Impacts of Distributed Generators on the Oscillatory Stability of Interconnected Power Systems

İstemihan GENÇ, Ömer USTA
İstanbul Technical University, Faculty of Electrical & Electronics Engineering, 34469, Maslak, İstanbul-TURKEY
e-mail: genc@elk.itu.edu.tr - usta@elk.itu.edu.tr

Abstract
Integration of distributed generation is continually and gradually affecting the stability of interconnected power systems. In this paper, the impacts of distributed synchronous generators on oscillatory stability are studied. In various parameter sub-spaces of interest, feasibility regions can be calculated to determine the conditions to sustain the stable operation of an interconnected power system. Through computations of the feasibility boundaries corresponding to Hopf bifurcations of electromechanical oscillatory modes, we determine the operating limits for a stable operation of the system under small and continual disturbances such as predictable changes in loading conditions of generators. Thus, with the case studies given in the paper, under different operating conditions, we investigate the effects of distributed synchronous generators on the oscillatory small-signal stability. It has been shown that penetrations of these generators can cause local or inter-area oscillatory instabilities depending on the system’s topology, operating point and control parameters.

Key Words: Oscillatory stability, distributed generation, Hopf bifurcation, feasibility boundaries.

1. Introduction
Deregulation in the power market has encouraged the move towards distributed generation, where many smaller generating plants located close to major loads, as opposed to a few large centrally located power stations, are penetrating into interconnected power systems. For many years, electric power systems have been operated within the boundaries defined by conservative reliability criteria. On the other hand, as higher efficiency is desired, especially as deregulation is encouraged, the operators are forced to operate these systems very close to their operating limits.

With the new trend in the power market, intra-area (local) and inter-area transactions of electricity have been greatly proliferated. Typically, these transactions are of considerable duration and larger variety than those in vertically integrated utility structures [1]. Naturally, this leads to frequently changing operating points and load flow patterns that may cause stability problems. Therefore, stability analysis of a system under such transformation becomes more critical and difficult than it was before.

Among many stability problems in power systems such as transient stability, where the stability under large disturbances are concerned, small signal stability analysis has also become an emerging problem. Stable
operation of an interconnected power system under small disturbances, such as predictable changes that are continually occurring in loading conditions and power transfers, involves small-signal stability analysis over a region or a set of changing operating points. Following such changes, the characteristics of the oscillatory behavior of the system depend on the local stability of the operating points. In this paper, oscillatory stability of power systems including distributed generation units is studied within this frame.

Oscillatory instability in nonlinear systems, such as power systems, is often related to Hopf bifurcations. Hopf bifurcation related instabilities in power systems have been studied for more than two decades. Early work on the Hopf bifurcation related instability in power systems can be found in [2], [3]. A fundamental platform for solving practical problems in large constrained nonlinear systems, mainly emphasizing power systems was developed in [4]. Specifically, voltage stability problems associated with bifurcations have been widely explored in the literature, e.g. [5]. In our work, we study stability problems related to Hopf bifurcations of the electromechanical modes involved in real-power-angle dynamics of power systems.

With the increase in distributed generation, impacts of the distributed generators, such as fuel-cells, micro-turbines, wind generators and solar panels, on the stability have become an emerging problem. Some examples of the studies in impacts of the distributed generation on dynamics and stability of power systems exist, for example, [6], [7]. The impacts of the distributed generation with different types of generation units including synchronous generators, on transient and voltage stability have been studied, e.g. [8], [9]. In this paper, the impacts of distributed synchronous generators on oscillatory stability are our main interest. Unlike the previous work on the stability problems related to distributed generation, we use the calculation of the feasibility boundaries associated with Hopf bifurcations, to determine and to study the conditions for a stable operation of distributed generators in accordance with the rest of the power system. However, this work does not include the stability problems related to the generation units other than synchronous generators. Since some of the distributed generation units, such as fuel cells and solar panels, are connected to the network through power electronics equipment, they have totally different dynamic and control characteristics and their stability problems, therefore, require another research effort.

The organization of the paper is as follows: In Section 2, we give some background information about the feasibility region defined for a differential-algebraic system and explain the use of feasibility boundary calculations in determining the oscillatory instabilities. In Section 3, the impacts of the distributed generators on the oscillatory stability and the methods of our analysis are summarized. In Section 4, simulations of selected case studies demonstrating the nature of inter-area or intra-area (local) oscillatory instabilities caused by the penetration and operation of distributed generators are given.

2. Oscillatory Stability and Feasibility Boundaries

The quasi-stationary dynamics of large electric power systems can be modeled by parameter dependent differential-algebraic equations (DAE) of the form

\[
\Sigma : \dot{x} = f(x, y, p) \quad f : \mathbb{R}^{n+m+p} \to \mathbb{R}^n, \\
0 = g(x, y, p) \quad g : \mathbb{R}^{n+m+p} \to \mathbb{R}^m, \\
x \in X \subseteq \mathbb{R}^n, y \in Y \subseteq \mathbb{R}^m, p \in P \subseteq \mathbb{R}^p,
\]

(1)

where in the state space \( X \times Y \), dynamic state variables, \( x \), and instantaneous state variables, \( y \), are distinguished. The parameter space and its variables are denoted by \( P \) and \( p \), respectively [10].
reduced Jacobian (system matrix) $A$ at nonsingular points for this system is calculated as

$$A = D_x f - D_y f(D_y g)^{-1} D_z g,$$

where $D_x f$ denotes the partial derivatives $\partial f/\partial x$, and so on.

The feasibility region for the differential-algebraic model of a power system has been introduced in [11] to distinguish the operating points of the system at which the system can be operated without loss of local stability. Within this region, the system operates at a locally stable equilibrium and can be driven to any point in the region by slow parametric variations without losing local stability. Feasibility region and boundary [11] calculations are crucial to determine preventive measures against an occurrence of instability. Designing control schemes and determining operating conditions require careful assessments of the feasibility regions. The feasibility boundary has been analyzed in Theorem 1 in [11] and it was shown that it corresponds to three zero-sets of functions which are related to the principal codimension one bifurcations, the saddle-node and Hopf-bifurcations, in case of a smooth induced dynamics and the singularity induced bifurcation in case of loss of stability along the singular set.

Oscillatory stability problem in power systems arises as a small signal stability problem of large interconnected systems, which are subject to disturbances such as changes in the system loading. Oscillatory instabilities observed in the real-power-angle dynamics of a power system are quite often related to Hopf bifurcations of electromechanical oscillatory modes. Essentially, Hopf bifurcation occurs as a simple pair of complex eigenvalues of the system operating at an equilibrium (operating point) transversally crosses in complex plane from the negative to the positive half-plane [12]. This process is connected to oscillatory stability since it generates (or diminishes) periodic orbits (limit cycles) around the operating point. Depending on the system parameters, the operating point and the structure of the system, two types of Hopf bifurcations can be experienced: supercritical and subcritical Hopf bifurcations. To study the possible nature of these kinds of oscillatory instabilities in power systems, examples of both supercritical and subcritical Hopf bifurcations on small power system models, for which explicit calculations of center manifold and curvature coefficients are feasible, are studied in [13].

An algorithm that calculates the feasibility boundary segments has, for example, been considered in [14]. This algorithm involves a procedure to find the equilibria which have eigenvalues on the imaginary axis. Basically, it has two stages: the calculation of a single point on the feasibility boundary and then the calculation of the feasibility boundary segment which is near the point calculated (“continuation” techniques). As the algorithm in [14] is used in this paper, during the calculation of the points on the feasibility boundary, eigenvalue sensitivities with respect to selected system parameters are computed. If $\lambda$ is an algebraically simple eigenvalue of the system matrix $A$ (see Eq. (2)), the sensitivity of the eigenvalue to the parameter $p$ at an operating point $(x_0, y_0, p_0)$ can be calculated as

$$\frac{d\lambda}{dp}(p_0) = \frac{w(p_0)^T \left( \frac{dA}{dp}(x_0, y_0, p_0) \right) v(p_0)}{w(p_0)^T v(p_0)},$$

where $w$ and $v$ are the left and right eigenvectors of $A$, respectively. The derivative $dA/dp$ for the system given in Eq. (1) can be given as
\[
\frac{dA}{dp} = \sum_{i=1}^{n} \frac{\partial A}{\partial x_i} \frac{dx_i}{dp} + \sum_{j=1}^{m} \frac{\partial A}{\partial y_j} \frac{dy_j}{dp} + \frac{\partial A}{\partial p}.
\]  

Since we study Hopf bifurcation related oscillatory instabilities, the feasibility boundary segments that are calculated in this paper correspond to the points where Hopf bifurcations occur. In our calculations, as we calculate the feasibility boundary segment corresponding to a selected oscillatory mode, by changing the parameters of interest, the eigenvalue pair is driven to the imaginary axis regardless of the trace of the other eigenvalues of the system which is initially operating at a stable operating point. Thus, after observing all the eigenvalues, we may be able to draw and define the complete feasibility boundary consisting of computed segments on a selected parameter space.

3. Impacts of Distributed Generators on Oscillatory Stability

Penetrations of distributed synchronous generators can change the oscillatory stability of the whole system, depending on how they change the structure or the topology of the interconnected system and its operating point. From a mathematical point of view, a connection of a generator to a power network results in an expansion of the degrees of the parameter and state spaces, see Eq. (1). A new operating point is established and the stability of this point should be studied through the calculation of the feasibility boundaries. Computation of the feasibility regions and their boundaries gives us important clues how the system must be operated without leading it to instability. In this work, we select the parameter sub-spaces of load reference set points [15] and speed droops of the governors to illustrate the feasibility regions. The calculation of the feasibility regions on the parameter sub-spaces of load reference set points directly shows the constraints on the real power transfers between the generator units and the other regions of the interconnected system.

The impact of distributed synchronous generators on the system stability will be studied in two different types of parallel operation with the utility power network:

i. Parallel operation without any power transfer

ii. Parallel operation with a power transfer

1. Parallel operation without any power transfer between the distributed generator and the rest of the system: Thinking of a situation in which no power transfer exists, could be considered that the operating points separately defined for the distributed generation and the rest of the system remain the same. But this does not necessarily mean that the stability of the whole system does not change, because the additional new parameters and states establish a new operating point for the interconnected system and certainly change the stability of the operating point. This implies that the interconnected system might also lose its stability after a connection of a distributed generator even if there doesn’t exist any power transfer between the generator and the rest of the system.

2. Parallel operation with a power transfer between the distributed generator and the rest of the system: The operating point can be continuously changing as it is adjusted in order to adapt to the new constraints, for example economical considerations. To assess the oscillatory stability for changing conditions, we can determine the oscillatory stability limits under different power transfer conditions through computations of feasibility boundaries calculated in parameter sub-spaces of load reference set points.
Both of the situations above, depending on the new operating point established and the control parameters selected or control actions taken, may have a degrading or a rewarding impact on the local or inter-area stability of the interconnected system. In the next section, we investigate the possible changes in the oscillatory stability of the system through calculations of feasibility regions and simulations for several case studies.

Clustering methods based on coherency and modal analysis, for example [16], [17], can be utilized for identifying the areas and thus distinguishing the local and inter-area modes. By the clustering approach [17] we use, the coherent groups of generators and the buses that are participating in different oscillatory modes can easily be determined. Hence, in a systematic way, integrating the clustering method which determines the areas that are playing role in the oscillatory stabilities associated with selected modes into the calculations of the feasibility boundary segments corresponding to those modes, we can approach the oscillatory stability problem caused by the penetration of distributed synchronous generators. As it will be observed in the case studies given in the next section, interconnection of a distributed generator can cause local oscillatory instabilities within the area to which it is connected or it can also cause inter-area oscillatory instabilities occurring between the areas of an interconnected power system. The oscillatory instabilities are related to Hopf bifurcations of electromechanical oscillatory modes. Depending on the type of the Hopf bifurcations, these instabilities can be experienced through sustained oscillations within the system where we can drive the system back to normal locally stable operating points or diverging oscillations as the system’s stability cannot be regained without disconnection of some part of the system. Both local and inter-area oscillatory instabilities caused by the connection of a distributed synchronous generator can be remedied or avoided by the local or system-wide control actions or selecting and establishing different operating points.

4. Simulation of Case Studies

As the interest in distributed generation grows, to supply the growing demand, multiple smaller generators are connected to the distribution system. We consider a system of two weakly connected areas to show the effect of penetrations of distributed synchronous generators on the local and inter-area oscillatory stability of the system. In Figure 1, a synchronous generator (generator 6) with a size quite smaller than the other generators is connected to the network with its local load. Here, the additional distributed generator (DG) can be considered as either an aggregate model for a group of multiple smaller generators that are connected to the system through a distribution system or a generator connected to the distribution system that is relatively small compared to the other generators within the transmission network.

Since electromechanical oscillations and the stability associated with these oscillations are of our main interest, the system is represented by a model that involves decoupled real-power/angle (real-power/frequency) dynamics [10]. In the models chosen for the case studies in this work, each generator is assumed to be a steam-turbine-generator modeled as in [1] where the generators are represented with a small number of state variables so that the interconnected system models will not be overly complex. Also, all the generators are considered to be connected to the system with their constant local loads. Since we study the small-signal stability problems, the linearized generator models around an operating point are used, whereas the nonlinearities come from the power-flow equations. More detailed description for the modeling and the parameters for the model are given in the Appendix.

A connection of a synchronous generator to Area 1 increases the number of local oscillatory modes of the area by one. The electromechanical modes (modes 1-4) inherent to the system before the generator is connected and the additional mode (mode 5) caused by the interconnection are listed in Table. By modal
analysis, modes 2, 3 and 5 are determined as the local modes of Area 1, and mode 4 is the local mode of Area 2, whereas mode 1, which has the lowest oscillation frequency, is distinguished as the inter-area mode.

The distributed generator can participate in some of the oscillatory modes at some extent. By coherency and modal analysis, we can distinguish the modes in which the distributed generator participates and the coherent groups of generators to which the generator will join with respect to these modes. In this case, the distributed synchronous generator belongs to Area 1 with respect to the inter-area oscillatory mode (mode 1), which has the lowest oscillation frequency. That implies, under a disturbance which excites the inter-area mode, the distributed synchronous generator will start to oscillate coherently with the other generators in Area 1 against the generators in Area 2, since a coherency within the areas exists with respect to the inter-area mode.

**Case study a - Parallel Operation without Power Transfer:** Parallel operation of a distributed generator with the utility network, even without a power exchange, may cause an oscillatory instability. To show this with an example, we compute the feasibility region and its boundary in the parameter sub-space $R_2 \times R_4$, where $R_i$ is the speed droop of the governor which belongs to the generator $i$, and we observe the location of the operating point before and after the connection. The feasibility boundary consists of Hopf bifurcation points associated with the inter-area mode and a local mode of Area 1.

As can be seen from Figure 2, the operating point OP before the connection falls in the feasible region whereas after the connection, the same operating point falls in the infeasible region as seen in Figure 3. The oscillatory instability in this case is associated with one of the local modes of Area 1. This instability mainly occurs as a result of an inconsistent selection of the control parameters for the additional distributed generator with the rest of the generators.

![Figure 1. Penetration of a distributed generator in Area 1.](image)

**Table.** Electromechanical oscillatory modes.

<table>
<thead>
<tr>
<th>Oscillatory Mode No.</th>
<th>Imaginary part of the corresponding eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±4.1i</td>
</tr>
<tr>
<td>2</td>
<td>±10.3i</td>
</tr>
<tr>
<td>3</td>
<td>±12.8i</td>
</tr>
<tr>
<td>4</td>
<td>±13.8i</td>
</tr>
<tr>
<td>5</td>
<td>±34.6i</td>
</tr>
</tbody>
</table>
Case study b- Parallel Operation with a Power Transfer: We consider the impact of a distributed generator on the oscillatory stability under the existence of a power transfer. In our case study, it is assumed that, for a period of time, the surplus generation at the location bus no. 6 will be used for the load increase in the region where normally fed by the generator 3 or vice versa.

In Figure 4, the feasibility region is depicted in the parameter sub-space \( P_{\text{ref},i} \times P_{\text{ref},6} \), where \( P_{\text{ref},i} \) is the load reference set point for the generator \( i \). The feasibility region is bounded by the Hopf bifurcation points associated with the local oscillatory modes of Area 1. The point OP corresponds to an operating point where no power exchange between generator 3 and generator 6 is present. The load reference set points are changed to increase or decrease the generations at each site. Any slow variation in the generations and thereby in the power transfer within the feasible region do not change the stability of the system. In this region, after small disturbances, the system returns to its stable operating point. If the operating point
passes through the feasibility boundary, depending on the type of the Hopf bifurcation, the system is led to an oscillatory instability. If the Hopf bifurcation is supercritical, there exist stable periodic solutions around the operating point and the solution approaches these limit cycles, thus resulting sustained oscillations. The amplitude of these oscillations depends on the closeness of the new operating point to the feasibility boundary [13]. Under these circumstances, the stability and the normal operation of the system can be regained by driving the operating point back to the feasible region. If the Hopf bifurcation is subcritical, the system loses its stability and it will be impossible to drive the operating point back to the feasible region after the region of attraction of a new properly selected operating point has been left.

As another possibility, an example of an operating condition for which the feasibility region is constrained by an inter-area oscillatory mode is given in Figure 5. A loading condition that changes and moves the operating point OP_A in the feasible region to the point OP_B in the infeasible region generates an inter-area oscillatory instability, since the crossing occurs through the feasibility boundary segment corresponding to the inter-area mode.

In our case study, the type of Hopf bifurcation during the displacement of the operating point is supercritical. This conclusion has been reached after making repeated time-domain simulations around the Hopf bifurcation point and it is observed that the amplitudes of the oscillations go to zero as the operating point approaches the Hopf bifurcation point. Therefore, sustained oscillations with a frequency close to the oscillation frequency of the inter-area mode will exist throughout the interconnected system. Since the generators within the areas are coherent with respect to the inter-area mode, these oscillations will be dominantly observed between the two areas, while they will be observed weakly within the areas. For example, sustained oscillations between the angles of the generators 1 and 4, $\delta_4 - \delta_1$, are given after crossing the feasibility boundary as seen in Figure 6. For a demonstration of how the generators within the areas oscillate coherently at the frequency of the inter-area mode against the generators of the opposing area after all the other modes are damped, time-domain simulations of $\delta_{4,1}$, $\delta_{5,1}$, and $\delta_{3,1}$, where $\delta_{i,j} = \delta_i - \delta_j$, are given in Figure 7. The generators of Area 1 (generators 4 and 5) are oscillating coherently with respect to the generator 1, which belongs to Area 1. Between the generators 1 and 3, the inter-area oscillatory mode is observed much less in magnitude since the two generators belong to the same area.
As a final remark, it should be noted that in our all case studies we have considered the possibilities of the impacts of small distributed generators on the oscillatory stability. We have not attempted to calculate the extent of the impact versus the penetration levels of the distributed generation, since this would be a research area for a specific power system. Therefore, it should not be concluded that the penetration of small distributed generation will always cause oscillatory instabilities, but instead it should be concluded that depending on the location or the closeness of the operating point to the feasibility boundary for the currently stable system, large or small penetration levels of distributed generation can cause local or inter-area oscillatory instabilities.
5. Conclusion

In this paper, the impacts of distributed synchronous generators on the oscillatory stability in interconnected power systems are analyzed. The proposed method for the analysis is involved with the calculation of feasibility regions and their boundaries that are defined by the constraints due to Hopf bifurcations of electromechanical oscillatory modes. As expected, penetrations of distributed generations change the operating point and the topology of the power system. A connection of a distributed generator may cause local or inter-area instabilities depending on many factors including the operating point intended, the location of the interconnection and the control parameters chosen. In the paper, through calculations of feasibility regions and boundaries, several case studies are given to demonstrate how the integration of the distributed synchronous generators can cause oscillatory instabilities. The case studies include parallel operation of the distributed generator with or without power transfer with the utility power network. The oscillatory instabilities can be experienced as either sustained oscillations (due to supercritical Hopf bifurcations) where the system’s stability can regained by proper control actions or as diverging unstable oscillations (due to subcritical Hopf bifurcations) where the system’s stability cannot be recovered without disconnection of some parts of the network. These results indicate that for a safe and reliable operation in agreement with the integration of distributed generators, careful assessments of the feasibility regions and their boundaries that also include the possibilities of oscillatory instabilities due to distributed generation are crucial.

Appendix

The power system example in the paper has been modelled by the differential algebraic equations (1) involving real-power-angle dynamics decoupled from voltage dynamics [10], [15]. In (1), the dynamic state vector $x$ consists of the states relevant to the generator units:

$$x = [x_1 \ x_2 \ \ldots \ x_m]^T,$$  

(5)
where $m$ is the number of generator units in the system. For each generator unit consisting of a generator, turbine, and governor, the local state vector $x_i$ is defined as $x_i = [\delta_i, \omega_i, p_{mi}, y_{vi}]$, where $\delta_i, \omega_i, p_{mi}, y_{vi}$ are the generator angle, generator’s per-unit (pu) angular speed, per-unit deviations in the mechanical input and in the valve opening regulated by the governor, respectively. The instantaneous state vector $y$ is formed by the load bus angles:

$$y = [\delta_{m+1}, \delta_{m+2}, \ldots, \delta_n]^T,$$

where $n$ is the number of buses in the system. The set of differential equations, $\dot{x} = f(x, y)$, is formed by the swing equations written for each generator and the primary control dynamics associated with turbine and governor:

$$\dot{x}_i = f_i(x, y) \quad i = 1, \ldots, m$$

where $f(.) = [f_1(.) \ldots f_m(.)]^T$. The swing equations are

$$\dot{\delta}_i = \omega_s (\omega_i - 1),$$

$$\dot{\omega}_i = \frac{1}{2H_i} (-D_i \omega_i + p_{mi} - P_{Li} - P_{ei}),$$

where $p_{mi}$, $P_{Li}$ and $P_{ei}$ (in pu) are the mechanical power input to the generator, real power flowing though the local load of the bus and the real power injected to the network at the bus, respectively and $\omega_s$ is the synchronous speed. $D_i$ is the damping coefficient and $H_i$ represents the inertia constant for the generator unit. The primary control dynamics is given by

$$\dot{p}_{mi} = \frac{1}{T_{ti}} (-p_{mi} + y_{vi}),$$

$$\dot{y}_{vi} = \frac{1}{T_{gi}} \left( P_{ref i} - \frac{1}{R_i} (\omega_i - \omega_{ref}) - y_{vi} \right),$$

where $T_{ti}$ and $T_{gi}$ are the time constants of the turbine and governor, respectively and $R_i$ is the governor speed-droop characteristic. The reference speed input to the governor is denoted by $\omega_{ref} = 1pu$ and the load reference set point is denoted by $P_{ref i}$.

The set of algebraic equations, $0 = g(x, y)$, is formed by the real power flow equations at each load bus,

$$0 = g_k(x, y) = P_{ei} + P_{L k+m} \quad k = 1, \ldots, n - m$$

where $g(.) = [g_1(.) \ldots g_{n-m}(.)]^T$. The injected real power at each bus, $P_{ei}$, is given by
\[ P_{ei} = \sum_{j=1}^{n} E_i E_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad i = 1, \ldots, n \]  

where \( E_i \) is the bus voltage and \( Y_{ij} = Y_{ij} \angle \theta_{ij} \) is the \((i,j)\)-th element of the network admittance matrix.

For the power system model (Figure 1) used in the case studies, inertias and damping coefficients for the generators are selected as follows:

- \( H_1 = 11 \text{ s} \)
- \( H_2 = 10 \text{ s} \)
- \( H_3 = 9 \text{ s} \)
- \( H_4 = 8 \text{ s} \)
- \( H_5 = 7 \text{ s} \)
- \( H_6 = 2 \text{ s} \)

- \( D_1 = 1.5 \text{ pu} \)
- \( D_2 = 1.2 \text{ pu} \)
- \( D_3 = 1.1 \text{ pu} \)
- \( D_4 = 1.1 \text{ pu} \)
- \( D_5 = 1.03 \text{ pu} \)
- \( D_6 = 1 \text{ pu} \)

**References**


