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A Novel MoM Approach for Obtaining Accurate and Efficient Solutions in Optical Rib Waveguide

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Abstract

The optical rib waveguide (ORW) plays an important role in the design of several integrated optical devices. Various methods have been proposed for obtaining the modal field solutions in ORW. However, to the best of our knowledge none of them is capable of providing accurate full-wave benchmark solutions. Here we present a novel MoM approach wherein the modes of a loaded rectangular waveguide are utilized as basis functions and demonstrate that this approach is very efficient and yields highly accurate full-wave results which can serve as benchmark solutions in assessing the quality of the data reported in the literature.

Key Words: *Method of moments, optical rib waveguide, integrated optics, rectangular waveguide, modal expansion, shielded waveguide*

1. Introduction

Analytical solutions can not be obtained for the dielectric waveguides utilised in optical integrated circuits. Lacking exact solutions one generally resorts to purely numerical techniques or to approximate modelling of these waveguides using techniques such as the application of slab guide solution successively in both cross-sectional directions [1-4], the beam propagation method [5], the variational approach [6] or equivalent circuit representations [7]. However, purely numerical techniques are rather inefficient and the applicability of the approximate methods is limited to certain simple geometries, and even in these cases the accuracy of the calculated results may rapidly deteriorate as the modal cut-off regions are approached and as the refractive index contrast between core and cladding/cap increases.

Accurate and efficient methods for computing the modes of Integrated Optical Waveguides (IOW) are therefore needed which may provide benchmark solutions for evaluating the validity of more efficient approximate methods of optical waveguide modelling.

In this paper we will introduce such an accurate and efficient full-wave method which is based on the Method of Moments (MoM). We coined our approach as MECH, which stands for Modal Expansion in Cartesian Harmonics. The specific problem addressed via MECH will be referred as the Investigated

Problem (IP). Conceptually, MECH is based on successive application of two rather straightforward steps. First, we assume that the IOW of interest is shielded by enclosing it into a rectangular domain Ω with conducting walls placed at some distance from the guiding regions. We then define a suitable Reference Problem (RP) in Ω and apply Galerkin version of MoM by using the modes of the RP in representing the sought-after solutions of the IOW.

As the generic RP we consider a rectangular waveguide with dielectric stratification in one of its cross-sectional directions. Although this RP is quite flexible and can be used for computing the modal fields in a wide class of IOW's, in the present paper we will restrict our attention solely to the problem of determining the propagation characteristics of bound modes in a Shielded Optical Rib Waveguide (SORW), i.e., assume the IP to be the SORW depicted in Figure 1b. For this problem a suitable choice of RP is the rectangular waveguide loaded as shown in Figure 1a, where the refractive indices of the various layers are yet to be specified.

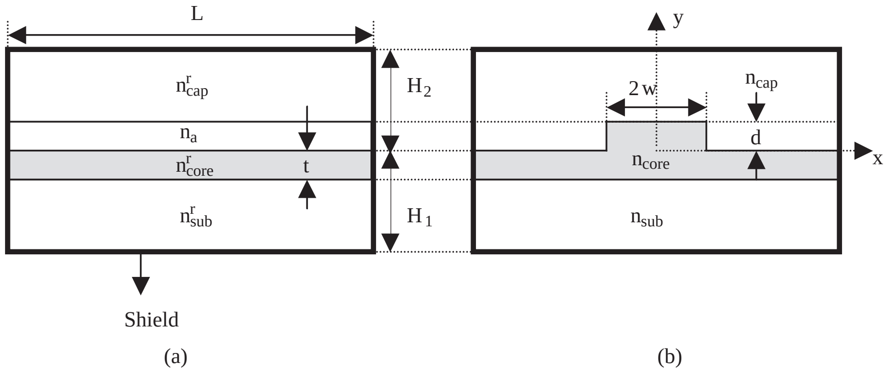

Figure 1. (a) The reference problem: inhomogeneously filled rectangular waveguide, (b) The investigated problem: Shielded Optical Rib Waveguide

The MoM approach we employ in converting linear partial differential equations into matrix eigenvalue equations is a standard technique. Several investigators have applied this technique for calculating the modal fields in various types of IOW's. As early as 1952, in his classic paper Schelkunoff [8] showed that by taking an empty waveguide as the RP one could reduce the problem of determining the fields in an IP into a matrix eigenvalue equation when the latter can be defined by the same waveguide, but filled with heterogeneous, non-isotropic media. 25 years later, in 1977, Ogusu [9] applied Schelkunoff's formulation for generating numerical solutions in IOW's. With the exception of some approximations in imposing the boundary conditions dictated by the IOW's, Ogusu's treatise is a full-wave approach which is based on the modes of an empty rectangular waveguide. Although the convergence properties of his formulation were rather poor, he was nevertheless able to correctly recover dominant features of modal fields in certain IOW's. More recently, a number of researchers followed Schelkunoff's original idea but, in order to improve the computational efficiency, utilised approximate boundary conditions by defining as RP an empty fictitious rectangular waveguide wherein **all** field components vanish on the boundaries and applied two dimensional Fourier analysis either to scalar [10,11] or to vectorial [12] wave equation.

The problem of determining the modal fields of IOW's remains to be an active area of research, however, to the best of our knowledge no full-wave method has yet appeared which is capable of providing "sufficiently accurate" results that can serve as "benchmark" solutions for evaluating the validity of more efficient approximate techniques. In this context, we will define the phrase "sufficiently accurate" as a minimum of five-digit accuracy in the modal eigenvalues of the IOW of interest. Although, this choice is necessarily somewhat arbitrary, we believe that this criterion is acceptable, since it can both serve as a benchmark in evaluating the results reported in the literature and can also meet even the most stringent requirements encountered in the design and optimisation of integrated optical devices. Our approach, which again derives from Schelkunoff's treatise, can serve for generating benchmark solutions for the modal fields in a fairly large class of IOW's. The main difference between our MECH approach and similar techniques reported in the literature is that we use a loaded rather than an empty rectangular waveguide as the RP. Although this choice leads to a somewhat less efficient representation for the RP, the efficiency gain in calculating the modes of the IP is generally much greater, particularly when the RP can be chosen "close" to the IP. As a result of this efficiency increase one can implement MECH on conventional PC's and compute modal solutions of certain IOW's within reasonable computer time, without having to recourse to any approximations.

In the remainder of this paper we will focus our attention to the specific problem of determining the modal fields in a SORW. The organisation of this paper is as follows: Theoretical framework of our approach is described in Section 2. In Section 3 we discuss some numerical experiments that we have performed for validation and optimisation of MECH using a simple canonical problem whose exact solutions are well known. Section 4 contains examples of our numerical solutions for the SORW, and Section 5 some concluding remarks.

2. Formulation

2.1. Modes of the reference problem

The reference problem (RP) considered in this work is a rectangular waveguide loaded with lossless, homogeneous dielectric strata along the cross-sectional direction y as depicted in Figure 1a. The modal fields of the RP can be decomposed into TM^y and TE^y type modes, and can easily be obtained via standard techniques [13]. We shall denote the transverse field components of the i-th TM^y mode with a prime ($\prime$) and write,

$$\vec{E}'_{ti}(x,y,z) = e^{-j\chi'_i z}\vec{e}_i{}'(x,y) \quad : \quad \vec{H}'_{ti}(x,y,z) = e^{-j\chi'_i z}\vec{h}'_i(x,y) \tag{1}$$

A double prime ($''$) will be used for the corresponding quantities of TE^y modes. When the propagating modes are normalised for unit power, the bi-orthonormality relations satisfied by these modes can be expressed as:

$$\int_S \vec{e}_i{}' \times \vec{h}_j{}'.\vec{u}_z dS = \int_S \vec{e}_i{}'' \times \vec{h}_j{}''.\vec{u}_z dS = \delta_{i,j} = \begin{cases} 1 & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases}$$

$$\int_S \vec{e}_i{}' \times \vec{h}_j{}''.\vec{u}_z dS = \int_S \vec{e}_i{}'' \times \vec{h}_j{}'.\vec{u}_z dS = 0 \tag{2}$$

where $\vec{u}_z$ denotes the unit vector along the axial direction z, and the integrations are performed over the cross-sectional area S of the rectangular waveguide.

2.2. Modes of the investigated problem

Assuming exp(-jβz) dependence, Maxwell's equations for the transverse modal fields $\vec{E}_t(x,y), \vec{H}_t(x,y)$ can be written as [14]:

$$\begin{aligned} \tfrac{\beta}{\omega\mu_o}\vec{E}_t &= \left[\bar{\bar{I}} + \tfrac{1}{k_o^2}\nabla_t \tfrac{1}{n^2}\nabla_t\right].\vec{H}_t \times \vec{u}_z \\ \tfrac{\beta}{\omega\varepsilon_o}\vec{H}_t &= \left[n^2\bar{\bar{I}} + \tfrac{1}{k_o^2}\nabla_t\nabla_t\right].\vec{u}_z \times \vec{E}_t \end{aligned} \tag{3}$$

where ∇_t denotes the transverse del operator, $\bar{\bar{I}}$ the unit dyad, and k$_o$ the free-space wavenumber. The axial field components are obtained via,

$$E_z = \frac{1}{j\omega\varepsilon_o n^2}\nabla_t.(\vec{H}_t \times \vec{u}_z)\ ; \quad H_z = \frac{1}{j\omega\mu_o}\nabla_t.(\vec{u}_z \times \vec{E}_t) \tag{4}$$

It should be noted that the IP is defined by the piecewise constant refractive index variation n(x,y) depicted in Figure 1b, and equations (3) apply for all x,y $\in$ S in the sense of distributions.

We now represent the transverse fields of the IP in terms of the modes of the RP as,

$$\vec{E}_t = V'_m\vec{e}_m{}' + V''_n\vec{e}_n{}'\ ; \quad \vec{H}_t = I'_m\vec{h}_m{}' + I''_n\vec{h}_n{}'' \tag{5}$$

where V and I stand for the (unknown) expansion coefficients, and summation over repeated indices is implied. Using these representations in (3) and noting that the equations satisfied by $\vec{e}$ and $\vec{h}$ are obtained by replacing β and n in (3) with χ and n$_r$, respectively, one obtains,

$$\begin{aligned} &(\beta V_m{}' - \chi'_m I'_m)\,\vec{e}_m{}' + (\beta V''_n - \chi''_n I''_n)\,\vec{e}_n{}'' = (\omega\varepsilon_o)^{-1}\nabla_t\left[\frac{-(\Delta n^2)}{n^2 n_r^2}\nabla_t\cdot(I'_m h'_m + I''_n\vec{h}_n{}'')\times\vec{u}_z\right] \\ &(\beta I'_m - \chi'_m V'_m)\,\vec{h}_m{}' + (\beta I''_n - \chi''_n V''_n)\,\vec{h}_n{}'' = \omega\varepsilon_o(\Delta n^2)\vec{u}_z \times (V'_m\vec{e}'_m + V''_n\vec{e}_n{}'') \end{aligned} \tag{6}$$

where (Δn^2) can be considered as a perturbation term which is defined via the refractive index variations representing the IP (n) and the RP (n$_r$), as

$$(\Delta n^2) = n^2 - n_r^2 \tag{7}$$

Following the standard Galerkin procedure we now dot multiply the first and second equations in (6), respectively, first with $\vec{h}_\nu{}' \times \vec{u}_z$ and $\vec{u}_z \times \vec{e}_\nu{}'$ (ν = 1,2, ...), and then with $\vec{h}_\nu{}'' \times \vec{u}_z$ and $\vec{u}_z \times \vec{e}_\nu{}''$ and integrate the resulting equations over S. Making use of (2) – (4) one then obtains the following matrix eigenvalue equation for the modal propagation factors β of the IP:

$$\beta^2\mathbf{V} = (\chi^2 + \mathbf{Q})\mathbf{V} \tag{8}$$

The derivation of (8) is outlined in Annex A. We will denote the number of TM^y and TE^y type modes of RP used in representing the fields of the IP in (5) by M and N, respectively, i.e., m =1,2,..M and n = 1,2 ... N, where m and n stand for the double indices which specify a particular mode of the RP. It should be noted that all terms in (8) are square matrices of order M+N, and that β and χ are diagonal matrices whose elements are the modal propagation constants of the IP and the RP, respectively. The i-th column of $\mathbf{V}$ yields the eigenvector for $\vec{E}_t(i)$ in (3) and the elements of $\mathbf{Q}$ represent the coupling induced between the modes of the RP due to the differences between RP and IP.

It is well known that the convergence properties of representations in the form of (5) are rather poor, especially when the fields of the IP undergo rapid changes and the modes of the RP fail to mimic them. It was this adverse convergence property of the Galerkin approach that has hindered earlier investigators [9-12] from obtaining accurate full-wave solutions for the modal fields of the SORW even with supercomputers.

In what follows we will demonstrate that by suitably modifying the RP one can greatly improve the convergence properties of the Galerkin technique. By choosing the RP for the SORW as in Figure 1a one can ensure that the modes of the RP represent the y dependence of the modal fields of the rib waveguide fairly well. Although the same argument does not hold in the x direction one would nevertheless expect to achieve substantial improvement in the convergence properties of the representation given in (5). Moreover, the freedom in choosing the refractive index n_a of the perturbed layer (see Figure 1) can be used at advantage to further improve the efficiency of the representation. To better illustrate these arguments it would be appropriate to consider first the canonical problem obtained by letting the height of the rib designated by d in Figure 1b tend to zero.

3. A Canonical Problem

When the rib height is made zero the IP becomes identical with one of the possible choices for RP. We would like to briefly investigate this canonical case in order to demonstrate some salient features of our approach. For this purpose let us consider the IP obtained by letting d $\rightarrow$ 0 in Figure 1b and define an RP by letting $n^r_{sub} = n_{sub}$, $n^r_{core} = n_{core}$, $n^r_{cap} = n_{cap}$ and $n_a = n_{core}$ in Figure 1a. With this choice (Δn^2) in (6) becomes

$$\Delta n^2 = \begin{cases} n^2_{cap} - n^2_{core} & \text{for } y \in 0, d \\ 0 & \text{elsewhere} \end{cases} \tag{9a}$$

Moreover, in this case the IP supports TE^y, TM^y type modes and, as mentioned in Annex A, the eigenvalue problem defined in (8) now simplifies into a trivial one. Our numerical tests utilizing this canonical structure have revealed that our approach has indeed the potential of substantially improving the convergence properties of the MoM when applied to IOW problems, particularly when, due to computational limitations, the number of expansion functions (modes of RP) used in the representation can not be increased at will. Figure 2 illustrates this point, where the error made in calculating an eigenvalue of the IP is plotted versus the number of modes used in the MoM calculations, both for the RP defined above(loaded guide) and also for an alternative choice of the RP as the empty guide. This alternative RP, which has been used in the literature for calculating modal fields of IOW, is obtained by choosing $n^r_{core} = n^r_{sub} = n_a = n^r_{cap} = n_{cap}$ in Figure 1a. With this choice (Δn^2) in (6) becomes:

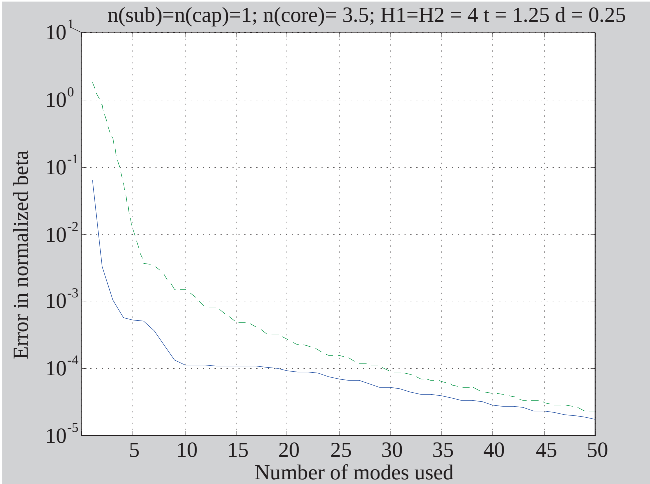

Figure 2. Error in MoM calculations vs. number of modes used in the representation; (Solid curve: Loaded guide described in the text, Dashed curve: Empty guide.)

$$\Delta n^2 = \begin{cases} n_{core}^2 - n_{cap}^2 & \text{for } y \in -t, 0 \\ n_{sub}^2 - n_{cap}^2 & \text{for } y \in -H_1, -t \\ 0 & \text{elsewhere} \end{cases} \tag{9b}$$

It can be seen from Figure 2 that the advantage of utilizing a loaded guide as the RP diminishes as the number of modes used in the MoM representation is increased. The reason for this becomes evident when one observes that although the fields of the bound modes of the RP mimic the y-dependence of the modes of the IP rather closely, higher order (non-bound) modes of the RP extend beyond the perturbation region, in much the same way as the modes of an empty waveguide, as depicted in Figures 3a and 3b.

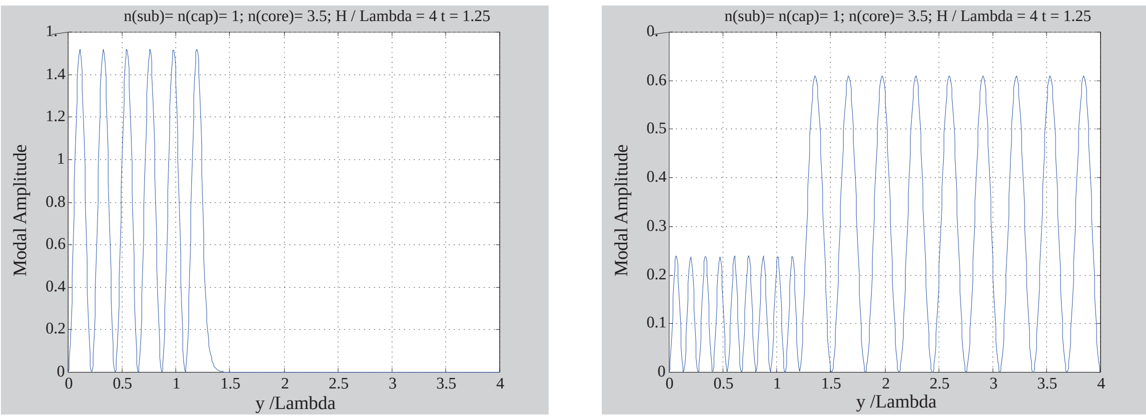

Figure 3. Variation of modal fields, (a) a bound mode, (b) a non-bound mode

Another point that can be demonstrated via this canonical problem concerns the difficulties in representing the discontinuity of the normal electric field components at core-substrate and core-cap transitions. Our calculations show that in order to reproduce this discontinuous transitions one may have to increase the number of modes used in the representation by as much as an order of magnitude beyond its value which yields an accurate estimate for the modal eigenvalue. This is the reason why the field plots for the SORW given in the next section fail to exhibit these discontinuity effects.

4. The Shielded Optical Rib Waveguide (SORW)

In representing the modal fields of the SORW we will utilize the RP depicted in Figure 1a by specifying the (Δn^2) in (6) as

$$\Delta n^2 = \begin{cases} n_{cap}^2 - n_{core}^2 & \text{for } y \in 0, d, and |x| \in w, L/2 \\ 0 & \text{elsewhere} \end{cases} \tag{10}$$

Using this RP we computed the modal refractive indices defined as $n_e = \beta / k_o$ for three specific rib waveguide structures defined in Table 1, which were previously investigated in the literature [6, 11].

In each case the modal indices were computed by MECH to a 5 digit accuracy. Using an equal mix of TE^y and TM^y modes, this required, depending on the mode of interest, the utilization of a total of 1000 to 3000 modes in the representation given in (5). For a single point the calculations took approximately one minute CPU time on a standard PC (Pentium III, 600 MHz) platform. This corresponds to a very substantial increase in the computational efficiency, since it is not possible to achieve comparable accuracy within acceptable CPU times when using the conventional representation which is based on the modes of the empty waveguide, even with supercomputers [11].

Table 1. Parameters of the three Rib Guides used in the computations

Guide	n_{cap}	n_{core}	n_{sub}	d(μm)	t(μm)	w(μm)	$\lambda(\mu m)$
1	1	3.44	3.34	1.1	0.2	1	1.55
2	1	3.44	3.36	0.1	0.9	1.5	1.55
3	1	3.43	3.37	0.5	0.8	2.5	0.85

The computed results expressed in terms of the normalized waveguide parameter b are given in Table 2, where

$$b = \frac{n_e^2 - n_{sub}^2}{n_{core}^2 - n_{sub}^2} \tag{11}$$

Table 2. Comparison of computed results with those reported in the literature

Guide 1			Guide 2			Guide 3	
Method	b_x	b_y	Method	b_x	b_y	Method	b_x
FD	0.4656	0.4638	FD	0.4346	0.3792	VM	0.80
VP	0.4804	0.4729	VP	0.4401	0.3925		
MECH	0.5099	0.4674	MECH	0.4338	0.3670	MECH	0.8019

In Table 2, b_x and b_y correspond, respectively, to quasi-x and quasi-y polarized modes. FD and VP stand, respectively, for semi-vectorial Finite Difference and Variational Perturbation techniques reported in [6] and VM stands for the TE^y mode expansion reported in [11]. As noted above, in our computations using MECH we have taken care to include appropriate number of modes of the RP in (5) so as to ensure a minimum of 5 digit accuracy in the calculated modal eigenvalue n_e. It should be noted that for the parameter values considered in this paper this corresponds to approximately 4 digit accuracy in the calculated b values listed in Table 2. We therefore believe that our values in Table 2 can safely be accepted as benchmark results which can be used in assessing the relative success of other approaches. It should be noted that the values given in Table 2 indicate that most of the previously reported results contain significant errors and the accuracies obtained with these methods would be unacceptable for a number of applications of practical significance.

Moreover, the computational efficiency gained when utilizing MECH can be used at advantage in investigating relevant properties of the modes of the SORW. An example is provided in Figure 4, which depicts the dispersion characteristics of the first two modes in Guide 3.

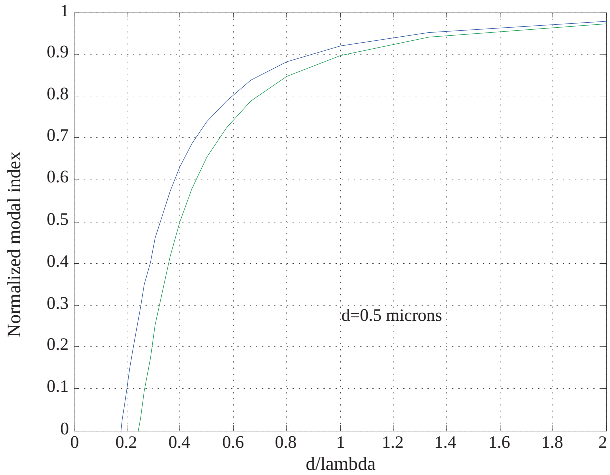

Figure 4. Dispersion characteristics of the first two modes in Guide 3.

On the other hand, being a full-wave solution MECH can also be used to investigate the cut-off transitions of the bound modes of the SORW, as illustrated in Figure 5. In this figure the field amplitude of first mode of Guide 3 which undergoes cut-off at approximately $d/\lambda = 0.15$ (see Figure 4) is plotted for two d/λ values chosen on both sides of the cut-off condition.

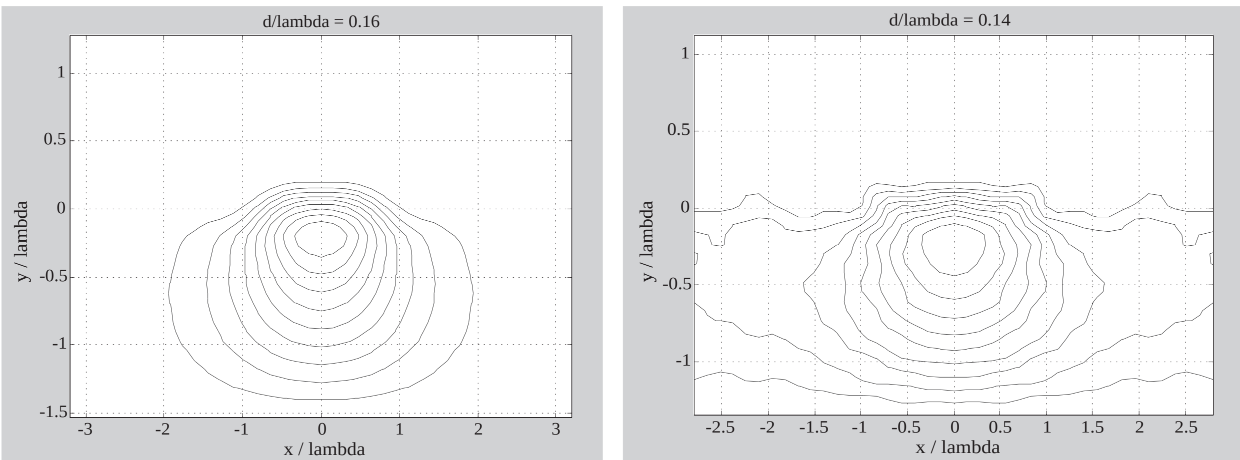

Figure 5. Contour plot depicting the cut-off transition of the first mode in Guide 3.

At this point it should be noted that at present we lack a simple criteria which could be used for automatic identification of the bound modes of SORW. If we consider Guide 3 at $\lambda = 0.85\,\mu$m, as an example, and following [11],use the values of L = 20 μm, $H_1 + H_2 = 8.8$ μm (see Figure 1) in the computations we have to identify the eigenvalues corresponding to a total of 10 bound modes which are supported by the structure, among the 40 eigenvalues obtained from (8) to have positive b values as defined in (11) and, hence are to be considered as candidates of bound modes. The modes which do not correspond to bound "rib modes" are, of course, not computational artifacts but legitimate solutions of the shielded structure, which, however, are of little or no interest to the designer of 1O devices. In the present work we have used a somewhat lengthy but safe approach of plotting the fields corresponding to all solutions of (8) which render positive b values. These plots can be used to identify and eliminate the "non-bound" modes since they yield field distributions which are not confined to the rib region. Two such plots, one depicting two bound modes and the other a non-bound mode are shown in Figures 6a and 6b.

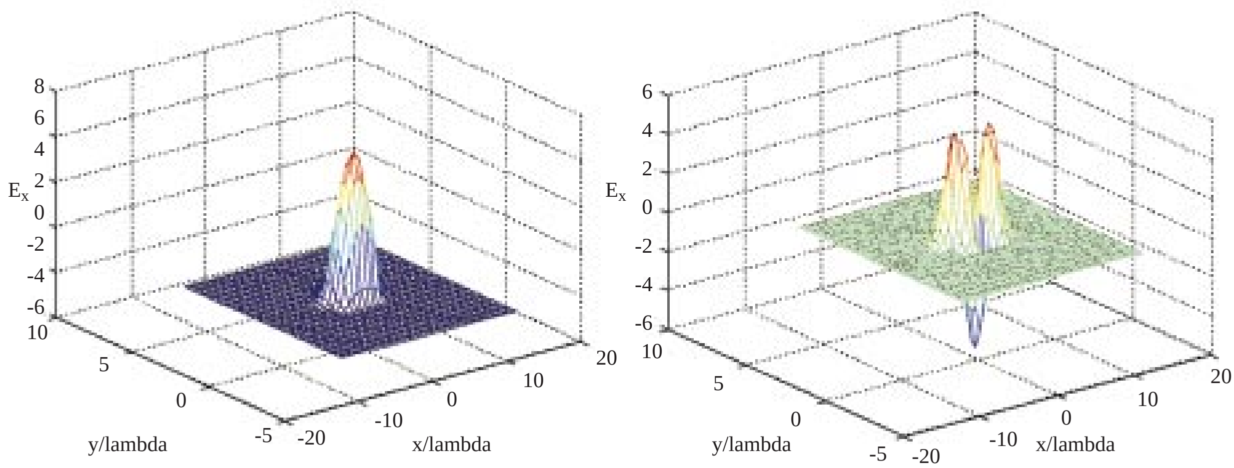

Figure 6a. Typical variation of the field amplitudes for bound modes

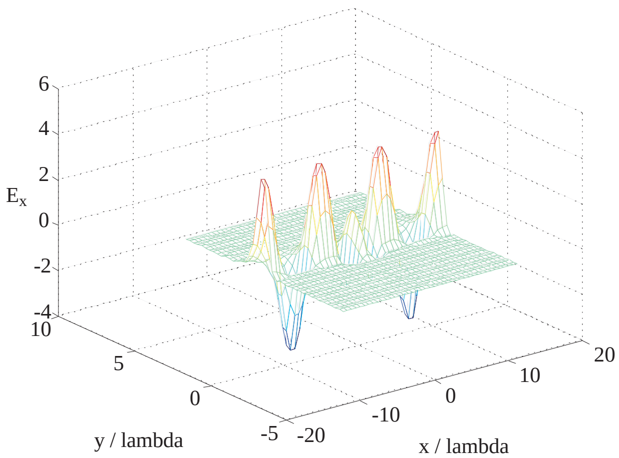

Figure 6b. Typical variation of the field amplitudes for non- bound modes with positive b values

As is evident from Figure 6 bound modes are characterized by the confinement of the EM field in the vicinity of the rib, whereas the non-guided modes with positive b values exhibit the typical behavior of slab modes along the y direction and that of rectangular waveguide modes in the x direction. Thus, once the variation of the modal fields of the shielded structure corresponding to positive b values are plotted one can uniquely identify the supported rib modes.

5. Conclusions

We have presented a MoM approach for the calculation of the modal fields in the SORW. Our approach, which was nick-named as MECH, differs from those reported in the literature in the choice of the basis functions used. We have demonstrated that this choice yields accurate full-wave solutions which can serve as benchmark results for assessing the validity of the various techniques reported in the literature. Our approach results in substantial improvement in the computational efficiency when compared to other full-wave techniques reported in the literature and can run on a standard PC.

Our approach is quite flexible and can also be used to calculate the modal fields of various other dielectric waveguides which conform to a cartesian frame of reference.

Annex A. The Matrix Eigenvalue Equation

In this Annex we will outline the derivation of the matrix eigenvalue equation (8) in the text. Dot multiplying the first equation in (6) with $\vec{h}_\nu{}' \times \vec{u}_z$ and the second equation with $\vec{u}_z \times \vec{e}_\nu{}'$ for $\nu = 1,2,..M$ and integrating over S one obtains

$$\begin{aligned} \beta\, \mathbf{V}' &= (\boldsymbol{\chi}' + \mathbf{A_1})\mathbf{I}' + \mathbf{C_1}\mathbf{I}'' \\ \beta\, \mathbf{I}' &= (\boldsymbol{\chi}' + \mathbf{A_2})\mathbf{V}' + \mathbf{C_2}\ \mathbf{V}'' \end{aligned} \tag{A-1}$$

Similar expressions are obtained by dot multiplying the first equation in (6) with $\vec{h}_\nu{}'' \times \vec{u}_z$ and the second equation with $\vec{u}_z \times \vec{e}_\nu{}''$ for $\nu = 1,2,..N$ and integrating over S,

$$\begin{aligned} \beta\, \mathbf{V}'' &= (\boldsymbol{\chi}'' + \mathbf{B_1})\mathbf{I}'' + \mathbf{D_1}\mathbf{I}' \\ \beta\, \mathbf{I}'' &= (\boldsymbol{\chi}'' + \mathbf{B_2})\mathbf{V}'' + \mathbf{D_2}\ \mathbf{V}' \end{aligned} \tag{A-2}$$

In these equations $\boldsymbol{\chi}'$ and $\boldsymbol{\chi}''$ are diagonal matrices of TM and TE mode propagation factors, respectively. The (ν,μ) th element of the matrices in (A-1) are obtained as:

$$A_1(\nu,\mu) = -(\omega\varepsilon_o)^{-1}\int_S \left[\nabla_t \frac{(\Delta n^2)}{n^2 n_r^2}\nabla_t \cdot (\vec{h}_\mu'' \times \vec{u}_z)\right] \cdot (\vec{h}_\nu' \times \vec{u}_z)dS \tag{A-3a}$$

$$A_2(\nu,\mu) = \omega\varepsilon_o \int_S (\Delta n^2)\vec{e}_\mu{}' \cdot \vec{e}_\nu{}')dS \tag{A-3b}$$

$$C_1(\nu,\mu) = -(\omega\varepsilon_o)^{-1}\int_S \left[\nabla_t \frac{(\Delta n^2)}{n^2 n_r^2}\nabla_t \cdot (\vec{h}_\mu{}'' \times \vec{u}_z)\right] \cdot (\vec{h}_\nu{}' \times \vec{u}_z)dS \tag{A-3c}$$

$$C_2(\nu,\mu) = \omega\varepsilon_o \int_S ((\Delta n^2)\vec{e}_\mu{}'' \cdot \vec{e}_\nu{}')dS \tag{A-3d}$$

The defining equations for the elements of the matrices in (A-2) are quite similar: $\mathbf{B}_1$ and $\mathbf{D_1}$ are obtained by replacing $\vec{h}_\nu{}'$ in respectively, (A-3c) and (A-3a) with $\vec{h}_\nu{}''$, and $\mathbf{D}_2$ and $\mathbf{B}_2$ are recovered when $\vec{e}_\nu{}'$ in, respectively, (A-3b) and (A-3d) is replaced with $\vec{e}_\nu{}''$. It should be noted that $\mathbf{A}_2$, χ' and $\mathbf{B}_1$, $\mathbf{B}_2$, χ'' are square matrices of order M$\times$M and N$\times$N, respectively; whereas the matrices $\mathbf{C}_1$, $\mathbf{C}_2$ are of order (M$\times$N) and $\mathbf{D}_2$ is of order (N$\times$M).

Eliminating $\mathbf{I}'$ and $\mathbf{I}''$ from (A-1) and (A-2) one obtains equation (8) in the text where the matrix $\mathbf{Q}$ is defined as,

$$\mathbf{Q} = \begin{pmatrix} \boldsymbol{\chi}'\mathbf{A}_1 + \boldsymbol{\chi}'\mathbf{A}_2 + \mathbf{A}_1\mathbf{A}_2 + \mathbf{C_1D_2} & \mathbf{C}_1\boldsymbol{\chi}'' + \mathbf{C}_1\mathbf{B}_2 + \boldsymbol{\chi}'\mathbf{C}_2 + \boldsymbol{\chi}'\mathbf{A}_1 \\ \boldsymbol{\chi}''\mathbf{D}_2 + \mathbf{B}_1\mathbf{D}_2 + \boldsymbol{\chi}'\mathbf{D}_1 + \mathbf{D}_1\mathbf{A}_{2+} & \mathbf{B}_1\boldsymbol{\chi}'' + \boldsymbol{\chi}''\mathbf{B}_2 + \mathbf{B}_1\mathbf{B}_{2+}\mathbf{D}_1\mathbf{C}_{2+} \end{pmatrix} \tag{A-4}$$

When only TM^y or TE^y modes of the RP are used in representing the modal fields of the IP then (A-4) reduces to:

$$\mathbf{Q} = \begin{cases} \boldsymbol{\chi}'\mathbf{A}_1 + \mathbf{A}_1\mathbf{A}_2 + \boldsymbol{\chi}'\mathbf{A}_2 & \textbf{for TM}^y\textbf{ modes} \\ \boldsymbol{\chi}''\mathbf{B}_2 + \mathbf{B}_1\boldsymbol{\chi}'' + \mathbf{B}_1\mathbf{B}_2 & \textbf{for TE}^y\textbf{ modes} \end{cases} \tag{A-5}$$

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