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Distance Spectra for Trellis Coded Modulation Schemes on Channels with Intersymbol Interference

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Abstract

The effect of intersymbol interference on distance spectra for trellis coded 8-PSK, 16-QAM and 32-AMPM modulation schemes is evaluated using the methods proposed by Schlegel. Distance spectra of 16-state 8-PSK scheme are computed for different intersymbol interference channels. It is seen that on channels with intersymbol interference the spectral lines are spread into a nearly continuous spectrum and the minimum Euclidean distance between codewords decreases severely. Hence, although the main contribution at large signal to noise ratios comes from the minimum Euclidean distance d_{free} *, higher spectral components also become very important at moderate values of the signal to noise ratio.*

Key Words: *Trellis Coded Modulation, Distance Spectrum, Intersymbol Interference.*

1. Introduction

In classical digital communication systems, error control is provided by transmitting additional redundant bits in the code, which has the effect of decreasing the information bit rate per channel bandwidth. Bandwidth efficient coding means that the use of the code saves transmitter power without increasing the signal bandwidth. This is achieved by combined coding and modulation. Trellis codes for band limited channels result from the treatment of modulation and coding as a combined entity rather than as two separate operations, and the corresponding combination is referred to as Trellis Coded Modulation (TCM). In the presence of Additive White Gaussian Noise (AWGN), maximum likelihood decoding of trellis codes consists of finding that particular path through the trellis with Minimum Squared Euclidean Distance (MSED) to the received sequence. Thus, in the design of trellis codes, the emphasis is on maximizing the Euclidean distance between transmitted symbol codewords, rather than maximizing the Hamming distance of an error correcting code.

TCM was first proposed by Ungerboeck in 1982 [1]. He proved that coding power gains at no bandwidth expense is possible. He designed trellis codes for Phase Shift Keying (PSK) and for Quadrature Amplitude Modulation (QAM).

The error probability is an important measure that determines the performance of a TCM code. Exact calculation of the error probability is not feasible even for simple trellis codes; however, TCM error probabilities can be estimated using simulations and performance bounds. Simulations often require long running times and are only useful for short constraint length codes and low Signal to Noise Ratios (SNR). Performance bounds are the most common means of predicting the error probability of codes at moderate to high SNR and of designing new coding schemes.

The set of squared Euclidean distances "d" together with corresponding multiplicities "N_d" is defined as the Distance Spectrum (DS) of the related code. An upper bound to the error probability can be obtained using DS. At high SNRs, the performance of the code is dominated by the first spectral term $d_1 = d_{\text{free}}$. However, for moderate values of the SNR, higher spectral lines of the code also become important, especially for trellis codes whose DS are relatively dense; and therefore, codes with the best free distance may not perform the best. The computation of the DS necessitates the existence of efficient algorithms. In general, the computational complexity of the algorithms depends on the degree of symmetry of the TCM schemes, which are classified according to the extent of symmetry they possess.

Rouanne and Costello [2] defined Quasi Regular Codes (QRC) and developed an algorithm for the DS computation of these codes. Rajab improved the Rouanne and Costello Algorithm (RCA) for TCM schemes having uncoded bits which cause parallel transitions in the trellis diagram [3, 4].

The RCA was originally developed to compute the DS of quasi regular codes on channels without Inter Symbol Interference (ISI). Schlegel makes an extended redefinition on quasi regularity. By his approach, it possible to evaluate the DS on channels with ISI by using the RCA [5].

In this paper, distance spectra (DS) of the 16-state 8-PSK TCM code on channels with various amounts of intersymbol interference (ISI) are computed and the effects of ISI on the DS are evaluated by applying the extended quasi regularity concept of Schlegel [5]. The organization is as follows: in Section 2, the design procedure of two dimensional TCM schemes given by Ungerboeck is summarized. How the TCM signal sequences are affected by the ISI on transmission channels is described in Section 3. The percentage contributions to the Error Event Probability (EEP) of the first five spectral lines of 16-state 8-PSK, 16-state 16 QAM, and 8-state 32 AMPM codes are presented in Section 4. The DS of the 16-state 8-PSK TCM code on channels with various amounts of ISI are presented and compared to the DS with no ISI in Section 5.

2. TCM Communications System

A rate $k/k+1$ trellis code can be generated by a binary convolutional encoder, generating a linear trellis, followed by a mapper as depicted in Figure 1. The convolutional encoder is a finite state automaton with 2^v possible states where v is the memory order of the encoder. At each signaling interval r, the encoder accepts k information bits $u_r = (u_r^1, \ldots, u_r^k)$ from an input sequence u.

The coding operation is performed using $\tilde{k}$ information bits $u_r^1, \ldots, u_k^{\tilde{k}}$, which are also denoted as the bits checked by the encoder. The remaining $k - \tilde{k}$ bits are called uncoded bits. Upon receiving u_r, the encoder makes a transition from state S_r to one of the $2^{\tilde{k}}$ possible successor states S_{r+1}, and outputs $k+1$ bits $v_r = (v_r^1)$, $\ldots, v_r^{k+1}$. The vector v_r which contains all the bits at the mapper input is called the signal selector. The $\tilde{k}+1$ bit-portion $v_r^1, \ldots, v_r^{\tilde{k}+1}$ of the signal selector v^r, which does not contain the uncoded bits, is called the subset selector. This subset selector is used to select one of the $^{\tilde{k}+1}$ subsets of a 2^{k+1}-ary signal set. The remaining $k - \tilde{k}$ uncoded information bits $u_r^{\tilde{k}+1}, \ldots, u_r^k$, called the point selector, determine which of the $2^{k-\tilde{k}}$ signals in the selected subset will be transmitted.

The convolutional encoder can be realized in systematic feedback form as shown in Figure 1. The encoder has v delay elements and $v-1$ modulo-2 adders where v is the memory order of the encoder. S, the state of the encoder, can be represented by a binary sequence $S^1, \ldots, S^v$. Since the content of the i^{th} delay element determines the bit S^i, the i^{th} delay element is labeled with S^i in Figure 1.

The encoder can be described by specifying the $\tilde{k}+1$ binary vectors h^i, $i=0,\ldots,\tilde{k}$ each containing $v+1$ bits. These vectors (usually given in octal form) completely specify the connections in the linear convolutional encoder. The intermediate elements of the vector hi corresponding to bits h^i_j, $i=0,\ldots,\tilde{k}, j = 1,\ldots,v-1$ determine whether the output bit v_r^{i+1} is connected to the modulo-2 adder between the j^{th} and $(j+1)^{th}$ delay elements. The additional bits h^i_0, $i=0,\ldots,\tilde{k}$ determine the connection between the output bit v_r^{i+1} and the output of the 1^{st} delay element, and the bit h^i_v, $i=0,\ldots,\tilde{k}$ determine the connection between the output bit v_r^{i+1} and the input of the v^{th} delay element. The bits h^0_0 and h^0_v are always 1, and h^i_0, h^i_v, $i=1,\ldots,\tilde{k}$ are always 0.

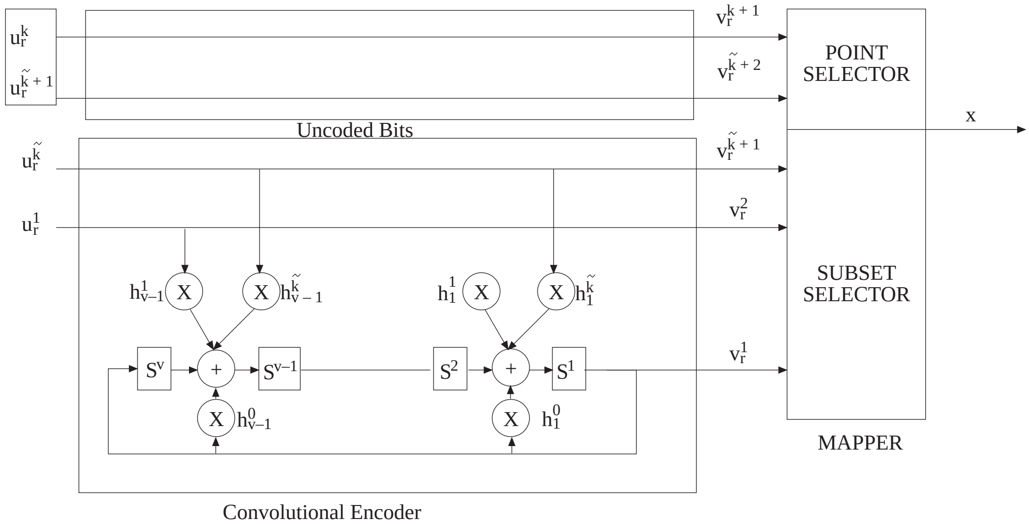

Figure 1. Trellis Coded Modulator

The signal selector v_r at the encoder output is fed into the mapper. The mapper selects the signal, x, from the set of 2^{k+1} signals forming the modulator alphabet. x is then transmitted through the channel. Realization of a trellis coded modulator by the structure given in Figure 1 was also introduced by Ungerboeck [1].

3. Channels with Intersymbol Interference

The signal x, generated by the trellis coded modulator as described in Section 2, is multiplied by the pulse shaping function $g(t)$ before transmission over the channel (Figure 2). If Pulse Amplitude Modulation (PAM) is used, x is real valued. If Pulse Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) is used, x is complex valued, since the signals have two dimensional representation.

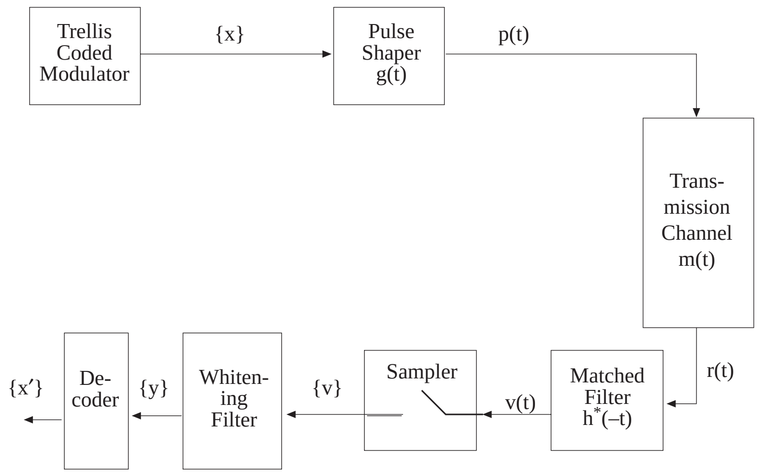

Figure 2. A Digital Communication System Using TCM

For a message sequence of length $L+1$, the sequence $\{x\} = (x_0, x_1, \ldots, x_{L-1}, x_L)$ transmitted over the channel can be represented by

$$p(t) = \sum_{r=0}^{L} x_r g(t - rT) \tag{1}$$

When $p(t)$ is transmitted over an AWGN channel whose impulse response is m(t), the received signal can be represented by

$$r(t) = \sum_{r=-L}^{L} x_r h(t - rT) + z(t) \tag{2}$$

where

$$h(t) = \int_{-\infty}^{\infty} g(\tau) m(t - \tau) d\tau \tag{3}$$

and $z(t)$ represents the AWGN of the channel.

The received signal is passed through a matched filter, and then sampled at a rate $1/T$ samples/s. The impulse response of the matched filter at time $T = 0$ is $h^*(-t)$. The output of the receiving filter is denoted as

$$v(t) = \sum_{r=0}^{L} c(t - rT) + n(t) \tag{4}$$

where $c(t)$ is the pulse representing the response of the receiving filter to the input pulse $h(t)$, i.e.,

$$c(t) = h(t) * h^*(-t) = \int_{-\infty}^{\infty} h(\tau) h^*(\tau - t) d\tau \tag{5}$$

and $n(t)$ is the response of the matched filter to the noise $z(t)$. The noise, $z(t)$, at the input of the matched filter is white noise; however, after passing through the matched filter, the noise $n(t)$ does not possess the properties of white noise.

If $v(t)$ is sampled at time $t = kT,\ k = 0, 1, 2...$, then

$$v(kT) \;=\; v_k = \sum_{r=0}^{L} x_r c(kT - rT) + n(kT)$$

$$v_k \;=\; \sum_{r=0}^{L} x_r c_{k-n} + n_k \tag{6}$$

The sample values can be expressed as $v_k = c_0(x_k + \frac{1}{c_0}\sum_{\substack{r=0\\ r\neq k}}^{L} x_r c_{k-r}) + n_k$. If c_0 is regarded as an arbitrary scale factor which is set to unity for convenience, then

$$v_k = x_k + \sum_{\substack{r=0\\ r\neq k}}^{L} x_r c_{k-r} + n_k \tag{7}$$

The term x_k represents the desired information symbol at the k^{th} sampling instant, the term $\sum_{\substack{r=-L\\ r\neq k}}^{L} xl_r c_{k-r}$ represents the intersymbol interference (ISI) and n_k is the noise variable at the k^{th} sampling instant.

The transmitter sends discrete time symbols at a rate $1/T$ symbols/second and the sampled output of the matched filter at the receiver is also a discrete-time signal with samples occurring at a rate $1/T$ per second. Therefore, the cascade of pulse shaper $g(t)$, the channel with impulse response $m(t)$, the matched filter $h^*(-t)$, and the sampler can be represented by an equivalent discrete time filter having tap gain coefficients $\{c_k\}$. It is assumed that $c_k = 0$ for all $|k| > M$, where M is a positive integer.

The noise sequence $\{n\}$ at the output of the matched filter is not white. It is possible to whiten the noise by filtering the sequence $\{v\}$.

Let $C(z)$ denote the z-transform of the impulse response of the filter that characterizes the channel. $C(z)$ can be expressed as

$$C(z) = \sum_{k=-M}^{M} c_k z^{-k} \tag{8}$$

where c_k is the filter tap gain coefficients given by equation (5). Since $c(t)$ is real and even $c_k = c_k^* = c_{-k} = c_{-k}^*$ (see equation (5)), $2M$ roots of $C(z)$ have symmetry, that is if q is a root $1/q^*$ is also a root. Hence, $C(z)$ can be factored as,

$$C(z) = F(z)F^*(z^{*\,-1}) \tag{9}$$

where $F(z)$ is a polynomial of degree M having the roots $q_1, \ldots, q_M$ and $F^*(z^{*\,-1})$ is a polynomial of degree M having the roots $1/q_1^*, \ldots, 1/q_M^*$. It can be shown [6] that if the sequence $\{v\}$ is passed through a digital filter, whose impulse response is $1/F^*(z^{*\,-1})$, the noise sequence at the output of the filter is white. Such a filter is called the noise whitening filter. Although there are $2M$ possible ways of making the decomposition in equation (9), in order to have a physically realizable, causal and stable noise whitening filter having all poles inside the unit circle, $F^*(z^{*\ -1})$ is chosen as the unique minimum phase filter with zeros $1/q_1^*, \ldots, 1/q_M^*$ inside the unit circle.

z-transform the sequence $\{y\}$ at the output of the whitening filter is given by

$$Y(z) = X(z)C(z)(1/F^*(z^{*\ -1})) = X(z)F(z)F^*(z^{*\ -1})(1/F^*(z^{*\ -1})) = X(z)F(z) \tag{10}$$

where $X(z)$ is the z-transform of the input sequence $\{x\}$ and $F(z)$ is the transfer function representing the cascade of the pulse shaper $g(t)$, the channel with impulse response $m(t)$, the matched filter $h^*(-t)$, the sampler and the discrete time noise whitening filter $F(z)$ is given by

$$F(z) = \sum_{k=0}^{M} f_k z^{-k} \tag{11}$$

where M is the degree of $F(z)$, and f_k is known as the filter tap gain coefficient.

Hence the cascade of pulse shaper $g(t)$, the channel with impulse response $m(t)$, the matched filter $h^*(-t)$, the sampler and the discrete time noise whitening filter can be represented by an equivalent discrete time filter, having tap gain coefficients $\{f_k\}$. The input of the filter is the transmitted sequence $\{x\}$. The sequence $\{y\}$ is the output of the filter. The additive noise sequence $\{w_n\}$ corrupting the output of the discrete time filter is a white Gaussian noise sequence.

The sequence $\{y\}$ at the output of the whitening filter is decoded by a sequence estimator. This sequence estimator finds the signal sequence $\{x'\}$ that most closely corresponds to the received sequence.

Let $\{x\} = (x_0, x_1, \ldots x_L)$ be the transmitted sequence of length $L+1$, $\{y\}$ be the sequence at the output of the whitening filter, and $\{x'\}$, $\{y'\}$ be the corresponding sequences estimated at the receiver.

Now, an equation which gives the squared Euclidean distance between the sequence $\{y\}$ and the estimated sequence $\{y'\}$ will be derived. The difference between $\{y\}$ and $\{y'\}$ is the sequence $\{e_y[k]\} = (y_0 - y'_0, \ldots, y_L - y'_L) = (e_{y0}, \ldots, e_{yL})$, the difference between $\{x_k\}$ and $\{x'_K\}$ is the sequence $\{e_x[k]\} = (x_0 - x'_0, \ldots, x_L - x'_L) = (e_{x0}, \ldots, e_{xL})$.

Let the sequence $\{t[n]\}$ denote the convolution of sequences $\{e_y[k]\}$ and $\{e * y[-k]\}$. t[n] is given by

$$t[n] = \sum_k e_{yk} e^*_{y(k-n)} \tag{12}$$

The squared Euclidean distance between sequences $\{y\}$ and $\{y'\}$, $d_{y,y'}$ is equal to $t[0]$, i.e.,

$$d_{y,y'} = \sum_{k=0}^{L} e_{yk} e^*_{yk} \tag{13}$$

Now, an equation which gives the sequence $\{t[n]\}$ in terms of sequences $\{e_x[k]\}, \{e_x^*[-k]\}$ and the channel filter gain coefficients c_k will be derived. The z transform of the sequence $\{t[n]\}, T(z)$, is the product of $E_y(z)$ and $E_y^*(z^{*-1})$. Since $E_y(z) = E_x(z)F(z)$ (see equation (10), $E_y^*(z^{*-1}) = F^*(z^{*-1})E_x^*(z^{*-1})$ and $C(z) = F(z)F^*(z^{*-1})$ (see equation (9)). $T(z)$ becomes

$$T(z) = E_x(z)F(z)F^*(z^{*-1})E_x^*(z^{*-1}) = E_x(z)C(z)E_x^*(z^{*-1})) \tag{14}$$

The inverse z-transform of $T(z)$, i.e., $\{t[n]\}$ is found by the convolution of inverse z-transforms of $E_x(z), C(z)$ and $E_x^*(z^{*-1})$. Hence $\{t[n]\}$ is given by

$$\begin{aligned}\{t[n]\} &= \{e_x[k]\} * \{c[k]\} * \{e_x^*[-k]\} \\ &= \{\sum_k e_{xk}c_{n-k}\} * \{e_x^*[-k]\} = \sum_p \sum_k e_{xk}c_{p-k}e^*_{x(-(n-p))}\end{aligned} \tag{15}$$

The term at $n=0$ gives the squared Euclidean distance between sequences $\{y\}$ and $\{y'\}$. As a result, the squared Euclidean distance between sequences $\{y\}$ and $\{y'\}$ is given by

$$d_{y,y'} = \sum_p \sum_k e_{xk}c_{p-k}e^*_{xp} \tag{16}$$

where e_{xk} is the difference between the k^{th} signals of transmitted sequence $\{x\}$ and the estimated sequence $\{x'\}$, and c_k is the tap gain coefficients of the equivalent discrete time filter given by equation (5). If the channel is ISI free, then $c_k = 0$, if $k \neq 0$, and the squared Euclidean distance between sequences $\{y\}$ and $\{y'\}$ is equal to the sum of the squared Euclidean distance between signals of sequences $\{x\}$ and $\{x'\}$, i.e.,

$$d_{y,y'} = \sum_{i=0}^{L} e_{xi}e^*_{xi} \tag{17}$$

where L is the length of the sequences $\{y\}$ and $\{y'\}$.

4. Percentage Contribution of Spectral Lines to the Error Event Probability

An error event is formed by a pair of distinct sequences that depart from the same state in the code trellis and merge in the same state some steps later. More precisely, an error event of length L is defined by two sequences $y = (\ldots, y_1, y_2, \ldots, y_L, ...)$ and $y' = (\ldots, y'_1, y'_2, \ldots, y'_L, \ldots)$, such that $y_i = y'_i$ for $i < 1, y_i \neq y'_i$ for $1 \leq i \leq L,\ y_i = y'_i$ for $i > L$.

The error event probability $P_e(e)$ is the probability of an error event starting at time t, given that the decoder has correctly identified the transmitter state at that time. Using the distance spectrum, an upper bound to P_e can be obtained as

$$P_e \leq \sum_{d=d_{\text{free}}}^{\infty} N_d Q\left(\sqrt{\frac{d}{2\eta_0}}\right) \tag{18}$$

where d represents the squared Euclidean distance between signal sequences, N_d is the average number (multiplicity) of codewords at distance d from a specific codeword where the average is taken over all codewords in the code, η_0 is the one-sided noise power spectral density, and $Q(.)$ is the Gaussian integral function $Q(\beta) \triangleq \left(\int_\beta^\infty e^{-x^2/2}dx\right)/(\sqrt{2\pi})$. It is assumed that the TCM signal x_r is sent through an AWGN channel with double-sided spectral noise density $\eta_0/2$. The noise variance in each dimension of signal space is $\sigma^2 = \eta_0/2$, and the signal to noise ratio (SNR) is defined as SNR $= \frac{E\{|x_r|^2\}}{2\sigma^2}$ where, $E\{|x_r|^2\}$ denotes the average signal energy. In order to compare different signal sets, is normalized and taken as unity.

The first 5 spectral lines of the Distance Spectrum (DS) of the 16-state 8-PSK, 16-state 16-QAM, 8-state 32 AMPM Ungerboeck type TCM codes are computed. Since all Ungerboeck type codes are quasi regular, the RCA can be implemented for all the above codes. The original RCA is used for the DS computation of 16-state 8-PSK TCM code which has no uncoded bit. For the codes which have uncoded bits, i.e., 16-state 16-QAM and 16-state 32-AMPM, the RCA modified by Rajab [3, 4] is used. The results of the computations are shown in Table 1. The parameters h_0, h_1, h_2, and (d_i, N_i) indicate the connection polynomials of the trellis encoder in octal form, the distance and the multiplicity of the i^{th} spectral line respectively.

Table 1. First Five Spectral Lines of the 16-State 8-PSK, 16-State 16-QAM and 8-State 32-AMPM

	16-State 8-PSK	16-State 16-QAM	8-State 32AMPM
h_0	23	23	11
h_1	04	04	02
h_2	16	16	04
(d_1, N_1)	(5.17, 2.25)	(2.4, 9.16)	(0.95, 6.89)
(d_2, N_2)	(5.76, 4.63)	(2.8, 27.21)	(1.14, 24.49)
(d_3, N_3)	(6, 1)	(3.2, 78.24)	(1.33, 85.09)
(d_4, N_4)	(6.34, 6.06)	(3.6, 314.69)	(1.52, 297.32)
(d_5, N_5)	(6.59, 4)	(4, 1000.88)	(1.71, 1026.47)

The percentage contribution of a spectral line (d_i, N_i) to the Error Event Probability (EEP) P_e of a TCM code is defined by

$$P_i/P_e \times 100 \tag{19}$$

where P_i is the term due to spectral line (d_i, N_i) in equation (18) and is given by

$$P_i = N_i Q(\sqrt{d_i/2\eta_0}) \tag{20}$$

Percentage contributions of the first five spectral lines of 16-state 8-PSK, 16-state 16- QAM, and 8-state 32-AMPM code are given in Figures 3, 4 and 5 respectively.

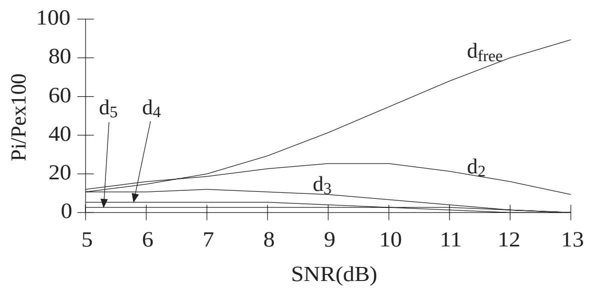

Figure 3. Relative Percentage Contribution of Various Lines to the Upper Bound on the EEP of the 16-State 8-PSK Code.

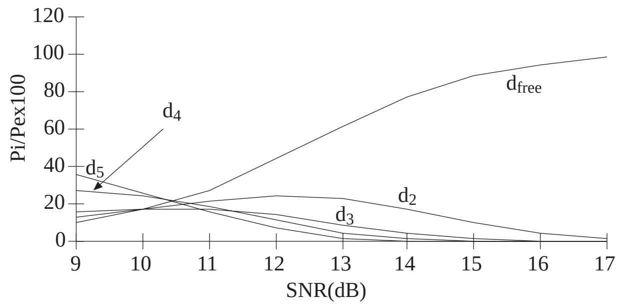

Figure 4. Relative Percentage Contribution of Various Lines to the Upper Bound on the EEP of the 16-State 16-QAM Code

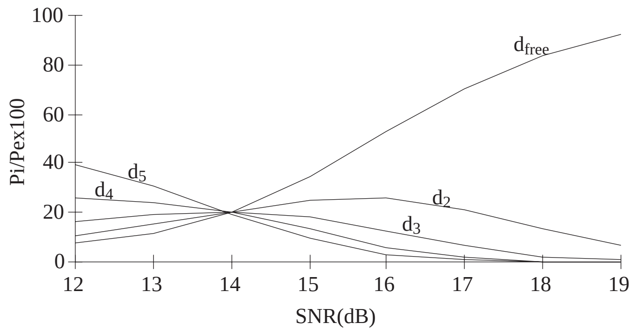

Figure 5. Relative Percentage Contribution of Various Lines to the Upper Bound on the EEP of the 8-State 32-AMPM Code.

The common property of Figures 3, 4 and 5 is that at high SNR's, the contribution due to the first spectral line d_{free} tends to 100%. So, at high SNR's, the performance of the codes can be accurately estimated by considering only d_{free}. However, at moderate and low SNR's, there is not a specific spectral line that dominates the others.

Figure 3 shows that for the 16-state 8-PSK, the contribution of the first three spectral lines are nearly the same, and around 10% for the SNR values between 5 and 6dB. As the SNR increases d_{free} dominates others. For the 16-state 16-QAM, Figure 4 shows that at low SNR's, the contribution of the fifth line is higher than the contribution of other lines and is 38% at 9dB, whereas the contribution of d_{free} is as low as 10% at the same SNR. It seems that there is an SNR value (10.5dB) below which the dominancy of the spectral line is proportional to its index (for i=1,...,5), and above which the roles are reversed, i.e., the smallest indexed line d_{free} (i=1) becomes the most dominant one. The relative contribution versus SNR curves depicted in Figure 5 for the 8-state 32-AMPM are similar to the curves obtained for the 16-state 16-QAM. The contribution of the fifth line is higher than other lines at SNR values lower than 14dB, where the d_{free} term has the lowest contribution. However, for SNR values higher than 14dB, the contribution of d_{free} is higher than the contribution of other spectral lines and reaches 90% at 19 dB.

5. Effect of Intersymbol Interference on the Distance Spectrum of 16-State 8PSK

In this section, the effect of intersymbol interference on the distance spectrum (DS) of the 16-state 8-PSK code is evaluated.

It is shown in Section 3 that a transmission channel can be modeled as a discrete time filter of duration 2MT, and filter tap gain coefficients c_k. These coefficients indicate the amount of intersymbol interference on the channel. The coefficient c_k determines the amount of interference on the transmitted signal at $t = T$ due to the symbol transmitted at $t = T - kT$.

The effect of ISI on the distance spectrum (DS) of 16-state 8-PSK code is investigated for four channels with different eye openings. To have a reference for comparison, the DS of the 16-state 8-PSK code on the ISI free channel, computed with error events of length up to 7, is also shown in Figure 6. In finding the distance spectra, only the error events with lengths lower than or equal to 7 are considered. This limitation does not affect the distances d_i in the spectrum, although corresponding multiplicities N_i are found slightly less than their actual values.

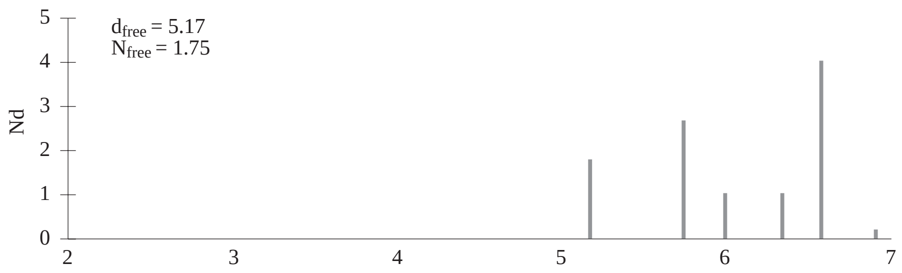

Figure 6. Distance Spectrum (formed by error events whose length is less than or equal to 7) of the 16-State 8-PSK Code on ISI Free Channel.

In order to model the ISI channel, we consider a simple channel impulse response $h[n]$, with $h[0] = 1$ and $h[1] = a$, whose eye opening is given as $(1-a)$. The corresponding combined channel $c[n]$ has coefficients $\{a,\ 1+a^2,\ a\}$, which can be normalized as $\{a/1+a^2,\ 1,\ a/1+a^2\}$. Four channels with different eye openings are obtained by assigning four different values to the parameter a.

Filter tap gains for the first channel are $c_0 = 1, c_1 = c_{-1} = 0.1$. Eye opening is reduced to 90% by ISI. The distance spectrum up to a distance of 7 (formed by error events of length less than or equal to 7) is plotted in Figure 7. The number of spectral lines plotted is 86. The free distance d_{free}, which is equal to 5.17 for the ISI free case, decreases to 4.76 for this channel; many new spectral lines appear and the distance spectrum becomes much more crowded than the ISI free case given in Figure 6. The ISI scatters the points in the 8-PSK signal constellation. The increase in the number of signal points causes the spread of the distance spectrum.

Filter tap gain coefficients for the second channel are $c_0 = 1, c_1 = c_{-1} = 0.2$. The opening of the eye pattern is reduced to 80% by ISI. The DS up to distance 7 (formed by error events of length less than or equal to 7) is plotted in Figure 8. The number of spectral lines is 96 and more than that for the previous channel, and the free distance d_{free} decreases to 4.36.

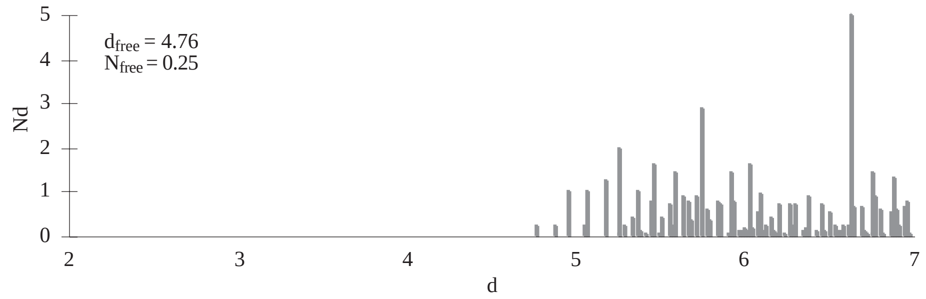

Figure 7. Distance Spectrum (formed by error events whose length is less than or equal to 7) of the 16-State 8-PSK Code on ISI Channel with 90% Eye Opening.

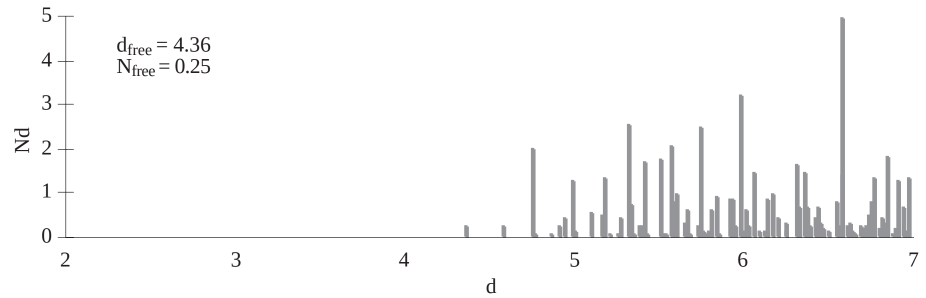

Figure 8. Distance Spectrum (formed by error events whose length is less than or equal to 7) of the 16-State 8-PSK Code on ISI Channel with 80% Eye Opening.

The amount of interference on the third channel is higher, the filter tap gain coefficients are chosen as $c_0 = 1, c_1 = c_{-1} = 0.3$. The opening of the eye pattern is reduced to 67% by the ISI. The DS up to distance 7 (formed by error events of length less than or equal to 7) is plotted in Figure 9. The number of spectral lines plotted is 113. The free distance d_{free} decreases to 3.97 for this channel. As expected, the decrease in d_{free} and the spread of the spectral lines are more serious than for the first two channels.

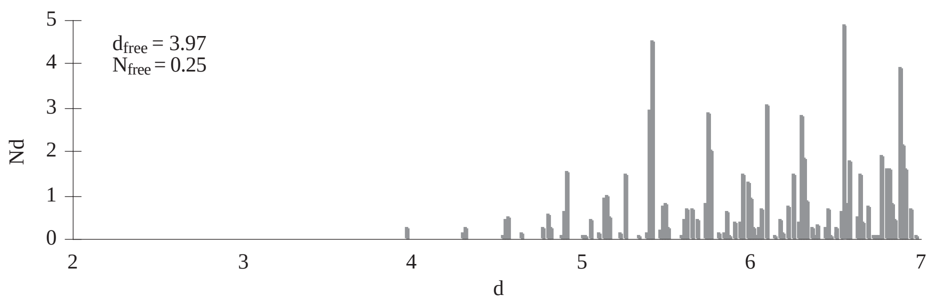

Figure 9. Distance Spectrum (formed by error events whose length is less than or equal to 7) of the 16-State 8-PSK Code on ISI Channel with 67% Eye Opening.

The last and the worst channel has filter tap gain coefficients $c_0 = 1, c_1 = c_{-1} = 0.49$. This channel model was used by Schlegel [5] where he shows how the DS can be computed on channels with intersymbol interference. The worst values of these parameters c_k for an interference over two symbol periods are

$c_0 = 1, c_1 = c_{-1} = 0.5$ [5]. The channel has severe ISI, and the corresponding eye pattern is nearly closed. The DS up to distance 7 (formed by error events of length less than or equal to 7) is plotted in Figure 10.

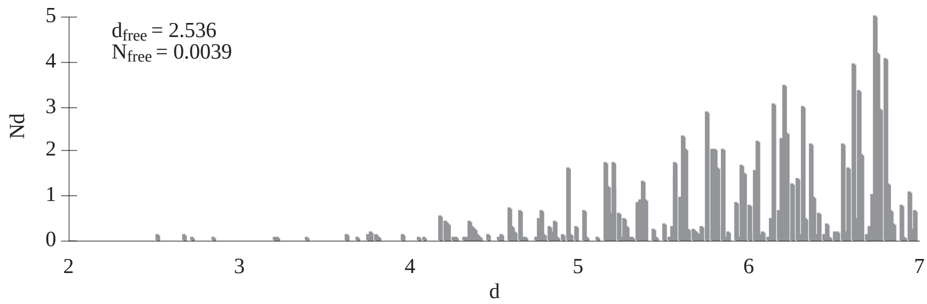

Figure 10. Distance Spectrum (formed by error events whose length is less than or equal to 7) of the 16-State 8-PSK TCM Code on ISI Channel with Nearly Closed Eye Pattern.

The number of spectral lines plotted in Figure 10 is 194. Intersymbol interference decreases the free distance of the code from 5.17 to 2.536. However, the multiplicity corresponding to d_{free} becomes as small as 0.00391. Spectral lines are spread into a nearly continuous spectrum with very small multiplicities. We observe that the original multiplicity (1.75) of the first spectral line corresponding to the ISI free case is almost equal to the sum of first 30 multiplicities of the ISI channel.

6. Conclusions

Percentage contributions to the error event probability (EEP) of the first five spectral lines of the 16-state 8-PSK, 16-state 16-QAM and 8-state 32-AMPM are computed and sketched versus channel SNR. It is observed how the performances of all TCM codes are dominated by the term d_{free} at high signal to noise ratios, while the contribution due to the first spectral line tends to 100% of the total error event probability. On the other hand, at moderate and low values of the signal to noise ratio, contribution of other terms may be even more important than that of the first term. Therefore, it is necessary to take the effects of higher spectral lines into account in order to estimate the performance of a TCM system accurately.

The effects of intersymbol interference on the distance spectrum of 16-state 8-PSK TCM code are investigated for four channels with different values of eye opening. It is seen that on channels with intersymbol interference, spectral lines are spread into a nearly continuous spectrum and the minimum Euclidean distance between codewords decreases severely. The effect of the intersymbol interference on the distance spectrum is a result of scattering in signal constellation points. Hence, the corresponding increase in the number of signal points causes the spread of the distance spectrum, which can be quite severe for channels with poor eye patterns.

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