

# The effect of the channel reliability factor in the MAP algorithm on turbo code performance in bluetooth systems

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## Abstract

*The effects of the channel reliability factor on the performance of turbo decoders that use a maximum a posteriori probability algorithm are investigated for wireless communication fading channels, such as Bluetooth systems. The channel reliability factor is related to the channel signal-to-noise ratio. For various values of the channel reliability factor, the obtained performance curves of the maximum a posteriori probability algorithm are given for the case of DMx Bluetooth data packets. When the channel reliability factor increases, the bit-error-rate performance of the DMx Bluetooth data packets also increases over fading channels.*

**Key Words:** Turbo coding, MAP algorithm, channel reliability factor ( $L_c$ ), bluetooth.

## 1. Introduction

The performance of turbo codes over Rayleigh fading channels has been examined by many researchers [1, 2, 3]. A turbo decoding system using the maximum a posteriori probability (MAP) algorithm requires knowledge of the channel reliability factor, a parameter related to the channel signal-to-noise ratio (SNR), to properly combine the a posteriori deformation of the separate decoders. Turbo codes are different from conventional codes because they are recursive and systematic. The 1/3-rate Turbo coder can be constructed by two recursive systematic coding (RSC) coders separated with a random interleaver.

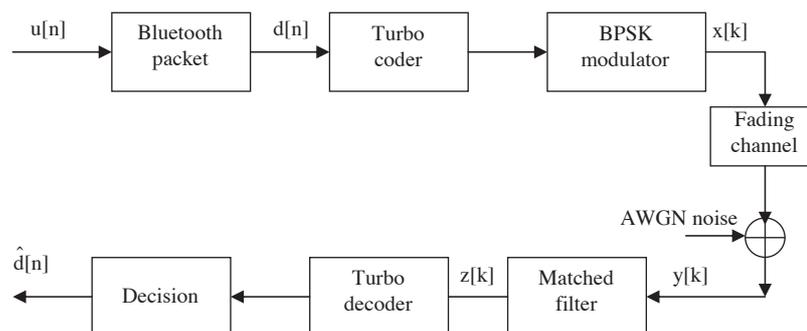
In practice, there are errors in the channel state information available to the receiver. In this paper, we propose a simple estimation structure of the channel reliability factor,  $L_c$ , for the intersymbol-interference (ISI) channel, which is based on the SNR at the receiver. Performance gains are obtained for various values of the channel reliability factor in case of different Bluetooth data packets.

The Bluetooth system is an industry specification for wireless short-range speech and data communication. Bluetooth has gained broad industry support, and the use of the international 2.4 GHz industrial-scientific-medical (ISM) band guarantees the operation on a worldwide basis. Bluetooth technology is advantageous

over other wireless local area network (LAN) connectivity technologies because of its low-cost, low-power and interoperability. Hence, Bluetooth is a technology of great interest for applications in indoor environments. The Bluetooth standard has been suggested and studied as a potential solution for mobility scenarios [4]. Since Bluetooth has both limited range and bandwidth, mobile environments raise questions about the amount of time it takes to make a connection between two Bluetooth units. In addition, the mobile environment presents different obstacles, such as fast/slow fading, multipath and the Doppler effects induced by platform motion. Compared with other systems operating in the same ISM frequency band, the Bluetooth radio hops faster and uses shorter packets, which makes a Bluetooth transmission very robust against a variety of typical interference effects. Bluetooth operates in the unlicensed 2.4 GHz ISM band. Gaussian-shaped frequency shift keying modulation is applied to minimize transceiver complexity. Each packet is transmitted on a different hop frequency. A packet nominally covers a single slot, but can be extended to cover up to 5 slots. Bluetooth can support an asynchronous data channel or a channel that simultaneously supports asynchronous data [5]. The Bluetooth system supports both point-to-point and point-to-multipoint connections. Piconets can be established and linked together in an ad-hoc fashion, where each piconet is identified by a different frequency hopping sequence. A piconet supports up to 8 devices, where one device acts as a master. The master controls traffic of up to a maximum of 7 units, defined as slaves in the piconet.

## 2. The Communication system model

The simulation system model is illustrated in Figure 1. Information symbols  $d[n]$ , which show Bluetooth data packet bits  $u[n]$ , are passed to the 1/3 rate Turbo encoder that consists of two identical recursive systematic convolutional encoders with generator polynomials of  $(7, 5)_8$ . Puncturing is not an issue. The two constituent encoders are separated by a random interleaver of length  $L$ . The coded symbols are interleaved by a channel interleaver, which makes the channel to appear like a random error channel at the receiver. Then, the symbols are modulated by a binary phase shift keying (BPSK) modulator and transmitted over a frequency Rayleigh fading channel in additive white Gaussian noise (AWGN).



**Figure 1.** The scheme of the simulation system model.

Channel matched filtering represents a process that uses a filter with coefficients that are the complex conjugate of the mirror image of the estimated channel profile. If an ISI corrupted signal is passed through the matched filter, the resulting transfer function of the channel becomes symmetrical around a real-valued central peak. Furthermore, channel matched filtering provides optimum symbol synchronization for the equalizer filter

[6]. At the receiver, after matched filtering and sampling at the perfect symbol timing rate of  $1/T$ , the received signal is given by

$$y[k] = h[k]x[k] + \eta[k], \quad (1)$$

where  $T$  is the symbol duration,  $k$  is an integer symbol subscript,  $x[k] \in \{-1, +1\}$  is a coded BPSK transmitted symbol,  $h[k]$  is a zero-mean complex Gaussian random variable that describes the multiplicative amplitude and phase distortion of the Rayleigh fading channel, and  $\eta[k]$  is zero-mean complex additive white Gaussian noise with variance  $\sigma_{noise}^2 = N_0/2$ . The Turbo decoder has two identical MAP decoders. The MAP algorithm used is the modified Bahl approach [7]. In order to prevent overflow during MAP decoder computations, the transition factors,  $\gamma(s', s)$ , which are computed from the channel transition probabilities are normalized. Independent normalization occurs for every value of the time index,  $k$ . This ensures that the largest value at each time index is one. The forward factors  $\alpha_k(s)$  are recursively calculated from the transition factors and the constituent coder's state transition matrix for the previous states. The backward factors  $\beta_k(s)$  are recursively calculated from the transition factors and the succeeding state transition matrix. At each recursion, factors that could cause underflow are set to zero.

### 3. The channel reliability factor ( $L_c$ ) for the MAP algorithm

Channel estimate statistics are needed for the MAP algorithm in the Rayleigh fading channel. The output of the matched filter is passed through the Turbo decoder after compensating for the effect of fading channels. The channel estimate,  $v[k]$ , for  $h[k]$  is expressed by

$$v[k] = h[k] + e[k], \quad (2)$$

where  $e[k]$  is the fading estimation error, assumed to be a zero-mean Gaussian random variable with variance  $\sigma_e^2 = \frac{1}{2}E\{e[k]e^*[k]\}$ . The variances of the channel estimate and the channel output are given as

$$\begin{aligned} \sigma_v^2 &= \frac{1}{2}E\{v[k]v^*[k]\} \\ \sigma_y^2 &= \frac{1}{2}E\{y[k]y^*[k]\} \end{aligned} \quad (3)$$

The cross-correlation coefficient between the received signal  $y[k]$  and the channel estimate  $v[k]$  is given as follows:

$$\begin{aligned} \rho_k &= \frac{\frac{1}{2}E\{y[k]v^*[k]\}}{\sqrt{\frac{1}{2}E\{y[k]y^*[k]\} \frac{1}{2}E\{v[k]v^*[k]\}}} \\ &= \frac{2\sigma_h^2}{\sqrt{(|x[k]|^2 2\sigma_h^2 + 2\sigma_{noise}^2)(2\sigma_h^2 + 2\sigma_e^2)}} x[k] \\ &= |\rho| e^{-j\varepsilon_k} \end{aligned} \quad (4)$$

In Equation (4),  $|\rho|$  and  $\varepsilon_k$  are the amplitude and phase of  $\rho_k$ , respectively. Clearly,  $\varepsilon_k$  depends on the transmitted data symbol,  $x[k]$ . The decision variable is given by

$$z[k] = y[k]v^*[k] \quad (5)$$

Since  $z[k]$  conditioned on  $v[k]$  and  $x[k]$  is Gaussian distributed, the channel reliability factor is only changed in the decoding algorithm and can be obtained from the log-likelihood ratio of  $z[k]$ .

$$\ln \left( \frac{p(z[k]|v[k], x[k] = +1)}{p(z[k]|v[k], x[k] = -1)} \right) = \frac{2|\rho|}{|\sigma_y \sigma_v (1 - |\rho|^2)|} z[k] \quad (6)$$

In Equation (6), the channel reliability factor differs from its conventional definition as a product of the fading amplitude and constant factor  $2/\sigma^2$ . The channel reliability factor required for MAP decoding when the Turbo decoder uses the BPSK modulation is given by [8]

$$\begin{aligned} L_c &= \frac{2|\rho|}{\sigma_y \sigma_h (1 - |\rho|^2)} \\ &= \frac{4\sqrt{E_s}}{N_0} \sigma_h^2 \left\{ \sigma_e^2 \left( \frac{2E_s}{N_0} \sigma_h^2 + 1 \right) + \sigma_h^2 \right\}^{-1} \end{aligned} \quad (7)$$

where  $E_s$  is the symbol energy,  $N_0 = 2\sigma_{noise}^2$  and  $|\rho| = \frac{E[y_k v_k^*]}{\sqrt{E[|y_k|^2]E[|v_k|^2]}} \text{sgn}(h_k)$  is the value of the cross-correlation coefficient between the received signal  $y[k]$  and the estimated channel output  $v[k]$ . Also,  $z[k]$  contains the estimate of the fading amplitude for the conventional definition in [9]. If channel estimation is perfect, then  $L_c$  becomes  $2/\sigma_{noise}^2$  in Equation (6). A channel reliability factor of  $2/\sigma_{noise}^2$  was used, even though the channel estimation is not perfect [10, 11]. As depicted in Equation (6), when channel estimation error exists, the channel reliability factor is dependent on channel estimate statistics, such as error variance  $\sigma_e^2$ , as well as the channel statistics,  $\sigma_h^2$  and  $\sigma_{noise}^2$ . Therefore, three statistical variables can be defined as follows:

$$\sigma_y^2 = \frac{1}{2} E\{y[k]y^*[k]\} \quad (8)$$

$$\eta_1 = \sigma_y^2$$

$$\sigma_v^2 = \frac{1}{2} E\{v[k]v^*[k]\} \quad (9)$$

$$\eta_2 = \sigma_v^2$$

and

$$E\{|z[k]|\} = \sigma_y \sigma_v (1 + |\rho|^2) \quad (10)$$

$$\eta_3 = \sigma_y \sigma_v (1 + |\rho|^2)$$

The channel reliability factor  $L_c$  can be expressed by using Equations (6), (8), (9) and (10) as

$$L_c = \frac{2\sqrt{\eta_3/\sqrt{\eta_1\eta_2} - 1}}{2\sqrt{\eta_1\eta_2} - \eta_3} \quad (11)$$

where  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  can be estimated by a block average over a Bluetooth data block of length  $N$  so as to obtain the estimate of  $L_c$ .

## 4. Simulation results

In this section, we provide simulation examples in order to illustrate the effects of the channel reliability factor on the performance of a MAP decoder algorithm for a Bluetooth data packet. The standard packet format of Bluetooth is given in Table 1. Table 2 gives the number of payload bits of Bluetooth data packets, such as DM1, DM3 and DM5 [5]. The DM1 packet is a packet that carries data information only. DM stands for Data-Medium rate. The payloads contains up to 18 information bytes (including a 1-byte header). The DM1 packet may cover up to a single time slot. The DM3 packet is a DM1 packet with an extended payload. The DM3 packet may cover up to three time slots. The payload contains 121 bytes (without a header). When a DM3 packet is sent or received, the RF hop frequency shall not change for a duration of three time slots. The DM5 packet may cover up to five time slots. The payloads contain up to 226 information bytes (including a 2-byte header).

**Table 1.** Standard packet format of Bluetooth [5].

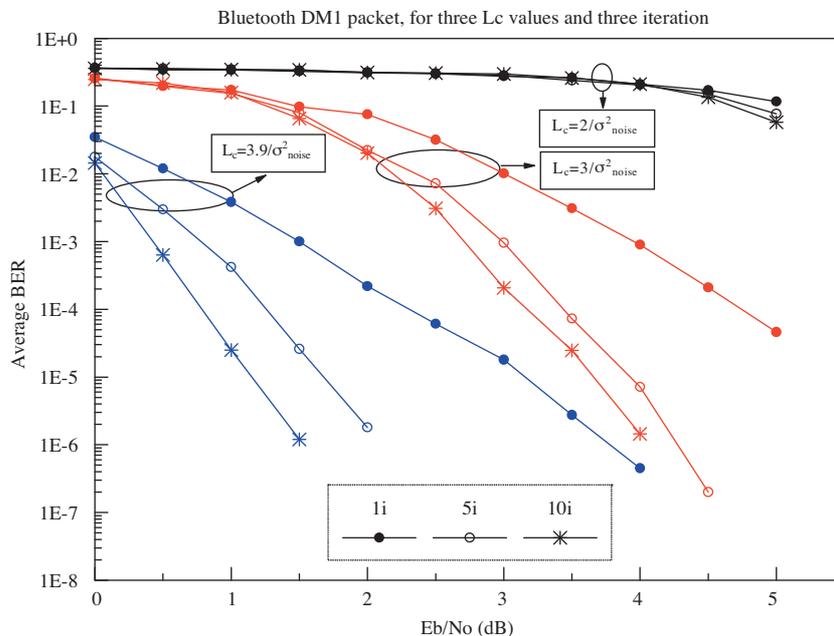
Access Code (72 bits)	Header (54 bits)	Payload (0-2745 bits)
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**Table 2.** Bluetooth medium-rate data packets.

Types of packet	Payloads (byte)
DM1	0 - 17
DM3	0 - 121
DM5	0 - 224

We consider a Rayleigh fading channel. The channel code is a rate 1/3 constraint length-four RSC code. The interleaver is generated randomly for all simulations. The block sizes of the information bits are randomly generated Bluetooth DMx packets. The code bits are BPSK modulated:  $x[k] \in \{-1, +1\}$ .

For the Bluetooth DM1 packet, the BER performance curves for various values of  $L_c$  are shown in Figure 2. The curves obtained for 1-iteration, 5-iterations and 10-iterations are indicated as 1i, 5i and 10i in the figure,



**Figure 2.** Average BER-Eb/No performance for the Bluetooth DM1 packet in Rayleigh channel.

respectively. In Figure 2, BER performance is the worst when the channel reliability,  $L_c$ , is  $2/\sigma_{noise}^2$ . When  $L_c$  is  $3/\sigma_{noise}^2$ , the performance improves. The best BER-Eb/No performance is possible when  $L_c$  is  $3.9/\sigma_{noise}^2$ . It can also be seen that as the number of iterations increase, the performance increases, as well.

Figure 3 shows BER performance curves for the Bluetooth DM3 packet. The performance gains in Figure 3 are better than those given in Figure 2.

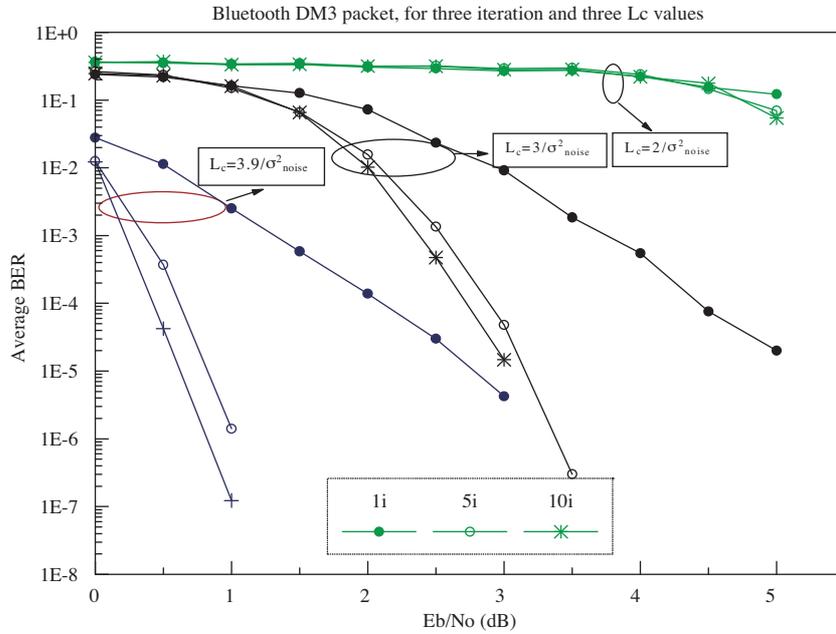


Figure 3. Average BER performance for the Bluetooth DM3 packet.

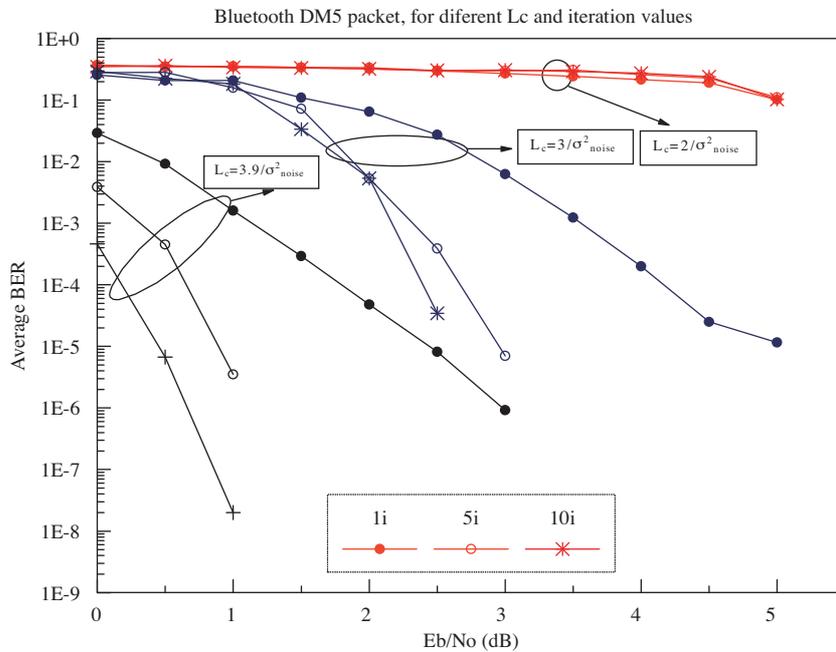


Figure 4. Average BER-Eb/No performances of the Bluetooth DM5 packet in a Rayleigh channel.

Finally, we illustrate BER-performance curves for various values of  $L_c$  in Figure 4 under the Bluetooth DM5 packet. The best performance is obtained when  $L_c$  is  $3.9/\sigma_{noise}^2$  with 10-iterations.

The best value for the channel reliability factor is the  $3.9/\sigma_{noise}^2$ . The number of  $3.9/\sigma_{noise}^2$  for optimum  $L_c$  is obtained by using a heuristic approach. Note that the worst BER performance was obtained for  $L_c$  is  $2/\sigma_{noise}^2$  in whole Bluetooth DMx data packets. All curves also show that increasing the Bluetooth data DMx packet length and the number of iterations besides the  $L_c$  value provides much better performance.

## 5. Conclusion

The effects of the channel reliability factor on MAP decoder algorithm performance have been investigated for a Rayleigh fading channel. In this paper, we evaluated BER-Eb/No performance for various  $L_c$  values under the DM1, DM3 and DM5 Bluetooth data packets.

Performance gains were found to range from about 0.2 to 1.5 dB at a BER of  $10^{-3}$ , depending on the  $L_c$  values, the number of iterations, and the Bluetooth data packet type used. The result show that performance significantly improves as both the channel reliability factor  $L_c$  value and the number of iterations increase.

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