A model based nonlinear adaptive controller for the passive bilateral telerobotic system

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Abstract

In this paper, we propose a new adaptive controller scheme for the bilateral telerobotic/teleoperation systems. The proposed controller achieves asymptotic tracking despite the parametric uncertainties associated with both master and slave robots while ensuring the passivity of the closed loop system. Extensive simulation studies are presented to illustrate the feasibility and efficiency of the proposed adaptive controller.

Key Words: Model Based Control, Nonlinear Systems, Passive Systems, Robotics, Tele-operation, Lyapunov Based Approaches

1. Introduction

A telerobotic system is basically a dual robot system in which a remote slave robot tracks the motion of a master robot that is commanded by a human operator. A typical telerobotic system is composed of a human operator maneuvering a master robot and a slave robot working on a remote and/or hard to reach environment. If only the master motion and/or forces are transmitted to the slave robot, the operation is referred as unilateral telerobotics. If, in addition, the motion of the slave robot with the forces applied by the environment are fed back to the master side, the operation is called bilateral telerobotics. This work will concentrate on bilateral telerobotic systems. Motivated by the large variety of applications, ranging from space explorations to mining, medical applications to nuclear waste decontamination telerobotics have been studied extensively. While this work is not intended to present a complete review of all aspects on control methods for telerobotic systems, a brief discussion on some of the important milestones related to our work is presented here. For a complete overview of the past research on various aspects of telerobotics readers are referred to [2], and [3].

In bilateral teleoperation the local side (where the operator and the master robot are present) and remote side (where the slave robot is placed) are connected via communication channels. This process often involves large distances and due to the limited data transfer rates between the local and remote sides substantial time

* A preliminary version of this work appeared in [1]
delays may occur and time delay might affect the overall stability of the system [4]. A novel solution to the aforementioned problem was proposed by [5], in which the authors proposed sending the scattering signals from transform delays into a passive transmission line connected to local and remote sides, defining passive force to velocity operators. Passivity of the closed loop system in a teleoperation system also ensures safe interaction and coupling stability of the teleoperation [5], [6], [7]. Therefore ensuring energetic passivity\(^1\) (passivity with the mechanical power as the supply rate) of the closed loop telerobotic system, as in [9] and [10], would be a way to ensure safe operation for both the human user and the slave robot’s environment. Another issue for telerobotics is the highly nonlinear dynamics of the system. Though there has been some successful control schemes for the linear telerobotic systems [11], [12], in order for a telerobotic system operate effectively, it is imperative that the nonlinear dynamics of master and slave robots are taken into account in the controller design [13]. Robot manipulators have well defined dynamic models; however exact knowledge of the parameters needed to complete their model are often not known precisely.

Therefore, a controller for a telerobotic system should not only compensate the nonlinearities of the overall system but also have to satisfy energetic passivity conditions of the closed-loop system in order to ensure safety. Recently researchers have paid some attention on the nonlinear controller design for a passive telerobotic system [14], [15].

In this study, we propose a new full-state feedback, adaptive controller scheme for the nonlinear bilateral telerobotic system. The proposed controller achieves asymptotic trajectory tracking despite the presence of parametric uncertainties in both the master and the slave arm while still ensuring the passivity of the overall closed-loop system. Introducing novel integral inequalities in conjunction with the Lyapunov type analysis we were able to satisfy all of the controller objectives (stability and passivity) at the same time via a simple controller structure. Lyapunov based techniques are used to ensure the stability and passivity of the closed loop system. Compared to the previous work, due to the use of hyperbolic functions the proposed method outputs a smoother response. Specifically when compared to that of [15] the proposed work our analysis takes the advantage of integral equations as opposed to high gain compensation terms (integral of sign of error terms) however while the controller of [15] can compensate for unmeasurable force measurements the proposed controller requires the full measurements of the master and slave robot tip forces.

The rest of the paper is organized as follows. Section 2 presents the overall system model and problem statement including the main and secondary controller objectives and the open loop error dynamics. The controller design with the desired end effector profile generation are given in Section 3. Simulation results are presented in Section 4, and Section 5 contains the concluding remarks.

## 2. System model and problem statement

The corresponding dynamics for a telerobotic system composed of an \(n\)-DOF master and an \(m\)-DOF slave robotic mechanisms (such as the one given in Figure 1), can be represented as [16]

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\(^1\)For further information about passivity and passivity based control of mechanical systems readers are referred to Chapter 6 of [8]
where the newly defined combined space representation of the dynamics given in equation (1) can be reformulated to have the advantageous form respectively. To ease the presentation of the controller design and the subsequent stability analysis, the joint where $\gamma > 0$ is a force scaling factor through the master (human operator) to the slave work environment. $M_i(q_i)$ are the symmetric positive definite inertia matrices, $C_i(q_i, \dot{q}_i)$ are the matrix containing the centripetal and Coriolis effects, $\tau_i$ are the control torque input vectors and $F_i$ are the vectors representing the joint space projection of the forces applied to the tip of the robots from the environment and/or the human operator, where $\{\cdot, \| i = m$ for the master robot, $i = s$ for the slave robot}. Assuming that both the forces applied by the human operator to the master robot and environment to the slave robot are measurable, the forces applied to the end effectors of the master and slave robots can be calculated as

$$F_H = \gamma (J_m)^{-T} F_m,$$
$$F_E = (J_s)^{-T} F_s,$$

where $F_H$ is the force applied to the end effector of the master by the operator and $F_E$ is the force applied to the slave robot by the surrounding environment, $J_m, J_s$ are the corresponding manipulator Jacobian matrices of the master and the slave robots respectively. It is also well known that, using the manipulator Jacobian, the end effector velocity can be calculated via the formulas

$$\dot{x}_m = J_m \dot{q}_m, \quad \dot{x}_s = J_s \dot{q}_s,$$

where $x_m(t)$ and $x_s(t)$ denote the end effector position and orientations of the master and the slave robots, respectively. To ease the presentation of the controller design and the subsequent stability analysis, the joint space representation of the dynamics given in equation (1) can be reformulated to have the advantageous form

$$\begin{align*}
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + F(q) + G(q) &= \tau + F,
\end{align*}$$

where the newly defined combined matrices $M(q), C(q, \dot{q}) \in \mathbb{R}^{(n+m)\times(n+m)}, F(q), F, \tau \in \mathbb{R}^{n+m}$ are explicitly defined as

$$\begin{align*}
M(q) &= \begin{bmatrix} \gamma M_m & 0_{n \times m} \\ 0_{m \times n} & M_s \end{bmatrix},
C(q, \dot{q}) &= \begin{bmatrix} \gamma C_m & 0_{n \times m} \\ 0_{m \times n} & C_s \end{bmatrix},
F(\dot{q}) &= \begin{bmatrix} \gamma F_m \\ F_s \end{bmatrix},
G(q) &= \begin{bmatrix} \gamma G_m \\ G_s \end{bmatrix},
F &= \begin{bmatrix} \gamma F_1 \\ F_2 \end{bmatrix},
\end{align*}$$

and $\tau = \begin{bmatrix} \gamma \tau_1 \\ \tau_2 \end{bmatrix}$.
and the combined joint space vector \( q \in \mathbb{R}^{n+m} \) is defined to have the form
\[
q = \left[ \begin{array}{c} (q_m)^T \\ (q_s)^T \end{array} \right]^T.
\]

It is straightforward to show that the newly formed combined dynamics (5) does have the same useful properties as the standard robot dynamics (as the boundedness of the inertia matrix, the skew symmetric relationship between the time derivative of the inertia matrix and the matrix representing the centripetal and Coriolis forces, and the linearly parameterizable property of the overall robot dynamics [13]). Furthermore, in order to ease the presentation of the stability analysis we will define a special vector function \( \text{Tanh}(\cdot) \in \mathbb{R}^n \) such that
\[
\text{Tanh}(\xi) \triangleq [\tanh(\xi_1), \ldots, \tanh(\xi_n)]^T
\]
for the vector \( \xi = [\xi_1, \ldots, \xi_n]^T \in \mathbb{R}^n \). Using the above definition and the properties of the hyperbolic functions it is straightforward to show that the following properties hold:
\[
\begin{align*}
\sum_{i=1}^{n} |\xi_i| - \xi^T \text{Tanh}(\xi) & \geq 0 \\
\int_{t_o}^{t} \sum_{i=1}^{n} |\xi_i| \, d\tau - \int_{t_o}^{t} \xi^T \text{Tanh}(\xi) \, d\tau & \leq \Omega < \infty.
\end{align*}
\]
Here, \( \Omega \in \mathbb{R}_+ \) is a positive bounding constant.

2.1. Control objective

The main control objective for the telerobotic system of equation (1) is to ensure that the end effector of the master arm follows a desired trajectory while, at the same time, ensuring that the end effector of the slave manipulator follows the trajectory formed by the master manipulator. To quantify this control objective we defined the tracking error signal \( e(t) \in \mathbb{R}^{2p} \), where \( p \) is the dimension of the operation space (containing both position and orientation information of the end effector) as
\[
e = \begin{bmatrix}
  x_d - x_m \\
  x_s - x_m
\end{bmatrix}
\]
where \( x_d(t) \in \mathbb{R}^p \) is the yet to be formulated desired end effector trajectory that the human operator has to follow and the terms \( x_m(t) \in \mathbb{R}^p \) and, \( x_s(t) \in \mathbb{R}^p \) were defined in equation (3). The error signal of equation (9) can also be represented to have the advantageous form
\[
e = \begin{bmatrix}
  x_d \\
  0_n
\end{bmatrix} - \begin{bmatrix}
  x_m \\
  x_m - x_s
\end{bmatrix} = \begin{bmatrix}
  x_d \\
  0_n
\end{bmatrix} - \begin{bmatrix}
  I_p & 0_{p \times p} \\
  I_p & -I_p
\end{bmatrix} \begin{bmatrix}
  x_m \\
  x_s
\end{bmatrix} = X_d - S \begin{bmatrix}
  x_m \\
  x_s
\end{bmatrix} = \begin{bmatrix}
  x_d \\
  0_n
\end{bmatrix} - \begin{bmatrix}
  I_p & 0_{p \times p} \\
  I_p & -I_p
\end{bmatrix} \begin{bmatrix}
  x_m \\
  x_s
\end{bmatrix}
\]
where \( X_d \) 
\[
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\]
where the auxiliary signals $X_d, x$ and $X$ are explicitly defined as

$$
X_d = \begin{bmatrix} (x_d)^T & (0_p)^T \end{bmatrix}^T,
$$

$$
x = \begin{bmatrix} (x_m)^T & (x_s)^T \end{bmatrix}^T,
$$

$$
X = Sx. \quad (11)
$$

Note that the auxiliary transformation $S \in \mathbb{R}^{p \times p}$, defined in equation (10), is always invertible due to its definition.

Our secondary control objective is to ensure that the closed-loop dynamics of the teleoperator system remains passive with respect to the user and the physical/virtual environment forces, which in operational space can be represented as [9]

$$
\int_{t_o}^t \left( \dot{x}_d^T F_E + \dot{x}_m^T F_H \right) d\tau \geq -c^2,
$$

(12)

where $c \in \mathbb{R}$ is a bounding scalar constant. Similarly, from equation (3), the fact that $F_H = \gamma J_m^T F_m$ and $F_E = J_s^T F_s$, equation (12) can also be represented as

$$
\int_{t_o}^t \left( \dot{q}_s^T F_s(\tau) + \gamma \dot{q}_m^T F_m(\tau) \right) d\tau \geq -c^2.
$$

(13)

### 2.2. Error Dynamics

Taking the time derivative of equation (10), the open loop dynamics of the error signal can be obtained to have the form

$$
\dot{e} = \ddot{X}_d - \dot{X} = \ddot{X}_d - \begin{bmatrix} \dot{x}_m \\ \dot{x}_m - \dot{x}_s \end{bmatrix}.
$$

(14)

Inserting (3) we can rearrange the error dynamics as

$$
\dot{e} = \ddot{X}_d - J \dot{q},
$$

(15)

where the generalized composite Jacobian matrix is defined as

$$
J = \begin{bmatrix} J_m & 0 \\ J_m & -J_s \end{bmatrix},
$$

(16)

and $\dot{q}(t)$ is the time derivative of the joint space position vector given in equation (6). Adding and subtracting $\alpha e$ to the right hand side of equation (15), where $\alpha$ is a positive definite diagonal gain matrix with proper dimension, we can further rearrange the open loop error dynamics as

$$
\dot{e} = -\alpha e + Jr,
$$

(17)

where the auxiliary signal $r(t) \in \mathbb{R}^{n+m}$, similar to that of [18], is explicitly defined as

$$
r = J^+(X_d + \alpha e) - \dot{q}.
$$

(18)
with $J^+$ being the pseudo inverse of the composite Jacobian matrix defined in equation (16). Based on the structure of equation (17), we are motivated to regulate $r(t)$ in order to regulate $e(t)$; hence we need to calculate the open-loop dynamics for $r(t)$. To this end we take the time derivative of (18), premultiply by the combined inertia matrix $M(q)$ of equation (5), and substitute equation (4) to yield the following advantageous form for the open-loop dynamics:

$$M(q) \ddot{r} = -C(q, \dot{q}) r + Y(q, \dot{q}, \ddot{x}_d, e, \dot{e}) \theta - u$$  \hspace{1cm} (19)$$

where the regression matrix parameter vector formulation $Y \theta$ is defined as

$$Y \theta = M(q) \frac{d}{dt} \left\{ J^+(X_d + \alpha e) \right\} + C(q, \dot{q}) \left\{ J^+(\dot{X}_d + \alpha \dot{e}) \right\} + F(q) + G(q),$$  \hspace{1cm} (20)$$

where $Y(q, \dot{q}, \ddot{x}_d, e, \dot{e}) \in \mathbb{R}^{(n+m)\times q}$ denotes the regression matrix, and $\theta \in \mathbb{R}^q$ denotes the constant but uncertain system parameters. $u(t) \in \mathbb{R}^{n+m}$ in equation (19) is the yet to be designed auxiliary control input signal formed as

$$u = \tau + F.$$  \hspace{1cm} (21)$$

Before going into the control design and analysis we also would like to define an auxiliary function denoting the combined user and physical/virtual environment forces as

$$F = S^T \begin{bmatrix} F_H \\ F_E \end{bmatrix} = \begin{bmatrix} F_H + F_E \\ -F_E \end{bmatrix},$$  \hspace{1cm} (22)$$

where the transformation $S$ was previously defined in equation (10).

3. Control design and analysis

3.1. Desired operation space trajectory

Similar to the design of [15], the desired end effector trajectory for the master robot $x_d(t)$ in equation (10) is generated using the dynamics

$$M_T \ddot{x}_d + B_T \dot{x}_d + K_T x_d = F_H + F_E,$$  \hspace{1cm} (23)$$

where $M_T, B_T, K_T$ are positive definite, constant, diagonal matrices. We will also utilize the standard assumption that the forces applied by the user on the master robot side, and by the environment on the slave robot side, are bounded functions of time (i.e. $F_H, F_E \in L_\infty$). From this assumption and (23) it is straightforward to prove that $\ddot{x}_d, \dot{x}_d, x_d$ are bounded functions of time (i.e. $\ddot{x}_d, \dot{x}_d, x_d \in L_\infty$). Now we are ready to propose the following Lemma for the passivity of the overall telerobotic system.

Lemma 1 For the combined system dynamics of (4), when the desired user trajectory signal, $x_d$, on the master robot is designed according to (23), and the time derivative of the error signal defined in equation (10) satisfies $\dot{e}(t) \in L_1$, then the overall passivity of the telerobotic system of (4) is passive with respect to the forces applied by user on the master robot and physical and/or virtual environment as given in equation (12).
Proof To prove the above Lemma we introduce a nonnegative scalar function $V_\theta(t) \in \mathbb{R}_+$ as

$$V_\theta = \frac{1}{2} \ddot{x}_d^T M_T \ddot{x}_d + \frac{1}{2} \ddot{x}_d^T K_T x_d,$$  

where the variables $M_T$ and $K_T$ were previously defined. Taking the time derivative of (24) and inserting for $M_T \ddot{x}_d$ from equation (23), we can upper bound the time derivative of $V_\theta(t)$ as

$$\dot{V}_\theta \leq \ddot{x}_d^T (F_H + F_E),$$  

where the fact that the term $\ddot{x}_d^T B_T \dot{x}_d$ is always positive is utilized. From equation (24) and equation (25), we can conclude that $V_\theta(t)$ is bounded function. Furthermore, integrating both sides of equation (25), the following integral inequality can be obtained

$$-c_1 \leq V_\theta(t) - V_\theta(t_0) \leq \int_{t_0}^{t} \ddot{x}_d^T (F_H + F_E) \, d\tau,$$  

where $c_1 \in \mathbb{R}$ is a positive scalar bounding constant. Using the definitions of equation (10) and equation (22) we can rearrange the integral inequality of (26) as

$$-c_1 \leq \int_{t_0}^{t} \dot{X}_d^T F d\tau.$$  

From its definition it is obvious that

$$\int_{t_0}^{t} X^T F d\tau = \int_{t_0}^{t} \dot{X}_d^T F d\tau - \int_{t_0}^{t} \dot{e}^T F d\tau.$$  

From the above definition and the assumption that the user and the environment forces are bounded, we can conclude that, whenever the integral of the error signal is bounded, i.e. $\dot{e}(t) \in \mathcal{L}_1$, the integral inequality

$$\int_{t_0}^{t} X^T F d\tau \geq -c_2,$$  

holds for some positive bounding constant $c_2$, which after same mathematical manipulations can be written in the form

$$\int_{t_0}^{t} x^T S^T S^T \begin{bmatrix} F_H \\ F_E \end{bmatrix} d\tau \geq -c_2.$$  

When the definitions equation (11) and $S^T S^T = I_{2p}$ are utilized, equation (12) is achieved, which proves the Lemma.

3.2. Controller design

As a consequence of Lemma 1, we have the additional control objective to ensure that $\dot{e}(t) \in \mathcal{L}_1$; for this purpose we propose the adaptive controller

$$u = Y \dot{\theta} + Kr + \beta_1 \text{Tanh}(r) + J^T (e + \text{Tanh}(e)),$$  

where

$$J = \begin{bmatrix} I_q \\ I_{2p} \end{bmatrix}, \quad \beta_1 = \beta_1(\tau), \quad \text{Tanh}(r) = \begin{bmatrix} \tanh(r_1) \\ \cdots \\ \tanh(r_{2p}) \end{bmatrix}.$$
where $K$ and $\beta_1$ are positive controller gain matrices with proper dimensions, $\hat{\theta}(t)$ denotes the dynamic parameter estimates of $\theta$ computed on-line according to the update rule

$$\dot{\hat{\theta}} = \Gamma Y^T r,$$  \hspace{1cm} (32)

with $\Gamma$ being the positive definite, diagonal gain matrix. In addition, the difference between the actual and estimated system parameters are defined as

$$\tilde{\theta} = \theta - \hat{\theta}(t).$$  \hspace{1cm} (33)

Substituting the adaptive controller of (31) into the open-loop error dynamics of (19), the closed-loop error system for $r(t)$ can now be obtained as

$$M(q) \dot{r} = -\zeta(q, \dot{q}) r + Y \dot{\tilde{\theta}} - K r - \beta_1 \text{Tanh}(r) - J^T (e + \text{Tanh}(e)).$$  \hspace{1cm} (34)

We are now ready to propose the following Theorem.

**Theorem 1** Given the telerobotic system dynamics of (4), the adaptive controller of (31), with the parameter estimates (32), ensures that the tracking error signal of (10) is asymptotically stable, in the sense that

$$\lim_{t \to \infty} e(t) = 0$$

while ensuring that the time derivative of equation (10) satisfies $\dot{e}(t) \in L_1$.

**Proof** To prove Theorem 1, a nonnegative scalar function $V(t)$ is defined as

$$V = \frac{1}{2} r^T M(q) r + \frac{1}{2} e^T e + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \sum_{i=1}^{2p} \ln(cosh(e_i)).$$  \hspace{1cm} (35)

Taking the time derivative of $V(t)$, inserting for the closed-loop dynamics of (34) and the dynamics of the parameter updates (32), one can obtain the expression

$$\dot{V} = -r^T K r - e^T \alpha e - \beta_1 r^T \text{Tanh}(r) - e^T \alpha \text{Tanh}(e),$$  \hspace{1cm} (36)

where the property

$$\frac{d}{dt} \left\{ \sum_{i=1}^{2p} \ln(cosh(e_i)) \right\} = \dot{e}^T \text{Tanh}(e)$$

has been utilized. Notice that, using the fact that

$$\xi^T \text{Tanh}(\xi) \geq 0 \ \forall \ \xi \in \mathbb{R}^n,$$

equation (36) can be upper bounded to the form

$$\dot{V} \leq -r^T K r - e^T \alpha e.$$  \hspace{1cm} (37)
From equation (35) and equation (37) we can conclude that \( r(t), e(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty \); and from the definition of (17) \( \dot{e}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty \) can be obtained; and finally from direct application of Barbalat’s Lemma [17], we can conclude that \( \lim_{t \to \infty} e = 0 \). Standard signal chasing arguments can be applied to show that all signals in the closed-loop dynamics are bounded (assuming that both robots operate inside a singularity-free operational region). Additionally, integrating both sides of equation (36) we obtain

\[
V(t_o) - V(t) \geq \int_{t_o}^{t} \beta_1 r^T \tanh(r) d\tau + \int_{t_o}^{t} e^T a \tanh(e) d\tau; \tag{38}
\]

and using the fact that \( V(t) \) is bounded, we can conclude that the integrals of the terms \( r^T \tanh(r) \) and \( e^T \tanh(e) \) are bounded. From direct application of the integral inequalities given in equation (8), we can conclude that

\[
\int_{t_o}^{t} \sum_{i=1}^{2n} |r_i| d\tau \leq \Omega_1, \quad \int_{t_o}^{t} \sum_{i=1}^{2n} |e_i| d\tau \leq \Omega_2, \tag{39}
\]

where \( \Omega_1, \Omega_2 \in \mathbb{R}_+ \) are positive bounding constants. Therefore we can conclude that the auxiliary tracking signal and the tracking error signal are integrable, i.e. \( r(t), e(t) \in \mathcal{L}_1 \). We can then use equation (17) to prove that \( \dot{e}(t) \in \mathcal{L}_1 \).

In conclusion, the controller proposed ensures that the tracking error signal asymptotically converges to zero (\( \lim_{t \to \infty} e = 0 \)) while also ensuring the passivity of the overall system of (4) when the desired end-effector profile for the master robot is generated through the dynamics given by equation (23), which concludes the proof.

\[\Box\]

4. Simulation results

To illustrate the performance and the effectiveness of the proposed adaptive controller, we simulated a telerobotic system composed of a 2-link planar master robot and a 3-link planar slave robot (the dynamics of the simulated robots are taken from [19] and [18], respectively). The corresponding constant system parameters of the master and slave robots were constructed as

\[
\theta_m = \begin{bmatrix} p_{m1} & p_{m2} & p_{m3} & f_{m1} & f_{m2} \end{bmatrix}^T,
\]
\[
\theta_s = \begin{bmatrix} \beta_{s1} & \beta_{s2} & \beta_{s3} & p_{s1} & p_{s2} & p_{s3} & f_{s1} & f_{s2} & f_{s3} \end{bmatrix}^T,
\]

where \( \theta_m \in \mathbb{R}^5 \) and \( \theta_s \in \mathbb{R}^9 \) contain the mass and friction parameters of the master and slave robots respectively. In the present simulations (similar to [19] and [18]), the following parameters had been selected:

\[
\begin{align*}
p_{m1} &= 3.31 \text{ kg} \cdot \text{m}^2, & p_{m2} &= 0.116 \text{ kg} \cdot \text{m}^2, & p_{m3} &= 0.16 \text{ kg} \cdot \text{m}^2, \\
f_{m2} &= 5.3 \text{ Nm} \cdot \text{sec}^2, & f_{m2} &= 1.1 \text{ Nm} \cdot \text{sec}^2, \\
\beta_{s1} &= 1.1956 \text{ kg} \cdot \text{m}^2, & \beta_{s2} &= 0.3946 \text{ kg} \cdot \text{m}^2, & \beta_{s3} &= 0.0512 \text{ kg} \cdot \text{m}^2, \\
p_{s1} &= 0.4752 \text{ kg} \cdot \text{m}^2, & p_{s2} &= 0.1280 \text{ kg} \cdot \text{m}^2, & p_{s3} &= 0.1152 \text{ kg} \cdot \text{m}^2, \\
f_{s1} &= 5.3 \text{ Nm} \cdot \text{sec}^2, & f_{s2} &= 2.4 \text{ Nm} \cdot \text{sec}^2, & f_{s3} &= 1.1 \text{ Nm} \cdot \text{sec}^2.
\end{align*}
\]
The desired end-effector trajectory of the master robot, $x_d$, were calculated according to equation (23) with the parameters $M_T = I$ and $B_T = K_T = 0$, while the forces applied by the human operator force and the actual/virtual environment were assumed to be measurable.

In order to mimic a realistic system, the control frequency used in the simulation studies were 1 KHz and a constant delay of 30 ms between the master and the slave robots were imposed to the overall simulation to include the effects of time delay. The adaptation and control gains are selected through a trial and error method and the best performance was obtained when the controller gains are selected as

$$
\alpha = \text{diag}\{2.1, 2.1, 2.8, 2.9\},
K = \text{diag}\{7.5, 6.5, 8, 7, 6\},
\beta_i = \text{diag}\{1, 1, 2, 1, 0.3\},
$$

(40)

and the adaptation gains were set to

$$
\Gamma = \text{diag}\{3, 0.1, 0.2, 0.4, 0.1, 0.1, 1, 0.3, 0.05, 5, 1.2, 5, 2, 3\},
$$

(41)

with the initial value of the system parameters set to zero.

Figure 2 illustrates the outcome of the desired and actual end-effector trajectories for the master and slave robots. Figures 3 and 4 illustrate the end-effector tracking error signals for the master and the slave robot, respectively, while the adaptation of the master and the slave robot system parameters are shown in Figures 5–9. The joint level control torque inputs are shown in Figures 10 and 11. As illustrated by the simulations, response of the controller output is smooth and tracking performance is satisfactory.

![Figure 2](image-url)

**Figure 2.** The desired and actual task space trajectories for the master and the slave robots: (a) the desired trajectory for the master robot (b) the actual master robot trajectory (c) the actual slave robot trajectory (d) the desired trajectory and the actual slave robot trajectory.
Figure 3. Task space tracking errors of the master robot.

Figure 4. Task space tracking errors of the slave robot.
Figure 5. Estimates of the mass related parameters of the master robot.

Figure 6. Estimates of the friction related parameters of the master robot.
Figure 7. Estimates of the $\beta$ parameters of the slave robot.

Figure 8. Estimates of the mass related parameters of the slave robot.
A second set of simulations were then performed to test the robustness of the proposed method against the changes in the network time delay effect. In these simulations the time delay between the local and the remote system were increased step by step from 0 to 600 milliseconds and the position tracking error performance of the system on both master and slave robots were recorded. The control and adaptation gains were same as the first simulations. Figures 12 and 13 illustrate the performance of the master robot while the error tracking performance of the slave manipulator is presented in Figures 14 and 15. Response of the master throughout all the simulations remains same. This is mainly due to the fact that there is nearly no delay at the master’s site. On the remote (slave robot) site degradations start around time delays of 250 milliseconds. However, even with time delays as high as 600 milliseconds, the controller is able to achieve practical tracking.
Figure 11. Control torque inputs for the slave robot.

Figure 12. Error tracking performance of the master with time delays 0, 30 and 100 milliseconds
Figure 13. Error tracking performance of the master with time delays 250, 500 and 600 milliseconds.

Figure 14. Error tracking performance of the slave for time delays 0, 30 and 100 milliseconds.
5. Conclusion

In this paper, we have presented a new adaptive controller scheme for the bilateral telerobotic system composed of an \( n \)-DOF master and \( m \)-DOF slave robot. The proposed method guarantees asymptotic end effector tracking despite the uncertainties in the dynamics of both the master and the slave robots provided that user and environment input forces are measurable. Due to the use of hyperbolic functions the response of the proposed controller is smoother compared to the similar controllers in the literature. Lyapunov-based techniques were used to prove the stability of the closed loop system. Simulation results were presented to illustrate the effectiveness of the proposed method.

Acknowledgment

This work was supported by the Turkish Scientific and Technical Research Council TUBITAK KARIYER Programme Project No: 104E061.

References


