The new empirical magnitude conversion relations using an improved earthquake catalogue for Turkey and its near vicinity (1900–2012)

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Abstract: Empirical magnitude conversion relationships are one of the important parameters for not only seismological studies but also seismic hazard analysis and development of the attenuation relationships. Particularly, for seismic hazard analysis, conversion of various types of magnitudes to moment magnitude, which is the most reliable and common magnitude scale, is a key requirement. Within this scope, different magnitude conversion equations have been derived by various researchers in the literature. In this study, new empirical magnitude conversion formulas for conversion from m_b, M_L, M_d, and M_S to M_w were derived by using a recently established earthquake catalogue. The most important feature of the new relationships is the use of the maximum data with respect to the literature. It is a well-known fact that having a greater number of data increases the sensitivity of the equations derived. Both orthogonal regression (OR) and ordinary least squares (OLS) were used to derive conversion equations, and the results obtained from these two methods were compared. In the derivation, 489 events with magnitudes in M_w scale taken from the Harvard GCMT Catalogue were used. Residual graphs created for both methods showed that the OR method gives better results than OLS for conversion from M_d to M_w. On the other hand, the OLS method showed preferable performance for conversions from m_b, M_L, and M_d to M_w. The equations proposed in this study were also compared with other empirical relations in the literature.

Key words: Moment magnitude, earthquake catalogue, orthogonal regression, ordinary least squares, empirical relations, magnitude scales

1. Introduction
One of the important parameters of the earthquake phenomenon is earthquake magnitude. In seismology, the magnitude term expresses the energy released during the rupture process. Occurrence of an earthquake consists of a wide range of physical parameters, such as rupture length, rupture area, surface displacement, particle velocity, ground acceleration, and released seismic energy. Although the size of an earthquake can be determined with a simple instrumental measurement in a short time, it is not possible to rapidly estimate these parameters. Earthquake magnitudes, which are simple empirical parameters, may not be directly relevant to the physical parameters of the earthquake source. On the other hand, rapid computations used in engineering studies are important for earthquake catalogues (Kanamori, 1983; Bormann, 2002). The most common empirical parameters used to express earthquake magnitude are M_L (local magnitude/Richter magnitude), M_d (duration/coda magnitude), M_s (surface wave magnitude), m_b (body wave magnitude, where m_b refers to the short period and m_l refers to the long period), and M_w (moment magnitude). M_w is particularly preferred for major earthquakes in recent years (McCalpin, 2012). The first magnitude type, M_L (local magnitude), was identified for local events in South California by Woods Anderson in torsion seismographs (Richter, 1935). Later on, M_d and m_b magnitudes were generated (Gutenberg, 1945a, 1945b, 1945c) and harmonized with the Richter magnitude scale. M_w (seismic moment/moment magnitude), which is widely used in recent years, is not only an instrumental parameter but is also associated with certain other physical parameters (such as slip rate) related to the earthquake source fault.

Different magnitude scales are computed by different formulas and they have varied saturation conditions. Selection of the magnitude type also depends on the earthquake size. For instance, while M_d (duration/coda) magnitude has been generally utilized for small and local events (for M ≤ 3.0), m_b and M_L have been used for major earthquakes (especially in teleseismic events) in any depth. M_w is recognized as the most credible parameter in seismology, and it is not saturated. In addition, wave
frequency range used for calculation of magnitude differs with magnitude scales. These frequencies are determined as $m_b: \sim 1$ s, $m_d: \sim 0.5–12$ s, $M_L: \sim 0.1–3$ s, $M_S: \sim 20$ s, and $M_w: \sim 10 \rightarrow \infty$ s in various studies (Kanamori, 1983). Many scientists have investigated the relationship between the above-mentioned empirical parameters using different methods, and several magnitude conversion relations have been derived to date. These empirical conversion relations provide homogeneity of the earthquake catalogue in terms of unified scale. For instance, different conversion relationships have been developed on a regional scale with different methods by Gutenberg and Richter (1956a, 1956b), Kanamori (1983), Ambraseys (1990), Papescu et al. (2003), Ulusay et al. (2004), Deniz (2006), Scordilis (2006), Kalafat et al. (2007), Grünthal (2009), Akkar et al. (2010), Das (2011), Çıvgın (2015), and Bayrak et al. (2005, 2009). On the other hand, various regression analyses have been performed for local scale by using different methods and databases. For instance, Köseoğlu et al. (2014) performed determination of spectral moment magnitude for the Marmara Region between 2006 and 2009 with magnitude $2.5 \leq M \leq 5.0$ by using differences between observed and synthetic source spectra calculated from S waves. As seen in the literature, the most common methods used to derive these relationships are ordinary least squares (OLS), orthogonal regression (OR), and maximum likelihood. Although each method has advantages and disadvantages as compared to the others, comparison of the residual graphs shows that different methods provide more reliable results for different magnitude scales.

In this paper, we derive a new empirical magnitude conversion relationship using an improved earthquake catalogue for Turkey and its near vicinity (Kadirioğlu et al., 2014). The improved earthquake catalogue covers the area bounded by $32^\circ$N and $45^\circ$N and by $23^\circ$E and $48^\circ$E, and it includes 12,674 events that occurred from 1900 to 2012. This catalogue comprises events reported in different magnitude scales (i.e. $M_s$, $m_b$, $M_d$, $M_L$, and $M_w$) from various catalogues. The magnitude range of the proposed catalogue varies between 4.0 and 7.9. For the regression analysis, an integrated database including approximately 37,000 earthquake parameters from Kadirioğlu et al. (2014) was prepared. From this integrated database, 489 events with magnitudes given in $M_w$ scale were selected. Among them, magnitudes in $m_b$, $M_d$, $M_L$, and $M_w$ scales were also determined for 488, 404, 462, and 208 events, respectively. Both OR and OLS methods were applied to derive conversion equations. In such a study, there are some uncertainties concerning the integrated catalogue. The most significant concern is the diversity in magnitude types and values. This may originate due to the operator calculating the earthquake parameters, the choice of the crustal model, or the use of various magnitude computing equations. For instance, in this study, for each event with $M_w$ magnitude, all other magnitude types (i.e. $M_s$, $m_b$, $M_d$, and $M_L$) are not provided in the integrated database. This situation can be identified as the epistemic uncertainty of the catalogue.

In this study, a new empirical relationship was developed and compared with the other empirical relations in the literature. These relationships are used in the “Updating of Turkey Seismic Hazard Map Project” supported by the National Earthquake Research Program of the Disaster and Emergency Management Authority (Turkish acronym: AFAD).

2. Dataset
In this study, the catalogue and integrated database of Kadirioğlu et al. (2014) that enable the creation of this catalogue were utilized. The catalogue contains 12,674 events with magnitudes $M \geq 4.0$ that occurred in Turkey and surrounding regions between 1900 and 2012 (Figure 1). Distribution of these earthquakes with respect to different magnitude types is given in Table 1. When selecting the earthquakes for the catalogue, the catalogues of ISC, EHB, EMSC, Harvard GCMT (Ekström et al., 2012), Alsan et al. (1975), Ayhan et al. (1981), Ambraseys and Finkel (1987), Ambraseys and Jackson (1998) Gutenberg and Richter (1954), Kalafat et al. (2011) and the AFAD Earthquake Department were primarily assessed with respect to the specific criteria. It should be noted that magnitudes in this catalogue are observed values, and any magnitude derived from empirical conversion equations is not taken into consideration in the catalogue.

The most important part of this and similar studies is the homogeneous catalogue that is used as a database for conversion. In this context, the integrated database used in this study was made homogeneous for the regression analysis with the following stages. Table 2 refers to an example of the integrated database. In this study, one of the major hurdles we faced was the regression analysis, such that different magnitudes were assigned by different agencies for the same event. The earthquake that occurred on 30 July 2009 at 0737 hours is a good example for this situation (Table 2). The magnitude of this earthquake is given as $M_s = 4.8$ and $m_b = 4.7$ by EMSC, $M_w = 5.0$ by HRVD, and $M_s = 4.8$ in the DDA and the ISC catalogues. In addition, $m_b = 4.9$ reported by the DJA agency was used in the ISC catalogue. The other difficulty concerning the integrated database is the significant difference between magnitudes for the same earthquake. Table 3 shows the parameters of the earthquake that occurred on 7 July 2009 at 0102 hours. For instance, $M_s$ and $M_w$ values provided by the NSSC agency are significantly lower than the values reported for other agencies. The integrated database was examined in order to eliminate these types of problems, and it was sorted out with regard to one type of magnitude
(M<sub>s</sub>, m<sub>b</sub>, M<sub>d</sub>, M<sub>L</sub>, and M<sub>M</sub>) for each event and made functional for this study. Thus, a homogeneous catalogue was created for the regression analysis.

During this process, the following steps were taken:
- If the same earthquake information was obtained from both the EMSC and ISC catalogues, the EMSC catalogue was taken into account and the corresponding information was deleted from the ISC catalogue.
- Repeated information on the ISC list was deleted.
- Contrary data (too small or greater values than the overall average) in the integrated database (like Table 3) were determined as outliers with the “expert opinion” method (Sims et al., 2008).
- Since the catalogue of Kalafat et al. (2011) includes magnitudes derived with various magnitude conversion relationships, it was included in the evaluation after 2011.
- Before taking the average of the magnitude values given for the same earthquake by different agencies in terms of same magnitude type (i.e. M<sub>s</sub>, m<sub>b</sub>, M<sub>d</sub>, and M<sub>L</sub>), upper and lower limits were specified with the method of “interquartile ranges and outliers”.
- The outliers method was not applied for earthquakes with less than 3 data and the average value was directly calculated.
- All steps in this process were separately performed for each magnitude scale (M<sub>s</sub>, m<sub>b</sub>, M<sub>d</sub>, M<sub>L</sub>). After the above-mentioned adjustments, we noticed that M<sub>s</sub>, m<sub>b</sub>, M<sub>d</sub>, and M<sub>L</sub> magnitudes were not complete for each M<sub>M</sub> value (Table 4). For regression, only one reference (Harvard GCMT Catalogue) is used for M<sub>M</sub>. Therefore, as we mentioned in Section 1, this situation can be explained as the epistemic uncertainty of the catalogue.

Table 1. Number of earthquakes in different magnitude types in the catalogue of Kadirioğlu et al. (2014).

<table>
<thead>
<tr>
<th>Magnitude type</th>
<th>Number of earthquakes</th>
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<td>M&lt;sub&gt;w&lt;/sub&gt;</td>
<td>489</td>
</tr>
<tr>
<td>M&lt;sub&gt;s&lt;/sub&gt;</td>
<td>2365</td>
</tr>
<tr>
<td>m&lt;sub&gt;b&lt;/sub&gt;</td>
<td>8390</td>
</tr>
<tr>
<td>M&lt;sub&gt;d&lt;/sub&gt;</td>
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</tr>
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<td>M&lt;sub&gt;L&lt;/sub&gt;</td>
<td>1218</td>
</tr>
<tr>
<td>Total</td>
<td>12,674</td>
</tr>
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</table>
As a result, for the regression analysis, 462 Mw–MS pairs, 488 Mw–mb pairs, 404 Mw–ML pairs, and 208 Mw–Md pairs were determined.

### 3. Methodology

In this study, magnitude conversion relationships were derived based on both OLS and OR methods via MATLAB software (Gilat, 2004). Standard error and regression residual parameters were calculated with the bootstrap method (Chernick, 1999) by means of both Excel and SPSS software (Argyrous, 2011). Residual graphs created for each magnitude type were assessed separately. As a result of the evaluation, negligible bias was observed in the formula derived by OR. This method is found more proper for the regression analysis of Mw to Mw conversion equation according to residuals. Although the OR method was also

**Table 2.** An example from the integrated database (30 July 2009 earthquake) (abbreviations: Ref., reference; Mo., month; Yr., year; Hr., hour; Mn., minute; Sec., second; Lat., latitude; Lon., longitude; D., depth).

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Day</th>
<th>Mo.</th>
<th>Yr.</th>
<th>Hr.</th>
<th>Mn.</th>
<th>Sec.</th>
<th>Lat. N</th>
<th>Lon. E</th>
<th>D. (km)</th>
<th>M_s</th>
<th>m_b</th>
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<th>M_l</th>
<th>M_w</th>
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*Agency magnitude information taken from the ISC (International Seismological Centre). Reference codes: EMSC, European-Mediterranean Seismological Centre, France; HRVD, Harvard Global Centroid Moment Tensor Catalogue, USA; DDA: AFAD, Disaster and Emergency Management Authority, Earthquake Department, Turkey; ISC - ISCJB: International Seismological Centre, United Kingdom; NEIC: National Earthquake Information Centre, USA; DIA: Badan Meteorologi, Klimatologi dan Geofisika, Indonesia; MOS: Geophysical Survey of Russian Academy of Sciences, Russia; KLT: Kalafat et al. (2011).

**Table 3.** An example from the integrated database (7 July 2009 earthquake).

<table>
<thead>
<tr>
<th>Ref.</th>
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<th>Hr.</th>
<th>Mn.</th>
<th>Sec.</th>
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<th>Lon. E</th>
<th>D. (km)</th>
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<th>m_b</th>
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*Agency magnitude information taken from the ISC catalogue. Reference code: NSSC, National Syrian Seismological Centre, Syria.
Table 4. Other scale magnitudes corresponding to observed $M_w$.

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<th>Hr.</th>
<th>Mn.</th>
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<th>Lat. N</th>
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</table>

Figure 2. Comparison of orthogonal regression (OR) and ordinary least squares (OLS) correlation plots for a) $M_s$ vs. $M_w$, b) $M_d$ vs. $M_w$, c) $m_b$ vs. $M_w$, and d) $M_L$ vs. $M_w$. Bolded formulas indicate proposed equations in this study.
used for derivation of the other magnitude conversion equations ($m_b$, $M_L$, and $M_d$ to $M_w$), the OLS method was preferred due to the significant bias.

According to the comparison of OR and OLS methods, the correlation plots demonstrate more or less the same results for the $M_w$ and $M_s$ relationship. On the other hand, appreciable dissimilarity could be observed for other relationships ($m_b$ vs. $M_w$, $M_d$ vs. $M_w$, $M_L$ vs. $M_w$) (Figures 2a–2d).

### 3.1. Orthogonal regression

OR is a standard linear regression method that has been used to correct the effects of measurement errors in estimation (Carroll and Ruppert, 1996). OR takes the error rates of dependent and independent variables into account. For this reason, it is considered to provide more reliable results. However, to obtain the most accurate results the eta ($\eta$) parameter, which indicates the error ratio between the dependent and independent variables, must be determined accurately. Especially in seismology, it is not possible to determine the error ratio between the

![Figure 3](image1.png)

Figure 3. Residual graphs of magnitudes that were calculated by OR: (a) $m_b$ to $M_w$, (b) $M_L$ to $M_w$, (c) $M_d$ to $M_w$. The graphs show significant bias in the linear trend. At this stage, it is clear that the OR has not performed well for $m_b$, $M_L$, and $M_d$ to $M_w$ conversion. Abbreviations: $M_w (obs)$, $M_w$ observed; $M_w (est)$, $M_w$ estimated.

![Figure 4](image2.png)

Figure 4. Plots of OR relations for $M_s$ to $M_w$ (OR).

![Figure 5](image3.png)

Figure 5. According to OR method, residual graph for all data.
magnitude types in the earthquake catalogues used for regression analysis because the earthquake magnitudes determined by different agencies have been affected by uncertainties from various seismic instruments, crustal methods, and several conversion relations. In addition, both dependent and independent variables contain a number of internal errors. For these reasons, the error ratio has not been calculated separately for each magnitude type, and in this study eta ($\eta$) was accepted as 1 for the OR method. In other words, it was considered that the error margin was equal in both variables. The formulas used for calculations are shown below. They were derived with the OR method and applied by MATLAB.

$$s_{xx} = \sum_{i=1}^{n} (X_i - X_{\text{mean}})^2$$

$$s_{yy} = \sum_{i=1}^{n} (Y_i - Y_{\text{mean}})^2$$

$$b = \frac{(syy - \eta s_{xx}) + \sqrt{(syy - \eta s_{xx})^2 + 4\eta s_{xy}^2}}{2s_{xy}}$$

$$a = Y_{\text{mean}} - b X_{\text{mean}}$$

$X$: Magnitudes that will be converted ($m_b, M_L, M_d, M_M$),
$Y$: Observed $M_w$,
$X_{\text{mean}}$: The average of the magnitudes that will be converted,
$Y_{\text{mean}}$: The average of the observed $M_w$.

In the residual graphs, corresponding to linear $m_b$, $M_L$, and $M_d$ to $M_w$ conversion relations obtained by OR, a significant slope was observed. This indicates a bias against conservative or nonconservative values for the above-mentioned magnitude calculations (Figures 3a–3c).

On the other hand, the OR conversion method was applied for $M_M$ magnitude. The formulas, standard errors, and residual scatters obtained from OR for $M_M$ to $M_w$ conversion are given below. When Figure 4 is examined, it is observed that the general trend deviates at $M_M = 5.4$. Therefore, bilinear relations were implemented for $M_M$ to $M_w$ conversion. In the residual graphs, there is almost no bias both for all data and data with $M_M \geq 4.0$ (Figures 5 and 6).

$$M_w = 0.5716 (\pm 0.024927) M_M + 2.4980 (\pm 0.117197)$$

$$3.4 \leq M_M \leq 5.4 \ (2a)$$

$$M_w = 0.8126 (\pm 0.034602) M_M + 1.1723 (\pm 0.208173)$$

$$M_M \geq 5.5 \ (2b)$$

The empirical conversion relationship for $M_M$ to $M_w$ derived with OR was compared with previously developed relations, and fairly compatible results were obtained (Figure 7).

### 3.2. Ordinary least squares

Although OLS is a frequently used simple method in empirical conversions, it is a method basically used to create a linear function between two dependent and independent variables. This method has some limitations, both mathematically and statistically. The most important limitation is that the dependent variable ($Y$) must be known with much more accuracy than the independent variable ($x$). Both dependent and independent variables are affected by uncertainty in the $Y = ax + b$ equation (Castellaro et al., 2006). In this study, while $M_M$, $m_b$, $M_d$, and $M_L$ magnitudes express independent variables ($x$), $M_w$ magnitude represents the dependent variable ($Y$). According to regression analysis, the results obtained from OLS are much better than those of OR for $m_b$, $M_d$, and $M_L$ to $M_w$ conversion. In the residual graphs, the trend line between the conservative and nonconservative values did not have a significant slope (Figure 8a–8c).

New empirical equations obtained from OLS and their standard errors are presented below.

$$M_w = 1.0319 (\pm 0.025) m_b + 0.0223 (\pm 0.130)$$

$$3.9 \leq m_b \leq 6.8 \ (3a)$$

$$M_w = 0.7947 (\pm 0.033) M_d + 1.3420 (\pm 0.163)$$

$$3.5 \leq M_d \leq 7.4 \ (3b)$$

$$M_w = 0.8095 (\pm 0.031) M_L + 1.3003 (\pm 0.154)$$

$$3.3 \leq M_L \leq 6.6 \ (3c)$$

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**Figure 6.** According to OR method, residual graph for $M_M \geq 4.0$.

**Figure 7.** Comparison of empirical equations with literature for magnitude conversion ($M_M$ to $M_w$).
Similarly, new empirical relationships were compared with other relations in the literature. According to this comparison, it was observed that the new relations between \( m_b \) and \( M_w \) obtained from OLS were similar to the results of Kalafat et al. (2011). However, the relations proposed by Ulusay et al. (2004) indicated appreciable differences. As seen in Figure 9a, Ulusay et al. (2004) overestimated \( M_w \) values for \( m_b \geq 5.0 \). On the other hand, although this study and that of Ulusay et al. (2004) provide similarly higher \( M_w \) estimations for \( M_L \) to \( M_w \) conversion, there were highly different results when compared with those of Grünthal et al. (2009) and Zaré and Bard (2002). They underestimate \( M_w \) values when compared to our results.

This study almost intersects with the results of Akkar et al. (2010) for \( M_d \geq 6.0 \) (Figure 9b). The same comparison was performed for \( M_d \) to \( M_w \) conversion relations and new empirical relations demonstrate results that are reasonably compatible with those of Akkar et al. (2010) and Ulusay et al. (2004). Moreover, this study overestimates \( M_w \) values for \( M_d \) between 3.5 and 6.0 compared to the literature (Figure 9c).

4. Discussion

New empirical equations are one of the important outputs of the Updating Seismic Hazard Map of Turkey project supported by the National Earthquake Research Program.
of AFAD. In this study, we aimed to derive conversion relations from the selected magnitude types (such as $M_S$, $m_b$, $M_L$, and $M_d$) to moment magnitude ($M_W$). The homogeneous catalogue used in this study includes the earthquakes with magnitudes greater than 4.0 that occurred in the region bounded by 32.00°N and 45.00°N and by 23.00°E and 48.00°E. Within the scope of this, 489 earthquakes with $M_w$ values obtained from the Harvard GCMT Catalogue were taken into consideration. Among these earthquakes, 462 events (between 1900 and 1982) had $M_S$ values, 488 events (between 1964 and 2012) had $m_b$ values, 404 events (between 1972 and 2012) had $M_L$ values, and 208 (between 1988 and 2009) had $M_d$ values.

For the regression analysis, both OR and OLS methods were used in this study. As we mentioned above, $\eta$ was accepted as 1 for the OR method, as the error ratio could not be calculated separately for each magnitude type in the catalogue (Eq. (1)). In the residual scatters for $M_S$ to $M_W$ conversions obtained from OR, almost no bias both for the complete data and for $M_S \geq 4.0$ was observed. Therefore, OR was determined as the suitable method for $M_S$ to $M_W$ conversion (Eqs. (2a) and (2b)). On the other hand, stronger physical correlation was

Figure 9. Comparison of empirical equations with literature for magnitude conversion: (a) $m_b$ to $M_w$, (b) $M_L$ to $M_w$, (c) $M_d$ to $M_w$.

Figure 10. Comparison between ISC $M_S$ and $M_W$ from HRVD GCMT.
observed between ISC $M_s$ and $M_w$ from HRVD GCMT. When it is considered that both magnitudes are measured in the long period, this is the expected result (Granville et al., 2005). Particularly, $M_s$ scales had good fit with $M_w \geq 5.8$ (Figure 10). As opposed to this, residual graphs for $m_b$, $M_L$, and $M_d$ to $M_w$ conversions performed by OR indicated a significant slope in linear trend between the conservative and nonconservative values. For this reason, the OR method was not approved for the conversion of the mentioned magnitudes to $M_w$. Therefore, the OLS method was applied for $m_b$, $M_L$, and $M_d$ to $M_w$ conversions, and in the trend line of residual graphs there was no significant slope (Eqs. (3a), (3b), and (3c)).

New empirical relationships that were derived by both OR and OLS gave compatible results with data set used. The relations used in this study were compared with the literature and generally consistent results were obtained for both $M_s$ to $M_w$ and $m_b$, $M_L$, and $M_d$ to $M_w$ conversions.

On the other hand, this study and that of Ulusay et al. (2004) indicate similarly higher estimations of $M_w$ values for $M_s$ than other studies and overestimate $M_w$ values for $M_d$ between 3.5 and 6.0.

Acknowledgments
This research is the mid-product of the “Updating of Seismic Hazard Map of Turkey” project supported by the National Earthquake Research Program and conducted by the Kandilli Observatory and Earthquake Research Institution (KRDEA), General Directorate of Mineral Research and Exploration (MTA), Prime Ministry Disaster and Emergency Management Authority (AFAD), Çukurova University, and Sakarya University. The authors would like to thank Prof Dr Semih Yücemen, Prof Dr Ayşen Akkaya, Research Assistant Sibel Balci, Prof Dr Sinan Akkar, and Assoc Prof Dr Mehmet Yılmaz for their time and valuable advice.

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