2$: if $x_1$ is class 1.1 and $x_2$ is class 2.1 and $x_3$ is class 3.2, then $(Y^2 = c_0^2 + c_1^2 x_1 + c_2^2 x_2 + c_3^2 x_3)$,

$R^3$: if $x_1$ is class 1.2 and $x_2$ is class 2.1 and $x_3$ is class 3.1, then $(Y^3 = c_0^3 + c_1^3 x_1 + c_2^3 x_2 + c_3^3 x_3)$,

$R^4$: if $x_1$ is class 1.2 and $x_2$ is class 2.1 and $x_3$ is class 3.2, then $(Y^4 = c_0^4 + c_1^4 x_1 + c_2^4 x_2 + c_3^4 x_3)$,

$R^5$: if $x_1$ is class 1.1 and $x_2$ is class 2.2 and $x_3$ is class 3.1, then $(Y^5 = c_0^5 + c_1^5 x_1 + c_2^5 x_2 + c_3^5 x_3)$,

$R^6$: if $x_1$ is class 1.1 and $x_2$ is class 2.2 and $x_3$ is class 3.2, then $(Y^6 = c_0^6 + c_1^6 x_1 + c_2^6 x_2 + c_3^6 x_3)$,

$R^7$: if $x_1$ is class 1.2 and $x_2$ is class 2.2 and $x_3$ is class 3.1, then $(Y^7 = c_0^7 + c_1^7 x_1 + c_2^7 x_2 + c_3^7 x_3)$,

$R^8$: if $x_1$ is class 1.2 and $x_2$ is class 2.2 and $x_3$ is class 3.2, then $(Y^8 = c_0^8 + c_1^8 x_1 + c_2^8 x_2 + c_3^8 x_3)$

This fuzzy system is represented by the ANFIS as shown in Figure 3.

![Figure 3. The ANFIS architecture.](image)
The functions of each node in five layers are defined as follows.

**Layer 1:** The output of node $h$ in this layer is defined by the membership function on $F_h$:

$$
\begin{align*}
    f_{1,h} &= \mu_{F_h}(x_1), \quad \text{for } h = 1, 2; \\
    f_{1,h} &= \mu_{F_h}(x_2), \quad \text{for } h = 3, 4; \\
    f_{1,h} &= \mu_{F_h}(x_3), \quad \text{for } h = 5, 6,
\end{align*}
$$

where fuzzy clusters are indicated by $F_1, F_2, \ldots, F_h$ and $\mu_{F_h}$ is the membership function related to $F_h$. Different membership functions can be defined for $F_h$.

In this ANFIS architecture, the membership function which is suitable for Gaussian distribution and the membership function which is suitable for Exponential distribution will be used whose parameters can be represented by $\{v_h, \sigma_h\}$ and $\{v_h\}$ respectively. Because the inputs $x_1$ and $x_3$ come from an exponential distribution and input $x_2$ comes from Gaussian distribution,

$$
\begin{align*}
\mu_{F_h}(x_1) &= \begin{cases} 
    2ce^{-\frac{x_1}{c}} & \text{if } x_1 > a(c)_i \quad \text{for } h = 1, 2 \\
    1 & \text{if } x_1 \leq a(c)_i
\end{cases} \\
\mu_{F_h}(x_2) &= \exp \left[-\left(\frac{x_2-a(c)_i}{\sigma_h}\right)^2\right] \quad \text{for } h = 3, 4 \\
\mu_{F_h}(x_3) &= \begin{cases} 
    2ce^{-\frac{x_3}{c}} & \text{if } x_3 > a(c)_i \quad \text{for } h = 5, 6 \\
    1 & \text{if } x_3 \leq a(c)_i
\end{cases}
\end{align*}
$$

The parameter sets $\{v_h, \sigma_h\}$ and $\{v_h\}$ in this layer are called premise parameters.

**Layer 2:** Each nerve in the second layer has input signals coming from the first layer and they are defined by the multiplication of their input signals. An output from this layer is said to be a fuzzy rule. This multiplied output forms the firing strength $w^L$ for rule $L$:

$$
\begin{align*}
    f_{2,1} &= w^1 = \mu_{F_1}(x_1) \cdot \mu_{F_3}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,2} &= w^2 = \mu_{F_1}(x_1) \cdot \mu_{F_3}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,3} &= w^3 = \mu_{F_1}(x_1) \cdot \mu_{F_3}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,4} &= w^4 = \mu_{F_1}(x_1) \cdot \mu_{F_3}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,5} &= w^5 = \mu_{F_1}(x_1) \cdot \mu_{F_4}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,6} &= w^6 = \mu_{F_1}(x_1) \cdot \mu_{F_4}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,7} &= w^7 = \mu_{F_3}(x_1) \cdot \mu_{F_4}(x_2) \cdot \mu_{F_5}(x_3), \\
    f_{2,8} &= w^8 = \mu_{F_3}(x_1) \cdot \mu_{F_4}(x_2) \cdot \mu_{F_5}(x_3).
\end{align*}
$$
Layer 3: The output of this layer is a normalization of the outputs of the second layer and nerve function is defined as

\[ f_{3,L} = \bar{w}^L = \frac{w^L}{\sum_{L=1}^{m=8} w^L} \quad L = 1, \ldots, 8. \] (31)

Layer 4: The output signals of the fourth layer are also connected to a function and this function is indicated by

\[ f_{4,L} = \bar{w}^L Y^L, \] (32)

where, \( Y^L \) denotes the conclusion part of the fuzzy if-then rule and it is indicated by

\[ Y^L = c_0^L + c_1^L x_1 + c_2^L x_2 + c_3^L x_3, \] (33)

where \( c_i^L \) are fuzzy numbers and denote posterior parameters.

Layer 5: There is only one node which computes the overall output as the summation of all the incoming signals [10, 24]:

\[ f_{5,1} = \hat{Y} = \sum_{L=1}^{8} \bar{w}^L Y^L. \] (34)

The algorithm which is suitable for path loss prediction problem, is based on the ANFIS and is defined as follows.

5.3. Proposed algorithm

Step 0: Optimal class numbers related to the data set belonging to independent variables are determined. Optimal value of class number \( l_i \) \((l_i = 2, l_i = 3, \ldots, l_i = \text{max})\) can be obtained by minimizing fuzzy clustering validity function \( S \):

\[ S = \frac{1}{n} \sum_{i=1}^{l_i} \sum_{j=1}^{n} \mu_{ij}^m \|v_i - x_j\|^2 \left/ \min_{i \neq j} \|v_i - v_j\|^2 \right., \] (35)

where \( \mu_{ij} \) denote the fuzzy membership, \( v_i \) denote the cluster center, \( n \) is the number of observations and \( m \) denotes the fuzziness index. As it can be seen in this statement, cluster centers, which are separated, well produce a high value for separation, so a smaller value of \( S \) is obtained. When the lowest \( S \) value is found, class number \( l_i \) giving this lowest \( S \) value is defined as the optimal class number.

Step 1: Priori parameters are determined.

Spreading is determined intuitively according to the space in which input variables gain value and to space in which the variables assume fuzzy-class values. Fuzzy values are defined in terms of Center parameters \( \max(X_i) \) and \( \min(X_i) \), which delimit the space in which the variable can assume value. A fuzzy value \( \nu_i \) is computed via the relation

\[ \nu_i = \min(X_i) + \frac{\max(X_i) - \min(X_i)}{(l_i - 1)} (i - 1) \quad i = 1, \ldots, p. \] (36)
Here, $l_i > 1$ denotes the optimal class number related to the variables, and $p$ indicates the number of independent variables.

**Step 2:** Weights $\bar{w}^L$ are counted, which are then used to form matrix $B$, to be used in forming the posteriori parameter set. When the exponential distribution function, which has the parameter set of $\{\nu_h\}$, and the membership function, which will be used in the calculation of these sets, are regarded, membership functions are as described in equation (27). The other hand, when the independent variables come from Gaussian distribution, membership functions are as defined in equation (29).

The $\bar{w}^L$ sets are normalizations of the sets which is indicated with $w^L$ and this is calculated with equation (31).

**Step 3:** On the condition that the independent variables are fuzzy and the dependent variables are crisp, a posteriori parameter set is obtained as crisp numbers in the form $c^L_i = (a^L_i, b^L_i)$. Under this condition, the equality

$$Z = (B^T B)^{-1} B^T Y$$

is used for determining the $a$ posteriori parameter set. Here, $B, Y$ and $Z$ defined as

$$B = \begin{bmatrix} \bar{w}^1_1, & \cdots, & \bar{w}^m_1, & \bar{w}^1_1 x_{11}, & \cdots, & \bar{w}^m_1 x_{11}, & \cdots, & \bar{w}^1_1 x_{1p}, & \cdots, & \bar{w}^m_1 x_{1p} \\ \vdots \\ \bar{w}^1_n, & \cdots, & \bar{w}^m_n, & \bar{w}^1_n x_{1n}, & \cdots, & \bar{w}^m_n x_{1n}, & \cdots, & \bar{w}^1_n x_{pn}, & \cdots, & \bar{w}^m_n x_{pn} \end{bmatrix},$$

$$Y = [y_1, y_2, \ldots, y_n]^T,$$

$$Z = [a^0_1, \ldots, a^m_1, a^0_1, \ldots, a^m_1, a^0_p, \ldots, a^m_p]^T.$$

**Step 4:** By using the posteriori parameter set $c^L_i = (a^L_i, b^L_i)$ obtained in Step 3, the system is model as indicated with equation (33). Setting out from the models and weights specified in Step 2, prediction values are obtained with the relation

$$\hat{Y} = \sum_{L=1}^m \bar{w}^L Y^L.$$  

**Step 5:** Error related to model is measured

$$\varepsilon = \frac{1}{n} \sum_{k=1}^n \varepsilon_k^2 = \frac{1}{n} \sum_{k=1}^n (y_k - \hat{y}_k)^2.$$  

If $\varepsilon < \phi$, then the posteriori parameter has been obtained as parameters of the models to be formed; the process is determined. If $\varepsilon \geq \phi$, then Step 6 begins. Here $\phi$ is a law stable value determined by decision maker.

**Step 6:** Central priori parameters specified in Step l, are updated with

$$\nu'_i = \nu_i \pm t,$$
in a way that it increases from the lowest value to the highest and decreases from the highest value to the lowest. Here, \( t \) is the step size:

\[
\begin{align*}
  t &= \frac{\max(x_{ji}) - \min(x_{ji})}{a} \\
  i &= 1, \ldots, n \\
  j &= 1, \ldots, p
\end{align*}
\]

(41)

and \( a \) is a stable value which is determinant of size of step and therefore iteration number.

**Step 7:** Predictions for each priori parameter obtained by change and error criterion related to these predictions are counted with

\[
\varepsilon_k = y_k - \hat{y}_k.
\]

(42)

Here, \( y_k \) is the \( k^{th} \) predicted outcome, and \( \hat{y}_k \) is the \( k^{th} \) network output of input vector.

The lowest of error criterion is defined. Priori parameters giving the lowest error specified, and prediction obtained via the models related to these parameters is taken as output.

### 5.4. Prediction path loss model

In this section, the above methodology is applied to develop the most suitable path loss model for signal data collected in the 900 MHz band in the Harbiye region of Istanbul. The obtained model is compared with the Bertoni-Walfisch model.

Harbiye region is an urban area and it has regular building structure. The gaps between buildings along the streets are small. Table lists the basic conditions characterizing the data collection. Additionally, the fraction of the area covered by the buildings in the region is 29%. Figure 4 shows a histogram of the building height \( h_b \), the center-to-center spacing of the rows of the buildings \( d_c \) and the \( \alpha \), respectively, in Figures 4(a), 4(b) and 4(c), which are the independent variables used in the constitute path loss model.

<table>
<thead>
<tr>
<th>Basic conditions characterizing the data collection.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations:</td>
</tr>
<tr>
<td>Total length of route:</td>
</tr>
<tr>
<td>Base Station antenna height:</td>
</tr>
<tr>
<td>Average building height:</td>
</tr>
<tr>
<td>Average center-to-center spacing between buildings rows:</td>
</tr>
</tbody>
</table>

In the histograms it appears building heights \( h_b \) have a Gaussian distribution and the center-to-center spacing of the rows of the buildings \( d_c \) and the propagation angle \( \alpha \) have exponential distribution. As such, ANFIS Gaussian path loss model equation (29) and membership function (27) are used.
The algorithm proposed in Section five was conducted with a program written in MATLAB using the Harbiye region dataset. From the program, the fuzzy rules governing the path loss model based on fuzzy inference system are:

\[
\begin{align*}
\hat{Y}_1 &= 8040 - 20x_1 + 859x_2 - 13042x_3 \\
\hat{Y}_2 &= -5361 + 4x_1 - 537x_2 + 20380x_3 \\
\hat{Y}_3 &= -28302 + 21x_1 + 872x_2 + 17232x_3 \\
\hat{Y}_4 &= 18001 - 13x_1 - 547x_2 - 26801x_3 \\
\hat{Y}_5 &= -1187 - 9x_1 - 197x_2 - 10220x_3 \\
\hat{Y}_6 &= 1060 + 5x_1 + 111x_2 - 7321x_3 \\
\hat{Y}_7 &= 5494 + 10x_1 - 181x_2 + 2747x_3 \\
\hat{Y}_8 &= -3052 - 6x_1 + 99x_2 + 7096x_3.
\end{align*}
\]

(43)

The input variable number, which is according to independent variables, are three and the fuzzy class number of each input variable is two, which is determinate in initial step in proposed algorithm. The fuzzy rules number is eight, from equation (16).
Comparison of the predictions is based on the error criterion given with equation (39). The error related to predictions obtained via the models given with equation (43), which are formed by ANFIS, is found as

$$\varepsilon_{ANFIS} = \frac{1}{n} \sum_{k=1}^{n} (y_k - \hat{y}_k)^2 = 31.0105,$$

and the error related to predictions obtained via the model given with equation (11), which is proposed by Bertoni-Walfisch, is found as

$$\varepsilon_{B-W} = \frac{1}{n} \sum_{k=1}^{n} (y_k - \hat{y}_k)^2 = 188.5351.$$

The relative error related to predictions that are obtained from ANFIS, was obtained as 0.2303 and the relative error related to predictions obtained from Bertoni-Walfisch Model, was obtained as 0.0795. These results were obtained by using the ratio between the sum of absolute value of errors and sum of actual value. Thus, can be say that the ANFIS was given approximately 15% better result for this study.

The graphs of errors obtained via proposed algorithm and Bertoni-Walfisch model are shown as compared and separated in Figure 5. In Figure 5(a), errors from fuzzy adaptive network which is related to proposed algorithm in this work, in Figure 5(b), errors from Bertoni-Walfisch model, in Figure 5(c), errors from both methods, are shown.

6. Conclusions

The path loss model prediction for the 900 MHz band is concluded on measurements from the Harbiye urban area of Istanbul. For each of the 745 different measurement points, the building height $h_b$, center-to-center
spacing of the rows of the buildings \( d_c \), propagation angle between base station antenna and mobile station antenna \( \alpha \) (in radians) are counted, and are applied as the input variables to the algorithm proposed in section five.

The buildings databases are an important factor for path loss measurements in urban areas. The Bertoni-Walfisch model is first model which takes into consideration the effect of buildings in path-loss modeling. As the measurements are collecting from an urban area, the predictions from the proposed algorithm are compared with the predictions from the Bertoni-Walfisch Model.

The predictions from algorithm, which is based on ANFIS, and the predictions from Bertoni-Walfisch Model are compared with the error criterion expressed in equation (40). According to the indicated error criterion, the errors obtained from the algorithm are less than the errors obtained from the Bertoni-Walfisch Model. As the proposed algorithm doesn’t necessitate the equality of the heights and distance of the buildings it can be used for the different areas which have similar characteristics to the area used in this study.

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**References**


